Huda Abdu Qaed
432203586
phys 505
4th homework

problem 1

$$R(k) = \frac{1}{4k^2} \left| \int_{-\infty}^{\infty} e^{2ikx} u(x) dx \right|^2$$

$$R(k) = \frac{4m^2}{4k^2\hbar^4} \left| \int_{-\infty}^{\infty} e^{2ikx} v(x)\, dx \right|^2$$

$$R(k) = \frac{4m^2 V_0^2}{4k^2\hbar^4} \left| \int_0^L e^{2ikx} dx \right|^2$$

$$R(k) = \frac{4m^2 V_0^2}{4k^2\hbar^4} \left| \frac{e^{2ikx}}{2ik} \Big|_0^L \right|^2$$

$$R(k) = \frac{4m^2 V_0^2}{4k^2\hbar^4} \left| \left[\frac{e^{2ikL}}{2ik} - \frac{1}{2ik} \right] \right|^2$$

$$R(k) = \frac{4m^2 V_0^2}{4k^2\hbar^4} \left[\frac{1}{2ik} (e^{2ikL} - 1) \cdot \frac{1}{-2ik} (e^{-2ikL} - 1) \right]$$

$$R(k) = \frac{4m^2 V_0^2}{4k^2\hbar^4} \left[\frac{1}{4k^2} (1 - e^{-2ikL} - e^{2ikL} + 1) \right]$$

$$R(k) = \frac{4m^2 V_0^2}{4k^2\hbar^4} \left[\frac{1}{4k^2} (2 - e^{-2ikL} - e^{2ikL}) \right]$$

$$R(k) = \frac{4m^2 V_0^2}{4k^2\hbar^4} \left[\frac{1}{4k^2} (2 - \cos 2kL + \cancel{i \sin 2kL} - \cos 2kL - \cancel{i \sin 2kL}) \right]$$

$$R(k) = \frac{4m^2 V_0^2}{4k^2\hbar^4} \left[\frac{1}{2k^2} (1 - \cos 2kL) \right.$$

$$R(k) = \frac{4m^2 V_0^2}{4k^2\hbar^4} \left[\frac{1}{k^2} \sin^2(kL) \right]$$

$$U_0 = \frac{2mV_0}{\hbar^2}$$

$$U_0^2 = \frac{4m^2 V_0^2}{\hbar^4}$$

$$\boxed{R(k) = \frac{U_0^2}{4k^4} \sin^2(kL)} \quad \text{or} \quad \boxed{R(k) = \frac{V_0^2}{4E^2} \sin^2\left(\frac{L}{\hbar}\sqrt{2mE}\right.} \rightarrow (1)$$

comparing (1) with relation at problem (1)

$$R = \frac{U_0^2 \sin^2(k'L)}{U_0^2 \sin^2(k'L) + 4k^2k'^2} \longrightarrow (2)$$

sub $k' = \sqrt{\frac{2m}{\hbar^2}(E - V_0)}$, $U_0^2 = \frac{4m^2V_0^2}{\hbar^2}$, $k'^2 = \frac{2m}{\hbar^2}(E - V_0)$, $k^2 = \frac{2mE}{\hbar^2}$ in (2)

$$R = \frac{\frac{4m^2V_0^2}{\hbar^4} \sin^2\left(\frac{L}{\hbar}\sqrt{2m(E-V_0)}\right)}{\frac{4m^2V_0^2}{\hbar^4} \sin^2\left(\frac{L}{\hbar}\sqrt{2m(E-V_0)}\right) + 4\frac{2mE}{\hbar^2}\frac{2m}{\hbar^2}(E-V_0)}$$

$$R = \frac{\frac{4m^2V_0^2}{\hbar^2} \sin^2\left(\frac{L}{\hbar}\sqrt{2m(E-V_0)}\right)}{\frac{4m^2V_0^2}{\hbar^2} \sin^2\left(\frac{L}{\hbar}\sqrt{2m(E-V_0)}\right) + \frac{16m^2E}{\hbar^4}(E-V_0)}$$

when $E \gg V_0$

$$R = \frac{\frac{\cancel{4m^2}V_0^2}{\cancel{\hbar^4}} \sin^2\left(\frac{L}{\hbar}\sqrt{2mE}\right)}{\frac{16\cancel{m^2}E^2}{\cancel{\hbar^4}}}$$

$$\boxed{R = \frac{V_0^2}{4E^2} \sin^2\left(\frac{L}{\hbar}\sqrt{2mE}\right)}$$ same relation (1)

