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Tut. Session 3

Revis. Lectures 8, 9 and 10

pb 4.3 p. 57

* Claims have a pareto distribution with $\alpha=2$ and θ unknown. Claims the following year are experience 6% uniform inflation. Let r be the ratio of the proportion of claims that will exceed d next year to the proportion of claims that exceed d this year. Determine the limit of r as d goes to infinity.

Ans:

For two parameter pareto distⁿ — α, θ

$$F(x) = 1-\left(\frac{\theta}{x+\theta}\right)^{\alpha}$$

$$\Rightarrow F(x) = 1-\left(\frac{\theta}{x+\theta}\right)^{2} \quad \text{at } \alpha=2, \theta$$

$$\Rightarrow pr(X>d) = 1-F(d) = \left(\frac{\theta}{d+\theta}\right)^{2} \qquad (1)$$

For next year, ∵ θ is a scale parameter

$$\theta \rightarrow 1.06\theta$$

$$pr(X>d) = \left(\frac{1.06\theta}{d+1.06\theta}\right)^{2} \qquad (2)$$

(1), (2)

$$\Rightarrow r = \frac{(1.06\cancel{\theta})^2}{(d+1.06\theta)^2} \Big/ \frac{\cancel{\theta^2}}{(d+\theta)^2}$$

$$\lim_{d\to\infty} r = \lim_{d\to\infty} \frac{(1.06)^2(d+\theta)^2}{(d+1.06\theta)^2}$$

$$\lim_{d\to\infty} r = \lim_{d\to\infty} \frac{1.1236d^2+2.2472\theta d+1.1236\theta^2}{d^2+2.12\theta d+1.1236\theta^2} \quad \begin{matrix}\div d^2 \\ \div d^2\end{matrix}$$

$$\therefore \lim_{d\to\infty} r = 1.1236$$

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pb 4.6 p. 57

Which of Models 1-5 could be considered to be from parametric distribution families? Which could be considered to be from variable-component mixture distributions?

Ans:

- For Model 1 $f_1(x) = 0.01, \quad 0 \le x < 100$

We couldn't consider it as being from a parametric distn family (although the uniform distn could be considered as a special case of the beta distn)

- For Model 2 $f_2(x) = \frac{3(2{,}000)^3}{(x+2{,}000)^4}, \quad x > 0$

$\Rightarrow$ Model 2 is a pareto distn and it's a member of the transformed beta family.

- For Model 3

$$f_3(x) = \begin{cases} 0.50, & x = 0 \\ 0.25, & x = 1 \\ 0.12, & x = 2 \\ 0.08, & x = 3 \\ 0.05, & x = 4 \end{cases}$$

$\because f_3(x)$ is defined at 0, 1, 2, 3 and 4

So, Model 3 could be considered only as a member of a discrete parametric distn family defined on a given number of non negative integers.

- For Model 4

$$f_4(x) = \begin{cases} 0.7, & x = 0 \\ 0.000003\,e^{-0.00001x} \end{cases}$$

Model 4 could be a member of the "exponential plus family" where the plus means the possibility of discrete probability at zero.

- For Model 5

$$f_5(x) = \begin{cases} 0.01, & 0 \le x < 50 \\ 0.02, & 50 \le x < 75 \end{cases}$$

Model 5 couldn't be a member of a parametric distn family but Model 5 could be variable-component mixture distribution (or, Mixture of uniform distributions).

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pb 4.4 p. 57

Determine the mean and second moment of the two-point mixture distribution in Example 4.4. The solution to this exercise provides general formulas for raw moments of a mixture distribution.

Ans: For general liability insurance models, the mixture of 2 paretos distns could be written as:

$$F(x) = 1 - a\left(\frac{\theta_1}{\theta_1 + x}\right)^{\alpha} - (1-a)\left(\frac{\theta_2}{\theta_2 + x}\right)^{\alpha+2} \quad (1)$$

1st comp. θ_1, α 2nd comp. $\rightarrow \theta_2, \alpha+2$

, see EX 4.4 p. 53

The m th moment of a k-point mixture distn is

$$E(Y^m) = \int y^m \left[a_1 f_{X_1}(y) + \dots + a_k f_{X_k}(y)\right] dy$$

$$E(Y^m) = a_1 E(Y_1^m) + \dots + a_k E(Y_k^m)$$

at $m = 1$ & two point Mixture distn

$$E(Y) = aE(Y_1) + (1-a)E(Y_2) \quad (2)$$

∵ For Pareto (α, θ)

$$E(X^k) = \frac{\theta^k k!}{(\alpha-1)\dots(\alpha-k)}$$, see p. 465, k is +ve integer

$$\Rightarrow E(X) = \frac{\theta}{\alpha - 1}, \alpha > 1 \ (*) \quad \text{and} \quad E(X^2) = \frac{2\theta^2}{(\alpha-1)(\alpha-2)}, \alpha > 2 \ (**)$$

(*), (1) → in (2)

$$\therefore E(Y) = a\frac{\theta_1}{\alpha-1} + (1-a)\frac{\theta_2}{\alpha+2-1}$$

$$\therefore E(Y) = a\frac{\theta_1}{\alpha-1} + (1-a)\frac{\theta_2}{\alpha+1}, \quad \alpha > 1$$

Similarly, for second moment

$m = 2$ & two point Mixture distn

(**), (1) → in (3)

$$E(Y^2) = aE(Y_1^2) + (1-a)E(Y_2^2) \quad (3)$$

$$\therefore E(Y^2) = a\frac{2\theta_1^2}{(\alpha-1)(\alpha-2)} + (1-a)\frac{2\theta_2^2}{(\alpha+2-1)(\alpha+2-2)}$$

$$\therefore E(Y^2) = a\frac{2\theta_1^2}{(\alpha-1)(\alpha-2)} + (1-a)\frac{2\theta_2^2}{(\alpha+1)\alpha}, \quad \alpha > 2$$

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4

pb 4.12 p. 57

Show that the Weibull distribution has a scale parameter

Ans:

For 2 parameter Weibull distn — θ, τ

$$F_X(x) = 1 - e^{-(x/\theta)^{\tau}}$$

See p. 469

For Weibull — θ, τ

$$F(x) = 1 - e^{-(x/\theta)^{\tau}}$$

let $Y = cX$, where $c > 0$

then $F_Y(y) = pr(Y \leq y)$

$$F_Y(y) = pr(cX \leq y) = pr\left(X \leq \frac{y}{c}\right)$$

$\therefore F_Y(y) = 1 - e^{-(y/c\theta)^{\tau}}$ which is also, a Weibull distribution, with parameters τ and $c\theta$

$\therefore \theta$ is a scale parameter.

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H.W (Assignment # 4 pb. 4.5 p. 57)

" Mixture of gammas distribution "

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