

King Saud university
Quiz. No.1

First semester, 1432H
Math 244

Maryam Al-Towaileb

Question No.1

Determine whether the following statements are true or false, and justify your answer:

- (a) The equation $\sqrt{3}x_1 + \frac{4}{5}x_2 - x_3 = 1$ is a linear homogeneous equation.

False: this equation is linear but not homogeneous. $[\sqrt{3}x_1 + \frac{4}{5}x_2 - x_3 = 0]$ is homogeneous.

- (b) The matrix $A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ is an elementary matrix.

True: add 2 times the third row of I_3 to the first row. $[I_3 \rightarrow_{2R_3} A]$.

- (c) If A is a matrix of size 3×3 and $\det(A) = 4$, then $\det(5A) = 20$.

False: $\det(5A) = 5^3 \det(A) = (125)(4) = 500$.

- (d) The determinant of the matrix $\begin{pmatrix} 2 & 3 & 1 & -5 \\ 1 & -4 & 5 & 1 \\ 4 & 6 & 2 & -10 \\ \sqrt{2} & 5 & 0 & 0 \end{pmatrix}$ equals to 0.

True: $R_3 = 2R_1$ (by theorem).

- (e) The homogeneous system of linear equations

$$5x_1 + x_2 - x_3 = 0, \quad x_1 - 2x_2 + 3x_3 = 0$$

has no solution.

False: this system has infinitely solutions; since this system of linear equations with more unknowns than equations.

(f) If $A^{-5} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -\frac{1}{32} & 0 \\ 0 & 0 & 243 \end{pmatrix}$, then $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 3 \end{pmatrix}$.

False: $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -\frac{1}{3} \end{pmatrix}$

(g) For any two invertible matrices A and B , $(AB^{-1})^{-1}$ is equal to BA^{-1} .

True: since $(BA^{-1})(AB^{-1}) = B(A^{-1}A)B^{-1} = BIB^{-1} = BB^{-1} = I$.

(h) If $A = \begin{pmatrix} 1 & 4 \\ 2 & 6 \end{pmatrix}$, then A^{-2} is equal to $\begin{pmatrix} 6 & -4 \\ -2 & 1 \end{pmatrix}$.

False: $A^{-1} = \frac{1}{-2} \begin{pmatrix} 6 & -4 \\ -2 & 1 \end{pmatrix} = \begin{pmatrix} -3 & 2 \\ 1 & -\frac{1}{2} \end{pmatrix}$, and then

$$A^{-2} = (A^{-1})^2 = \begin{pmatrix} -3 & 2 \\ 1 & -\frac{1}{2} \end{pmatrix} \times \begin{pmatrix} -3 & 2 \\ 1 & -\frac{1}{2} \end{pmatrix} = \begin{pmatrix} 11 & -7 \\ \frac{-7}{2} & \frac{9}{4} \end{pmatrix}.$$

.....