(1) and (2) agree provided $E \gg V_0$

another way

$$R = \frac{U_0^2 \sin^2(L\sqrt{\varepsilon - U_0})}{U_0^2 \sin^2(L\sqrt{\varepsilon - U_0} + 4\varepsilon(\varepsilon - U_0)}$$

when $\varepsilon \gg U_0$

$$R = \frac{U_0^2 \sin^2(L\sqrt{\varepsilon})}{4\varepsilon^2}$$

$$R = \frac{U_0^2 \sin^2(Lk)}{4k^4}$$

problem 2

$$V(r) = \frac{A}{r^2}$$

$$F_B(\theta) = -\frac{2m}{q\hbar^2}\int_0^\infty r\, V(r) \sin(qr)\, dr$$

$$f_B(\theta) = \frac{-2mA}{q\hbar^2}\int_0^\infty \frac{1}{r}\sin(qr)\, dr$$

$$F_B(\theta) = \frac{-2mA}{q\hbar^2}\int_0^\infty \frac{1}{r}\sin(qr)\, dr$$

$x = qr$
$dx = q\,dr \Rightarrow dr = \frac{dx}{q}$
$r = 0 \Rightarrow x = 0$
$r = \infty \Rightarrow x = \infty$

$$F_B(\theta) = \frac{-2mA}{q\hbar^2}\int_0^\infty \frac{\sin x}{x}\, dx$$

$$F_B(\theta) = \frac{-\cancel{2}mA}{\hbar^2 q}\frac{\pi}{\cancel{2}}$$

$$F_B(\theta) = \frac{-m\pi A}{\hbar^2 q}$$

$q = 2k\sin(\frac{\theta}{2})$

$$F_B(\theta) = \frac{-m\pi A}{2\hbar^2 k \sin(\frac{\theta}{2})}$$

$$\frac{d\sigma}{d\Omega} = \sigma(\theta) = \left|F_B(\theta)\right|^2 = \frac{m^2\pi^2A^2}{\hbar^4 q^2}$$

$$\boxed{\sigma(\theta) = \frac{m^2A^2\pi^2}{4\hbar^4 k^2 \sin^2(\frac{\theta}{2})}}$$ differential cross-section

problem 3

$$V(r) = V(r-R)$$
$$V(r-R) = V_0 R\,\delta(r-R)$$
$$V(r) = V_0 R\,\delta(r-R)$$

scattering potential has a translation invariance property $\boxed{V(r) = V(r-R)}$ where R is a constant vector

so

$$\frac{-2m}{q\hbar^2}\int_0^\infty r\,V(r)\sin(qr)\,dr = \frac{-2m}{q\hbar^2}\int_0^\infty r\,V(r-R)\sin(qr)\,dr$$

$$F_B(\theta) = \frac{-2m}{q\hbar^2}\int_0^\infty r\,V(r-R)\sin(qr)\,dr$$

$$F_B(\theta) = \frac{-2mV_0R}{q\hbar^2}\int_0^\infty r\,\delta(r-R)\sin(qr)\,dr$$

$$F_B(\theta) = \frac{-2mV_0RR}{q\hbar^2}\sin(qR)$$

$$\boxed{F_B(\theta) = \frac{-2mV_0R^2}{q\hbar^2}\sin(qR)}$$

$$q = 2k\sin(\tfrac{\theta}{2})$$

$$\sigma(\theta) = \frac{d\sigma}{d\Omega} = |F_B(\theta)|^2$$

$$\boxed{\frac{d\sigma}{d\Omega} = \frac{4m^2V_0^2R^4}{q^2\hbar^4}\sin^2(qR)} \quad q = 2k\sin\tfrac{\theta}{2}$$

$$\frac{d\sigma}{d\Omega} = \frac{4m^2V_0^2R^4}{4k^2\sin^2(\frac{\theta}{2})\hbar^4}\sin^2(4Rk^2\sin^2\tfrac{\theta}{2})$$

$$\frac{d\sigma}{d\Omega} = \frac{m^2V_0^2R^4}{\hbar^4k^2\sin^2(\frac{\theta}{2})}\sin^2(4Rk^2\sin^2\tfrac{\theta}{2})$$

