KING SAUD UNIVERSITY

Math Department

First Semester 40/41

October28th 2019

Time 90mn

Math106 Midterm1

Question 1(2+3+3)

a) Find the number c so that $\sum_{11}^{30}(k^2-c)=0$

b) Approximate the integral $\int_0^{2\pi}(\cos x)^4 dx$ using Simpson's rule with n=8

c) Use Riemann sums to evaluate $\int_0^1 x^3 dx$

Question 2(3+2+3)

a) Evaluate the integral $\int \frac{5^{tanx}}{(cosx)^2}dx$

b) If $y = 2^{(sinx)^2} + x^\pi \pi^x$ find y'

c) Compute $\int \frac{dx}{\sqrt{x}(2+x)}$

Question 3(3+3+3)

a) Find $\int \frac{(lnx+1)dx}{\sqrt{16(xlnx)^2-9}}$

b) Evaluate the integral $\int \frac{dx}{x\sqrt{x^5-4}}$

c) Compute $\int \frac{2e^{-3x}dx}{1-e^{-6x}}$

106Midterm1grading scheme(Sem1-40/41)

<u>Question1</u> a) $\sum_{11}^{30} k^2 = \sum_{1}^{30} k^2 - \sum_{1}^{10} k^2 = 20c \quad (\mathbf{1})$

$$9455 - 385 = 20c \;\; so\; c = \frac{907}{2} = \;\; 453.5 \;\; (\mathbf{1})$$

b) $S_6 = \frac{2\pi - 0}{3.8}(1 + 4\left(\frac{\sqrt{2}}{2}\right)^4 + 2.0 + 4\left(\frac{\sqrt{2}}{2}\right)^4 + 2(-1)^4 + 4\left(\frac{\sqrt{2}}{2}\right)^4 + 2.0$

$$+ 4\left(\frac{\sqrt{2}}{2}\right)^4 \;\; + 1) \;\; (\mathbf{2})$$

$$= \frac{2\pi}{3} \approx 2.09439 \;\; (\mathbf{1})$$

c) $P = \{0, \frac{1}{n}, \frac{2}{n}, \ldots\ldots.1\}\; x_k = \frac{k}{n}\; and\; \Delta x_k = \frac{1}{n}$, take $u_k = x_k \;\; 1 \le k \le n \;\; (\mathbf{1})$.

$$R_P = \sum_{1}^{n} (\frac{k}{n})^3 \frac{1}{n} = \frac{1}{n^4}(\frac{n(n+1)}{2})^2 \rightarrow \frac{1}{4} \;\; as\; n \rightarrow \;\; so \;\; \int_0^1 x^3 dx = \frac{1}{4} \;\; (\mathbf{2})$$

<u>Question2</u> a) $\int \frac{5^{tanx}}{(cosx)^2} dx = \int 5^u du \;\; u = tanx \;\; (\mathbf{2})$

$$= \frac{1}{ln5} 5^{tanx} + C \; (\mathbf{1})$$

$b)\; y' = 2ln2sinxcosx2^{(sinx)^2} + \pi x^{\pi-1}\pi^x + ln\pi x^\pi \pi^x$

$$(\mathbf{1}) + \frac{\mathbf{1}}{\mathbf{2}} + (\frac{\mathbf{1}}{\mathbf{2}})$$

$$c) \int \frac{dx}{\sqrt{x}(2+x)} = 2\int \frac{du}{2+u^2} \quad u = \sqrt{x} \;\; (\mathbf{2})$$

$$= \sqrt{2}\tan^{-1}(\sqrt{x/2}) + C \; (\mathbf{1})$$

Question3 a) $\int \frac{\ln x+1}{\sqrt{16(x\ln x)^2-9}}dx = \frac{1}{4}\int \frac{du}{\sqrt{u^2-9}}$ $u = 4x\ln x$ $\mathbf{(2)}$

$= \frac{1}{4}\cosh^{-1}(\frac{4x\ln x}{3}) + C$ $\mathbf{(1)}$

b) $\int \frac{dx}{x\sqrt{x^5-4}} = \frac{2}{5}\int \frac{du}{u\sqrt{u^2-4}}$ $u = x^{5/2}$ $\mathbf{(2)}$

$= \frac{2}{10}\sec^{-1}(\frac{x^{5/2}}{2}) + C$ $\mathbf{(1)}$

c) $\int \frac{2e^{-3x}}{1-e^{-6x}}dx = \frac{-2}{3}\int \frac{du}{1-u^2}$ $u = e^{-3x}$ $\mathbf{(2)}$

