Tutorial Session ①

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Pb 2.3 p.17 Textbook

A random variable X has density function $f(x) = 4x(1+x^2)^{-3}$, $x > 0$

Determine the mode of X

Ans: $f(x) = 4x(1+x^2)^{-3}$

$f'(x) = 4(1+x^2)^{-3} + 4x(-3)(1+x^2)^{-4}(2x)$

$= 4(1+x^2)^{-3} - 24x^2(1+x^2)^{-4}$

Setting the derivative equal to zero

$4(1+x^2)^{-3} - 24x^2(1+x^2)^{-4} = 0 \quad \times (1+x^2)^4$

$4(1+x^2) - 24x^2 = 0$

$1 + x^2 - 6x^2 = 0$

$5x^2 = 1$

$\therefore x = \pm \frac{1}{\sqrt{5}}$

The only positive solution is the mode of $1/\sqrt{5}$ #

Pb 2.4 p.17

A nonnegative random variable has a hazard rate function of $h(x) = A + e^{2x}$, $x \geq 0$. You are also given $S(0.4) = 0.5$. Determine the value of A.

Ans:

$\because S(b) = e^{-\int_0^b h(x)\,dx}$

$\therefore 0.5 = S(0.4) = e^{-\int_0^{0.4} [A + e^{2x}]\,dx}$

$0.5 = e^{-Ax - 0.5e^{2x} \Big|_0^{0.4}}$

$\therefore e^{-0.4A - 0.5e^{0.8} + 0.5} = 0.5$

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By taking logarithms for both sides,

$$-0.4A - 0.5e^{0.8} + 0.5 = \ln 0.5$$

$$\therefore \boxed{A = 0.2009}$$

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X has a pareto distribution with parameter $\alpha = 2$ and $\theta = 10{,}000$. Y has a Burr distribution with parameters $\alpha = 2$, $\gamma = 2$ and $\theta = \sqrt{20{,}000}$. Let r be the ratio of

$$pr(X > d) \text{ to } pr(Y > d).$$

Determine $\lim\limits_{d \to \infty} r$.

Ans: $X \sim$ pareto (α, θ) (pareto II, lomax)

$$\Rightarrow f(x) = \frac{\alpha\theta^{\alpha}}{(x+\theta)^{\alpha+1}},$$ See p. 465

$$S(x) = \left(\frac{\theta}{x+\theta}\right)^{\alpha}$$

$$pr(X > x) = S(x) = \left(\frac{\theta}{x+\theta}\right)^{\alpha}$$

$$pr(X > d) = \left(\frac{10{,}000}{d + 10000}\right)^{2} \quad (1)$$

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$$Y \sim \text{Burr}(\alpha, \theta, \gamma)$$

, See P. 464

$$\Rightarrow f(y) = \frac{\alpha \gamma (y/\theta)^{\gamma}}{x\left[1+(y/\theta)^{\gamma}\right]^{\alpha+1}}$$

$$S(y) = \left[\frac{1}{1+(y/\theta)^{\gamma}}\right]^{\alpha}$$

$$pr(Y>d) = S(d)$$

$$pr(Y>d) = \left[\frac{1}{1+(d/\sqrt{20,000})^{2}}\right]^{2}$$

$$pr(Y>d) = \left(\frac{20,000}{20,000+d^{2}}\right)^{2} \qquad (2)$$

(1), (2) $\Rightarrow$

$$r = \frac{\left(\frac{10,000}{10,000+d}\right)^{2}}{\left(\frac{20,000}{20,000+d^{2}}\right)^{2}}$$

$$r = \frac{1}{4}\left(\frac{20,000+d^{2}}{10,000+d}\right)^{2} = \left(\frac{20,000+d^{2}}{20,000+2d}\right)^{2}$$

$$r = \frac{(20,000)^{2}+40,000d^{2}+d^{4}}{(20,000)^{2}+80,000d+4d^{2}}$$

$$\frac{\div d^{4}}{\div d^{4}}$$

$$\lim_{d\to\infty} r = \lim_{d\to\infty} \frac{\frac{(20,000)^{2}}{d^{4}} + \frac{40,000}{d^{2}} + 1}{\frac{(20,000)^{2}}{d^{4}} + \frac{80,000}{d^{3}} + \frac{4}{d^{2}}}$$

$$= \infty$$

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H.W Pb 2.2 P. 17 Textbook.