problem 4

$$V(r) = V_0 e^{-a^2 r^2}$$

$$F_B(\theta) = -\frac{2m}{q\hbar^2}\int_0^\infty r\, V(r)\sin(qr)\,dr$$

$$F_B(\theta) = -\frac{2mV_0}{q\hbar^2}\int_0^\infty r e^{-a^2r^2}\sin(qr)\,dr$$

$$F_B(\theta) = -\frac{2mV_0}{\cancel{q}\hbar^2}\frac{\sqrt{\pi}}{4a^3}\cancel{q}\, e^{-\frac{q^2}{4a^2}}$$

$$\int_0^\infty r e^{-a^2r^2}\sin(qr)\,dr = \frac{\sqrt{\pi}}{4a^3}\, q e^{-\frac{q^2}{4a^2}}$$

$$F_B(\theta) = -\frac{mV_0\sqrt{\pi}}{2\hbar^2 a^3} e^{-\frac{q^2}{4a^2}}$$

$$\sigma(\theta) = |F_B(\theta)|^2 = \frac{m^2V_0^2\pi}{4\hbar^4a^6} e^{-\frac{q^2}{2a^2}}$$

$$q = 2k\sin\left(\frac{\theta}{2}\right)$$

$$\boxed{\frac{d\sigma}{d\Omega} = \sigma(\theta) = \frac{m^2V_0^2\pi}{4\hbar^4a^6} e^{-\frac{4k^2\sin^2\frac{\theta}{2}}{2a^2}}} \rightarrow \text{differential cross-section}$$

$$\sigma_{total} = \int \overbrace{\frac{d\sigma}{d\Omega}}^{|F(\theta)|^2} \underbrace{d\Omega}_{\sin\theta\, d\theta\, d\phi}$$

$$\sigma_{total} = \frac{m^2V_0^2\pi}{4\hbar^4a^6}\int_0^{2\pi} d\phi \int_0^{\pi} \sin\theta\, e^{-\frac{4k^2\sin^2\frac{\theta}{2}}{2a^2}}\,d\theta$$

$$\sigma_{total} = \frac{m^2V_0^2\pi\, 2\pi}{4\hbar^4a^6}\int_0^{\pi} \sin\theta\, e^{-\frac{4k^2\sin^2\frac{\theta}{2}}{2a^2}}\,d\theta$$

$$\sin^2\left(\frac{\theta}{2}\right) = \frac{1}{2}(1-\cos\theta)$$

$$\sigma_{total} = \frac{m^2V_0^2\pi^2}{2\hbar^4a^6}\int_0^{\pi} \sin\theta\, e^{-\frac{2k^2(1-\cos\theta)}{2a^2}}\,d\theta$$

$$\sigma_{total} = \frac{m^2V_0^2\pi^2}{2\hbar^4a^6}\int_0^{\pi} \sin\theta\, e^{-\frac{k^2}{a^2}} e^{\frac{k^2\cos\theta}{a^2}}\,d\theta$$

$u = \cos\theta$

$du = -\sin\theta\, d\theta$

$\theta = 0 \Rightarrow u = 1$

$\theta = \pi \Rightarrow u = -1$

$$= -\frac{m^2V_0^2\pi^2}{2\hbar^4a^6} e^{-\frac{k^2}{a^2}}\int_1^{-1} e^{\frac{k^2u}{a^2}}\,du$$

$$= -\frac{m^2V_0^2\pi^2}{2\hbar^4a^6} e^{-\frac{k^2}{a^2}}\left[\frac{a^2}{k^2}e^{-\frac{k^2}{a^2}} - \frac{a^2}{k^2}e^{\frac{k^2}{a^2}}\right]$$

$$\sigma_{total} = \frac{-m^2 V_0^2 \pi^2}{2\hbar^4 a^6}\left[\frac{a^2}{k^2} e^{-\frac{2k^2}{a^2}} - \frac{a^2}{k^2}\right]$$

$$\sigma_{total} = \frac{m^2 V_0^2 \pi^2}{2\hbar^4 a^6}\left[\frac{a^2}{k^2} - \frac{a^2}{k^2} e^{-\frac{2k^2}{a^2}}\right]$$

$$\boxed{\sigma_{total} = \frac{m^2 V_0^2 \pi^2}{2\hbar^4 k^2 a^4}\left[1 - e^{-\frac{2k^2}{a^2}}\right]}$$

total differential cross-section.