$= \frac{-2}{3}\tanh^{-1}(e^{-3x}) + C$ $\mathbf{(1)}$

Note:the last answer implicitly assumes $x > 0$. Also accept the answer

$\frac{-2}{3}\coth^{-1}(e^{-3x}) + C$

KING SAUD UNIVERSITY **First Semester 40/41**

Math Department **November 25th 2019**

Time 90mn

Math106 Midterm2

Question 1(2+3+3)

a) Find $\lim_{x\to 0}(1+8x^2)^{\frac{1}{x^2}}$

b) Compute the integral $\int e^{4x} sinx dx$

c) Evaluate $\int (sinx)^2 (cosx)^2 dx$

Question 2(3+3+2)

a) Evaluate the integral $\int \frac{\sqrt{x^2-25}}{x} dx$

b) Find $\int \frac{3x^2+7x+2}{(x+1)^2(x+3)} dx$

c) Compute $\int \frac{dx}{\sqrt{x(x+1)+1}}$

Question 3(3+3+3)

a) Find $\int \frac{dx}{x^{1/2}+x^{1/3}}$

b) Does the integral $\int_0^{\infty} \frac{xdx}{1+x^4}$ converge? Find its value if it does.

c) Compute the area of the region bounded by the curves: $y = x^2, y = x - 1$

$y = 0$, and $y = 4$.

106Midterm2grading scheme(Sem1-40/41)

<u>Question1</u> a) $y = (1+8x^2)^{\frac{1}{x^2}}$

$\lim_{x\to 0} lny = \lim_{x\to 0} \frac{\ln(1+8x^2)}{x^2} = \lim_{x\to 0} \frac{16x}{(1+8x^2)(2x)} = 8 \quad (\mathbf{1,5})$

So $\lim_{x\to 0} y = e^8 \quad (\mathbf{0,5})$

b) $\int e^{4x} sinxdx = \frac{1}{4} e^{4x} sinx - \frac{1}{4}\int e^{4x} cosxdx \quad (\mathbf{1})$

$$= \frac{1}{4} e^{4x} sin4x - \frac{1}{4}\left(\frac{1}{4} e^{4x} cosx + \frac{1}{4}\int e^{4x} sinxdx\right) \quad (\mathbf{1})$$

Thus $\int e^{4x} sin4xdx = \frac{1}{17} e^{4x}(4sinx - cosx) + C \quad (\mathbf{1})$

c) $\int (sinx)^2(cosx)^2 dx = \frac{1}{4}\int (sin2x)^2 dx = \frac{1}{8}\int (1 - cos4x)dx \quad (\mathbf{2})$

$$= \frac{1}{8}\left(x - \frac{sin4x}{4}\right) + C \quad (\mathbf{1})$$

<u>Question2</u> a) $\int \frac{\sqrt{x^2-25}}{x} dx = 5\int (tan\theta)^2 d\theta \qquad x = 5sec\theta \quad (\mathbf{1})$

$$= 5(tan\theta - \theta) + C \quad (\mathbf{1})$$

$$= \sqrt{x^2 - 25} - 5\sec^{-1}\left(\frac{x}{5}\right) + C \quad (\mathbf{1})$$

$$b) \quad \frac{3x^2 + 7x + 2}{(x+1)^2(x+3)} = \frac{1}{x+1} - \frac{1}{(x+1)^2} + \frac{2}{x+3} \quad (\mathbf{1.5})$$

$$\int \frac{3x^2 + 7x + 2}{(x+1)^2(x+3)} dx = \ln|x+1| + \frac{1}{x+1} + 2\ln|x+3| + C \quad (\mathbf{1.5})$$

$I = \int e^{4x} \sin x dx$

$= \frac{1}{17} e^{4x}[4\sin x - \cos x] + C$

$e^{4x}(-\cos x) - \int 4e^{4x}(-\cos x) dx$

$= -e^{4x}\cos x + 4\left[e^{4x}\sin x - \int 4e^{4x}\sin x\, dx\right]$

$I = -e^{4x}\cos x + 4e^{4x}\sin x - 16I$

$17I = e^{4x}[4\sin x - \cos x]$

$$c)\ \int \frac{dx}{\sqrt{x^2+x+1}} = \int \frac{dx}{\sqrt{\left(x+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}} \quad \mathbf{(1)}$$

