3rd homework

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(1)

$j_1 = \frac{1}{2}$ $\quad j_2 = \frac{1}{2}$

$m_1 = -\frac{1}{2}, \frac{1}{2}$ $\quad , \quad m_2 = -\frac{1}{2}, \frac{1}{2}$

$J = j_1 + j_2 = \frac{1}{2} + \frac{1}{2} = 1$ $\qquad$ and $\qquad J = |j_1 - j_2| = 0$

$\Downarrow$ $\qquad\qquad\qquad\qquad\qquad\qquad \Downarrow$

$m_j = m_1 + m_2 = 1, 0, -1$ $\qquad\qquad\qquad m_j = 0$

* The state space of the compound system is a $(2j_1+1)(2j_2+1)$ - dimensional space:

$(2(\frac{1}{2})+1)(2(\frac{1}{2})+1)$

$= (2)(2) = 4$ states..

* $j=1$ $\qquad m_j = \frac{1}{2} + \frac{1}{2} = 1 \quad , \quad m_j = -\frac{1}{2} + \frac{1}{2} = 0 \quad , \quad m_j = -\frac{1}{2} - \frac{1}{2} = -1$

$$\boxed{|1,1\rangle = |\tfrac{1}{2}, \tfrac{1}{2}\rangle = |++\rangle}$$

To obtain $|1,0\rangle \rightarrow$ apply the lowering operator J_-

$J_- = J_{1-} + J_{2-}$

$J_-|j\, m_j\rangle = \hbar\sqrt{j(j+1) - m_j(m_j-1)}\; |j, m_j - 1\rangle$

$J_{1-}|m_1 m_2\rangle = \hbar\sqrt{j_1(j_1+1) - m_1(m_1-1)}\; |m_1 - 1, m_2\rangle$

$J_{2-}|m_1 m_2\rangle = \hbar\sqrt{j_2(j_2+1) - m_2(m_2-1)}\; |m_1, m_2 - 1\rangle$

$$J_-|1,1\rangle = J_{1-}|\tfrac12,\tfrac12\rangle + J_{2-}|\tfrac12,\tfrac12\rangle$$

$$\hbar\sqrt{1(1+1)-1(1-1)}\;|1,0\rangle = \hbar\sqrt{\tfrac12(\tfrac12+1)-\tfrac12(\tfrac12-1)}\;|-\tfrac12,\tfrac12\rangle + \hbar\sqrt{\tfrac12(\tfrac12+1)-\tfrac12(\tfrac12-1)}\;|\tfrac12,-\tfrac12\rangle$$

$$\sqrt{(2)}\;|1,0\rangle = \sqrt{(1)}\;|-\tfrac12,\tfrac12\rangle + \sqrt{(1)}\;|\tfrac12,-\tfrac12\rangle$$

$$\boxed{|1,0\rangle = \frac{1}{\sqrt2}|-\tfrac12,\tfrac12\rangle + \frac{1}{\sqrt2}|\tfrac12,-\tfrac12\rangle = \frac{1}{\sqrt2}\left(|-+\rangle + |+-\rangle\right)}$$

To obtain $|1,-1\rangle$:

$$J_-|1,0\rangle = (J_{1-}+J_{2-})\left[\frac{1}{\sqrt2}|-\tfrac12,\tfrac12\rangle + \frac{1}{\sqrt2}|\tfrac12,-\tfrac12\rangle\right]$$

$$\hbar\sqrt{1(1+1)-0}\;|1,-1\rangle = \underset{0}{J_{1-}\frac{1}{\sqrt2}|-\tfrac12,\tfrac12\rangle} + J_{1-}\frac{1}{\sqrt2}|\tfrac12,-\tfrac12\rangle + J_{2-}\frac{1}{\sqrt2}|-\tfrac12,\tfrac12\rangle + J_{2-}\frac{1}{\sqrt2}|\tfrac12,-\tfrac12\rangle$$

