## Tut. Sess. 4 : Ch 5

Pb 5.1 p. 68 Textbook

Let $X$ have cdf $F_X(x) = 1 - (1+x)^{-\alpha}$, $x, \alpha > 0$. Determine the pdf and cdf of $Y = \theta X$.

Ans:

$$F_Y(y) = 1 - \left(1 + \frac{y}{\theta}\right)^{-\alpha}$$

$F_Y(y) = F_X\left(\frac{y}{\theta}\right)$ for $Y = \theta X$ theorem

$F_Y(y) = 1 - \left(\frac{\theta}{\theta+y}\right)^{\alpha}$ which is the cdf of the pareto dist$\underline{n}$

$\Rightarrow$

$$\text{pdf is } f_Y(y) = \frac{d}{dy}\left[F_Y(y)\right]$$

$$= -\alpha\left(\frac{\theta}{\theta+y}\right)^{\alpha-1}\left[\frac{-\theta}{(\theta+y)^2}\right]$$

$$f_Y(y) = \frac{\alpha\theta^{\alpha}}{(\theta+y)^{\alpha+1}}$$

, See p. 465 Appendix A

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Ans:

| 0 – 300 | 300 – 350 | 350 – 400 | 400 – 450 | 450 – 500 |
|---|---|---|---|---|
| 42 | 3 | 5 | 5 | 0 |

| 500 – 600 | 600 – |
|---|---|
| 5 | 40 |

For the next three years, all claims are inflated by 10% per year

In 1996 $\longrightarrow 1.1X$

In 1997 $\longrightarrow 1.21X$

In 1998 $\longrightarrow 1.331X$

where $X$ be the r.v of the claim in 1995 and $\boxed{Y = 1.331X}$ be the r.v of the claim in 1998

$Pr(Y > 500)$ ?? المطلوب The required

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$$pr(Y > 500) = pr(X > 500/1.331) = pr(X > 376)$$

From given data, $pr(X > 350) = \frac{55}{100} = 0.55$ and

$$pr(X > 400) = \frac{50}{100} = 0.50$$

$\therefore$ the required probability must be between two values 0.50 and 0.55.

i.e $0.50 < pr(Y > 500) < 0.55$

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Let X have a pareto distribution. Determine the cdf of the inverse, transformed, and inverse transformed distributions. Check Appendix A to determine if any of these distributions have special names.

Ans:

$$Y = X^{1/\tau}$$

* Revise Lecture 12

- $\tau > 0$: transformed distn
- $\tau = -1$: inverse distn
- negative $\tau$: inverse transformed distn

For pareto $\alpha, \theta$ distn,

$$F_X(x) = 1 - \left(\frac{\theta}{x+\theta}\right)^{\alpha} \qquad (1)$$

* For $\tau > 0$, $F_Y(y) = F_X(y^{\tau})$ Theorem

$$\therefore F_Y(y) = 1 - \left(\frac{\theta}{y^{\tau}+\theta}\right)^{\alpha}$$

$$F_Y(y) = 1 - \left(\frac{\theta}{\theta + y^{\tau}}\right)^{\alpha} = 1 - \left(\frac{1}{1+(y/\theta^{1/\tau})^{\tau}}\right)^{\alpha} \qquad (2)$$

which is the Burr distn with parameters $\alpha, \theta^{1/\tau}, \tau$

See p. 464 Appendix A

For $\tau = -1$

$$F_Y(y) = 1 - F_X(y^{-1}) \quad \text{Theorem}$$

$$① \Rightarrow \quad F_Y(y) = \cancel{1} - \left[\cancel{1} - \left(\frac{\theta}{y^{-1}+\theta}\right)^{\alpha}\right]$$

$$= \left(\frac{\theta}{y^{-1}+\theta}\right)^{\alpha} \qquad \frac{\times y\theta^{-1}}{\times y\theta^{-1}}$$

$$\therefore F_Y(y) = \left(\frac{y}{y+\theta^{-1}}\right)^{\alpha}$$

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which is the inverse Pareto dist$\underline{n}$ with parameters $\alpha, \theta^{-1}$

For negative $\tau$ (Revise formula (*) in Lecture 12)

$$F_Y(y) = 1 - F_X(y^{-\tau})$$

$$① \Rightarrow \quad F_Y(y) = \cancel{1} - \left[\cancel{1} - \left(\frac{\theta}{\theta+y^{-\tau}}\right)^{\alpha}\right]$$

$$= \left(\frac{\theta}{\theta+y^{-\tau}}\right)^{\alpha} \qquad \frac{\times y^{\tau}\theta^{-1}}{\times y^{\tau}\theta^{-1}}$$

$$= \left(\frac{y^{\tau}}{y^{\tau}+\theta^{-1}}\right)^{\alpha}$$

$$= \left(\frac{y^{\tau}}{y^{\tau}+(\theta^{-1/\tau})^{\tau}}\right)^{\alpha}$$

$$\therefore F_Y(y) = \left[\frac{(y/\theta^{-1/\tau})^{\tau}}{1+(y/\theta^{-1/\tau})^{\tau}}\right]^{\alpha}$$

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which is the inverse Burr dist$\underline{n}$ with three parameters $\alpha, \theta^{-1/\tau}, \tau$