$$= \sinh^{-1}\left(\frac{(2x+1)}{\sqrt{3}}\right) + C \quad \mathbf{(1)}$$

Question3

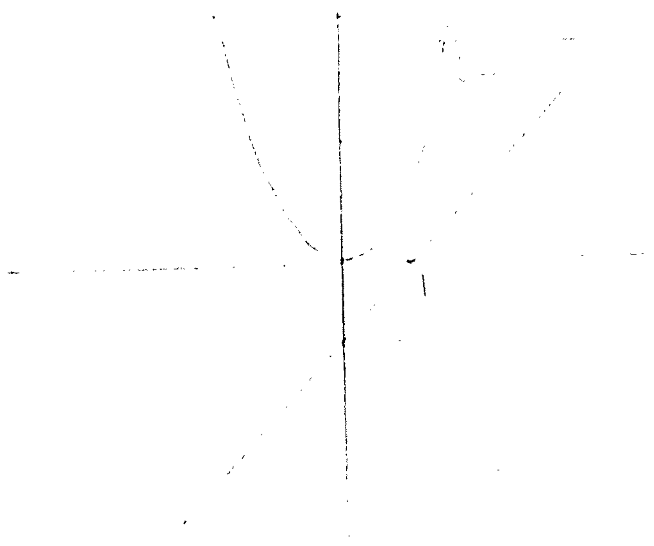
a) $\int \frac{dx}{x^{1/2}+x^{1/3}} = 6\int \frac{u^3\,du}{1+u} = 6\int \left(u^2 - u + 1 - \frac{1}{u+1}\right)du$ $\mathbf{(2)}$

$$= 2x^{\frac{1}{2}} - 3x^{\frac{1}{3}} + 6x^{\frac{1}{6}} - 6\ln\left|1 + x^{\frac{1}{6}}\right| + C \quad \mathbf{(1)}$$

b$\int_0^c \frac{x\,dx}{1+x^4} = \frac{1}{2}\int_0^{c^2} \frac{du}{1+u^2} = \frac{1}{2}\tan^{-1}(c^2) \to \frac{\pi}{4}\ as\ c \to \infty$ $\mathbf{(2.5)}$

Thus $\int_0^\infty \frac{x\,dx}{1+x^4}$ *converges and is equal to* $\frac{\pi}{4}$ $\mathbf{(0.5)}$

c) graph$\mathbf{(1)}$



$$A = \int_0^4 (y + 1 - \sqrt{y})dy = [\frac{y^2}{2} + y - \frac{2}{3}y^{\frac{3}{2}}]_0^4 = \frac{20}{3} \quad (2)$$

KING SAUD UNIVERSITY

Math Department

First Semester 41

December 15 2019

Final exam106

Time: 180mn

Question 1(2+2+3)

a) Find the number c in the mean value theorem for $f(x) = -x^2 + 4x$ on [0, 3]

b) Compute the integral $\int \frac{dx}{\sqrt{5^x-16}}$

c) Evaluate $\int \frac{cotxdx}{\sqrt{9-(sinx)^4}}$

Question 2(3+3+3)

a) Compute $\lim_{x\to 3}\left(\frac{1}{x-3} - \frac{1}{\ln(x-2)}\right)$

b) Find $\int x^2 \tan^{-1}(x)dx$

c) Evaluate the integral $\int (tanx)^4(secx)^6 dx$

Question 3(3+3+3)

a) Compute the following integral $\int \frac{x^2dx}{(x^2+9)^{3/2}}$

b) Find the integral $\int \frac{(3x-2)dx}{(x^2+4)(x+2)}$

c) Evaluate the integral $\int \frac{dx}{3-sinx+cosx}$

Question 4(3+2+1)

a) Sketch the region bounded by the curves: $y = 4 - x^2, y = x + 2$
$x = -3, x = 0$ and find its area.

b) Find the volume obtained by revolving the region bounded by the curves $y = -x^2 + 2,\; y = 1$ about the line of equation $x = 3$

c) Set up an integral for the volume obtained by revolving the region in part b) about the line of equation $y = 4$.

Question5(3+3+3)

a) Find the length of the curve given by $r = (\cos(\frac{\theta}{2}))^2$, $0 \le \theta \le \pi$.

b) Sketch the region R that lies inside the curve $r = 1 - sin\theta$ and outside the curve $r = 1$ and find its area.

c) Find the area of the surface obtained by revolving the curve $r = 2cos\theta \quad 0 \le \theta \le \pi/4$ about the y-axis.