$$\sqrt2\cancel{\hbar}\;|1,-1\rangle = \frac{\cancel{\hbar}}{\sqrt2}\Big[\cancel{\sqrt{\tfrac12(\tfrac12+1)+\tfrac12(-\tfrac12-1)}}\;|-\tfrac32,\tfrac12\rangle + \sqrt{\tfrac12(\tfrac12+1)-\tfrac12(\tfrac12-1)}\;|-\tfrac12,-\tfrac12\rangle$$
$$+\sqrt{\tfrac12(\tfrac12+1)-\tfrac12(\tfrac12-1)}\;|-\tfrac12,-\tfrac12\rangle + \underset{0}{\cancel{\sqrt{\tfrac12(\tfrac12+1)+\tfrac12(-\tfrac12-1)}}}\;|\tfrac12,-\tfrac32\rangle\Big]$$

$$\sqrt2\;|1,-1\rangle = \frac{1}{\sqrt2}\left[\sqrt{\tfrac34+\tfrac14}\;|-\tfrac12,-\tfrac12\rangle + \sqrt{\tfrac34+\tfrac14}\;|-\tfrac12,-\tfrac12\rangle\right]$$

$$|1,-1\rangle = \frac{2}{\sqrt2.\sqrt2}\;|-\tfrac12,-\tfrac12\rangle$$

$$\boxed{|1,-1\rangle = |-\tfrac12,-\tfrac12\rangle = |--\rangle}$$

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* $j=0$ $\quad m_j=0 \quad +\frac{1}{2}-\frac{1}{2} \; , \; -\frac{1}{2}+\frac{1}{2}$

$$|0,0\rangle = a\left|\tfrac{1}{2},-\tfrac{1}{2}\right\rangle + b\left|-\tfrac{1}{2},\tfrac{1}{2}\right\rangle$$

the orthonormality $|jm\rangle$:

$$\langle 1,0|0,0\rangle = 0$$

$$\left(\frac{1}{\sqrt{2}}\left\langle -\tfrac{1}{2},\tfrac{1}{2}\right| + \frac{1}{\sqrt{2}}\left\langle \tfrac{1}{2},-\tfrac{1}{2}\right|\right)\left(a\left|\tfrac{1}{2},-\tfrac{1}{2}\right\rangle + b\left|-\tfrac{1}{2},\tfrac{1}{2}\right\rangle\right) = 0$$

$$0 + \frac{b}{\sqrt{2}}(1) + \frac{a}{\sqrt{2}}(1) + 0 = 0$$

$$\frac{b}{\sqrt{2}} + \frac{a}{\sqrt{2}} = 0$$

$$\frac{1}{\sqrt{2}}(a+b) = 0 \quad \Rightarrow \quad a = -b$$

$$\langle 0,0|0,0\rangle = 1 \quad \Rightarrow |a|^2 + |b|^2 = 1$$

$$2|a|^2 = 1$$

$$a = \frac{1}{\sqrt{2}} \quad , \quad b = -\frac{1}{\sqrt{2}}$$

$$\boxed{|0,0\rangle = \frac{1}{\sqrt{2}}\left|\tfrac{1}{2},-\tfrac{1}{2}\right\rangle - \frac{1}{\sqrt{2}}\left|-\tfrac{1}{2},\tfrac{1}{2}\right\rangle = \frac{1}{\sqrt{2}}\left(|+-\rangle - |-+\rangle\right)}$$

* The states of total spin for two particles ($s=1/2$):

$$|1,1\rangle = |1/2,1/2\rangle = |++\rangle$$

$$|1,0\rangle = \frac{1}{\sqrt{2}}|-1/2,1/2\rangle + \frac{1}{\sqrt{2}}|1/2,-1/2\rangle = \frac{1}{\sqrt{2}}\left(|-+\rangle + |+-\rangle\right)$$

$$|1,-1\rangle = |-1/2,-1/2\rangle = |--\rangle$$

$$|0,0\rangle = \frac{1}{\sqrt{2}}|1/2,-1/2\rangle - \frac{1}{\sqrt{2}}|-1/2,1/2\rangle = \frac{1}{\sqrt{2}}\left(|+-\rangle - |-+\rangle\right)$$