<u>Final Dec19 Grading scheme</u>

<u>Question</u> 1

a) $\int_0^3 (4x-x^2)dx = 3(4c-c^2) \Leftrightarrow c^2-4c+3=0 \ \ \mathbf{(1.5)}$

$$c = 1 \ or \ c = 3 \ \mathbf{(0,5)}$$

b) $\int \frac{dx}{\sqrt{5^x-16}} = \frac{2}{ln5}\int \frac{du}{u\sqrt{u^2-16}} = \frac{1}{2ln5}\sec^{-1}\left(\frac{5^{\frac{x}{2}}}{4}\right) + C \ \ \mathbf{(1.5) + (0.5)}$

c) $\int \frac{cotxdx}{\sqrt{9-(sinx)^4}} = \frac{1}{2}\int \frac{du}{u\sqrt{9-u^2}} \quad u = (sinx)^2 \quad \mathbf{(2)}$

$$= \frac{-1}{6}\,\mathrm{sech}^{-1}\left(\frac{(sinx)^2}{3}\right) + C \quad \mathbf{(1)}$$

<u>Question 2</u>

a) $\lim_{x\to 3} \frac{\ln(x-2)-x+3}{(x-3)\ln(x-2)} = \lim_{x\to 3} \frac{\frac{1}{x-2}-1}{\ln(x-2)+\frac{x-3}{x-2}} = \lim_{x\to 3} \frac{\frac{-1}{(x-2)^2}}{\frac{1}{x-2}+\frac{1}{(x-2)^2}}$

$$= -\frac{1}{2} \quad \mathbf{(1) + (1) + (1)}$$

b) $\int x^2 \tan^{-1}(x)dx = \frac{x^3}{3}\tan^{-1}(x) - \frac{1}{3}\int \frac{x^3}{1+x^2}dx$

$$= \frac{x^3}{3}\tan^{-1} x - \frac{1}{3}\int \left(x - \frac{x}{1+x^2}\right)dx$$

$$= \frac{x^3}{3}\tan^{-1} x - \frac{1}{6}x^2 + \frac{1}{6}\ln(1+x^2) + C$$

$$\mathbf{(1) + (1) + (1)}$$

c) $\int (tanx)^4 (secx)^6 dx = \int u^4(u^2+1)^2 du$

$$= \frac{(tanx)^9}{9} + \frac{2}{7}(tanx)^7 + \frac{1}{5}(tanx)^5 + C$$

$$\mathbf{(1.5) + (1.5)}$$

Question 3

a) $\int \frac{x^2 dx}{(x^2+9)^{3/2}} = \int \frac{(tan\theta)^2}{\sec(\theta)} d\theta = \int (sec\theta - cos\theta) d\theta \quad x = 3tan\theta \quad (\mathbf{1.5})$

$$= \ln\left|\frac{\sqrt{x^2+9}}{3} + \frac{x}{3}\right| - \frac{x}{\sqrt{x^2+9}} + C \quad (\mathbf{1.5})$$

~~$\ln|x+2|$~~

b) $\frac{3x-2}{(x+2)(x^2+4)} = \frac{-1}{x+2} + \frac{x+1}{x^2+4} \quad (\mathbf{1.5})$

$$\int \frac{(3x-2)dx}{(x+2)(x^2+4)} = -\ln|x+2| + \frac{1}{2} ln(\overset{4}{1} + x^2) + \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + C \quad (\mathbf{1.5})$$

c *d)* $\int \frac{dx}{3-sinx+cosx} = \frac{1}{2}\int \frac{du}{2-u+u^2} = \frac{1}{2}\int \frac{du}{(u-\frac{1}{2})^2+\frac{7}{4}} \quad (\mathbf{2})$

Put $u = \tan\frac{x}{2}$, $dx = \frac{2\,du}{1+u^2}$, $\sin x = \frac{2u}{1+u^2}$, $\cos x = \frac{1-u^2}{1+u^2}$

$$= \frac{1}{\sqrt{7}}\tan^{-1}\left(\frac{2u-1}{\sqrt{7}}\right) + C = \frac{1}{\sqrt{7}}\tan^{-1}\left(\frac{2\tan\left(\frac{x}{2}\right)-1}{\sqrt{7}}\right) + C(\mathbf{1})$$

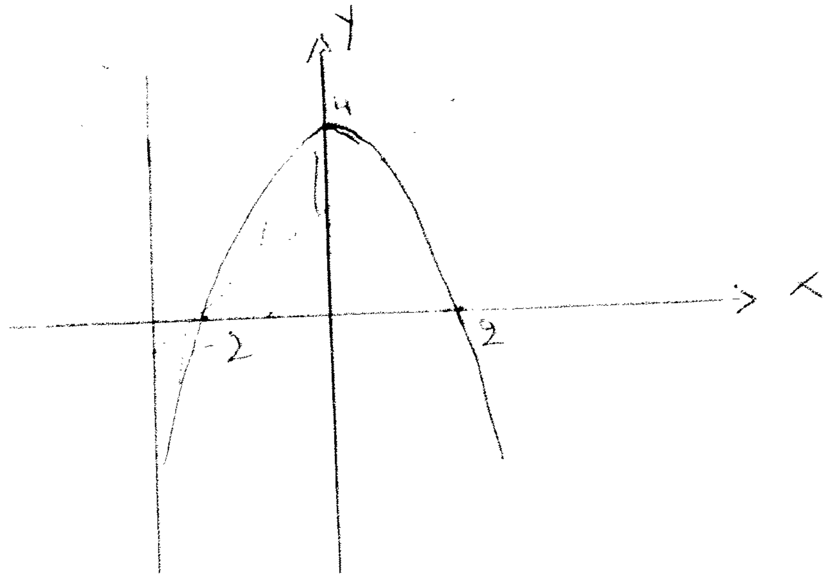
Question 4

a)$x + 2 = 4 - x^2 \Leftrightarrow x = -2 \; or \; x = 1$

$$A = \int_{-3}^{-2} x + 2 - (4 - x^2)dx + \int_{-2}^{0} 4 - x^2 - (x+2)dx \quad (\mathbf{1.5})$$

$$= \frac{11}{6} + \frac{10}{3} = \frac{31}{6} \quad (\mathbf{0.5})$$

graph $(\mathbf{1})$



b) $V = \int_{-1}^{1} 2\pi(3-x)(-x^2+1)dx = 8\pi$ **(2)**

c) $V = \int_{-1}^{1} \pi(9-(2+x^2)^2)dx$ **(1)**

Question5

a) $L = \int_0^{\pi} \sqrt{(\cos(\frac{\theta}{2}))^4 + (\cos(\frac{\theta}{2}))^2(\sin\left(\frac{\theta}{2}\right))^2}\, d\theta$ **(1.5)**

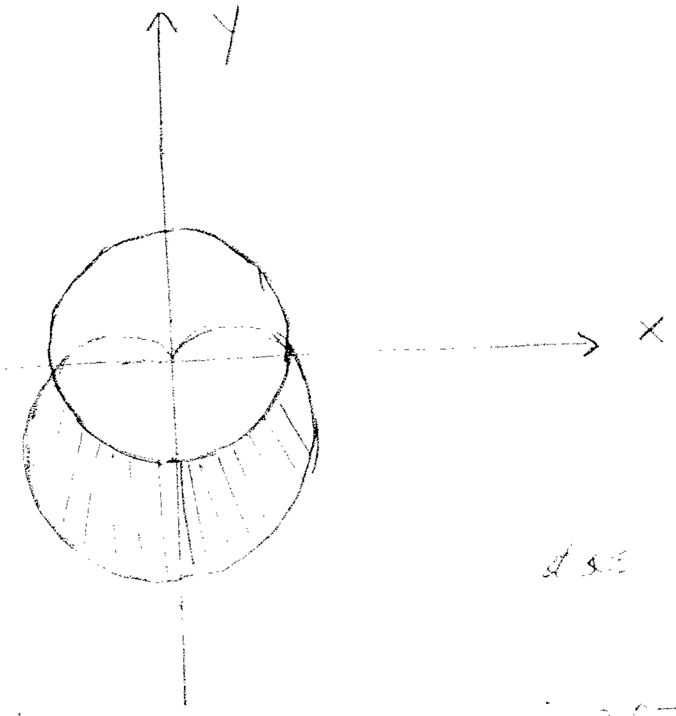
$= \int_0^{\pi} \cos\left(\frac{\theta}{2}\right) d\theta = 2$ **(1.5)**

b) $1 - \sin\theta = 1 \Rightarrow \theta = 0 \text{ or } \theta = \pi \text{ Or } \theta = 2\pi$

$$A = \frac{1}{2}\int_{\pi}^{2\pi} [(1-\sin\theta)^2 - 1]d\theta\ )\ \mathbf{(1)}$$

$= \frac{\pi}{4} + 2$ **(1)** $\simeq 2.785$

graph **(1)**



c) $S = 8\pi \int_0^{\pi/4} \cos^2\theta\, d\theta = 4\pi \left[\theta + \frac{\sin 2\theta}{2}\right]_0^{\pi/4}$

$= 2\pi\left(\frac{\pi}{4} + 1\right)$ (1.5)+(1.5)

$= 2\pi(2.57) = (6.28)(2.57) = 16.1396$