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(2) prove the relation (9): $|j_1=1, j_2=1, J=2, M=-1\rangle$

applying the lowering operator on eq. (8):

$$L_-|j_1=1, j_2=1, J=2, M=0\rangle = (L_-^{1}+L_-^{2})\frac{1}{\sqrt{6}}\Big\{|j_1=1, m_1=-1\rangle|j_2=1, m_2=1\rangle + 2|j_1=1, m_1=0\rangle|j_2=1, m_2=0\rangle$$

$$+|j_1=1, m_1=1\rangle|j_2=1, m_2=-1\rangle\Big\}$$

$$\sqrt{2(2+1)-0}\,|j_1=1, j_2=1, J=2, M=-1\rangle = \frac{1}{\sqrt{6}}\Big\{L_-^{1}|j_1=1, m_1=-1\rangle|j_2=1, m_2=1\rangle + 2L_-^{1}|j_1=1, m_1=0\rangle|j_2=1, m_2=0\rangle$$

$$+L_-^{1}|j_1=1, m_1=1\rangle|j_2=1, m_2=-1\rangle + L_-^{2}|j_1=1, m_1=-1\rangle|j_2=1, m_2=1\rangle$$

$$+2L_-^{2}|j_1=1, m_1=0\rangle|j_2=1, m_2=0\rangle + L_-^{2}|j_1=1, m_1=1\rangle|j_2=1, m_2=-1\rangle\Big\}$$

$$\sqrt{6}\,|j_1=1, j_2=1, J=2, M=-1\rangle = \frac{1}{\sqrt{6}}\Big\{\cancelto{0}{\sqrt{1(1+1)+1(-2)}}\,|j_1=1, m_1=-2\rangle|j_2=1, m_2=1\rangle + 2\sqrt{1(1+1)-0}\,|j_1=1, m_1=-1\rangle|j_2=1, m_2=0\rangle$$

$$+\sqrt{1(1+1)-1(1-1)}\,|j_1=1, m_1=0\rangle|j_2=1, m_2=-1\rangle + \sqrt{1(1+1)-0}\,|j_1=1, m_1=-1\rangle|j_2=1, m_2=0\rangle$$

$$+2\sqrt{1(1+1)-0}\,|j_1=1, m_1=0\rangle|j_2=1, m_2=-1\rangle + \cancelto{0}{\sqrt{1(1+1)+(-2)}}\,|j_1=1, m_1=1\rangle|j_2=1, m_2=-2\rangle\Big\}$$

$$|j_1=1, j_2=1, J=2, M=-1\rangle = \frac{1}{\sqrt{6}\cdot\sqrt{6}}\Big\{2\sqrt{2}\,|j_1=1, m_1=-1\rangle|j_2=1, m_2=0\rangle + \sqrt{2}\,|j_1=1, m_1=0\rangle|j_2=1, m_2=-1\rangle$$

$$+\sqrt{2}\,|j_1=1, m_1=-1\rangle|j_2=1, m_2=0\rangle + 2\sqrt{2}\,|j_1=1, m_1=0\rangle|j_2=1, m_2=-1\rangle\Big\}$$

$$|j_1=1, j_2=1, J=2, M=-1\rangle = \frac{\sqrt{2}}{6}\Big\{3|j_1=1, m_1=-1\rangle|j_2=1, m_2=0\rangle + 3|j_1=1, m_1=0\rangle|j_2=1, m_2=-1\rangle\Big\}$$

$$|j_1=1, j_2=1, J=2, M=-1\rangle = \frac{1}{\sqrt{2}}\Big\{|j_1=1, m_1=0\rangle|j_2=1, m_2=-1\rangle + |j_1=1, m_1=-1\rangle|j_2=1, m_2=0\rangle\Big\}$$

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To find $|j_1=0, j_2=1, J=1, M=0\rangle$

applying L_- on eq. (13)..

$$L_-|j_1=1, j_2=1, J=1, M=1\rangle = \frac{1}{\sqrt{2}}\Big\{L_-^1|j_1=1, m_1=1\rangle|j_2=1, m_2=0\rangle - L_-^1|j_1=1, m_1=0\rangle|j_2=1, m_2=1\rangle$$

$$+ L_-^2|j_1=1, m_1=1\rangle|j_2=1, m_2=0\rangle - L_-^2|j_1=1, m_1=0\rangle|j_2=1, m_2=1\rangle\Big\}$$

$$\sqrt{1(1+1)-1(1-1)}\,|j_1=1, j_2=1, J=1, M=0\rangle = \frac{1}{\sqrt{2}}\Big\{\sqrt{1(1+1)-0}\,|j_1=1, m_1=0\rangle|j_2=1, m_2=0\rangle - \sqrt{1(1+1)-0}\,|j_1=1, m_1=-1\rangle|j_2=1, m_2=1\rangle$$

$$+ \sqrt{1(1+1)-0}\,|j_1=1, m_1=1\rangle|j_2=1, m_2=-1\rangle - \sqrt{1(1+1)-0}\,|j_1=1, m_1=0\rangle|j_2=1, m_2=0\rangle\Big\}$$

$$\sqrt{2}\,|j_1=1, j_2=1, J=1, M=0\rangle = \frac{1}{\sqrt{2}}\Big\{\cancel{\sqrt{2}\,|j_1=1, m_1=0\rangle|j_2=1, m_2=0\rangle} - \sqrt{2}\,|j_1=1, m_1=-1\rangle|j_2=1, m_2=1\rangle$$

$$+ \sqrt{2}\,|j_1=1, m_1=1\rangle|j_2=1, m_2=-1\rangle - \cancel{\sqrt{2}\,|j_1=1, m_1=0\rangle|j_2=1, m_2=0\rangle}\Big\}$$

$$|j_1=1, j_2=1, J=1, M=0\rangle = \frac{\cancel{\sqrt{2}}}{\sqrt{2}\cdot\cancel{\sqrt{2}}}\Big\{|j_1=1, m_1=1\rangle|j_2=1, m_2=-1\rangle - |j_1=1, m_1=-1\rangle|j_2=1, m_2=1\rangle\Big\}$$

$$|j_1=1, j_2=1, J=1, M=0\rangle = \frac{1}{\sqrt{2}}\Big\{|j_1=1, m_1=1\rangle|j_2=1, m_2=-1\rangle - |j_1=1, m_1=-1\rangle|j_2=1, m_2=1\rangle\Big\}$$

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$$\langle j_1=1, m_1=0, j_2=1, m_2=0\,|\,J=2, M=0\rangle$$

$$= \langle j_1=1, m_1=0|\langle j_2=1, m_2=0|\,\frac{1}{\sqrt{6}}\,|j_1=1, m_1=-1\rangle|j_2=1, m_2=1\rangle$$

$$+ \langle j_1=1, m_1=0|\langle j_2=1, m_2=0|\,\frac{2}{\sqrt{6}}\,|j_1=1, m_1=0\rangle|j_2=1, m_2=0\rangle$$

$$+ \langle j_1=1, m_1=0|\langle j_2=1, m_2=0|\,\frac{1}{\sqrt{6}}\,|j_1=1, m_1=1\rangle|j_2=1, m_2=-1\rangle$$

* from properties of the clebsch-Gordan coefficient:

$$\langle j_1 j_2 j m | j_1 j_2 j' m' \rangle = \delta_{jj'} \delta_{mm'}$$

$$\langle j_1=1, m_1=0, j_2=1, m_2=0 | J=2, M=0 \rangle = \frac{1}{\sqrt{6}}(0) + \frac{2}{\sqrt{6}}(1) + \frac{1}{\sqrt{6}}(0)$$

$$= \frac{2}{\sqrt{6}} = \sqrt{\frac{\cancel{4}^{\,2}}{2\times 3}} = \sqrt{\frac{2}{3}}$$