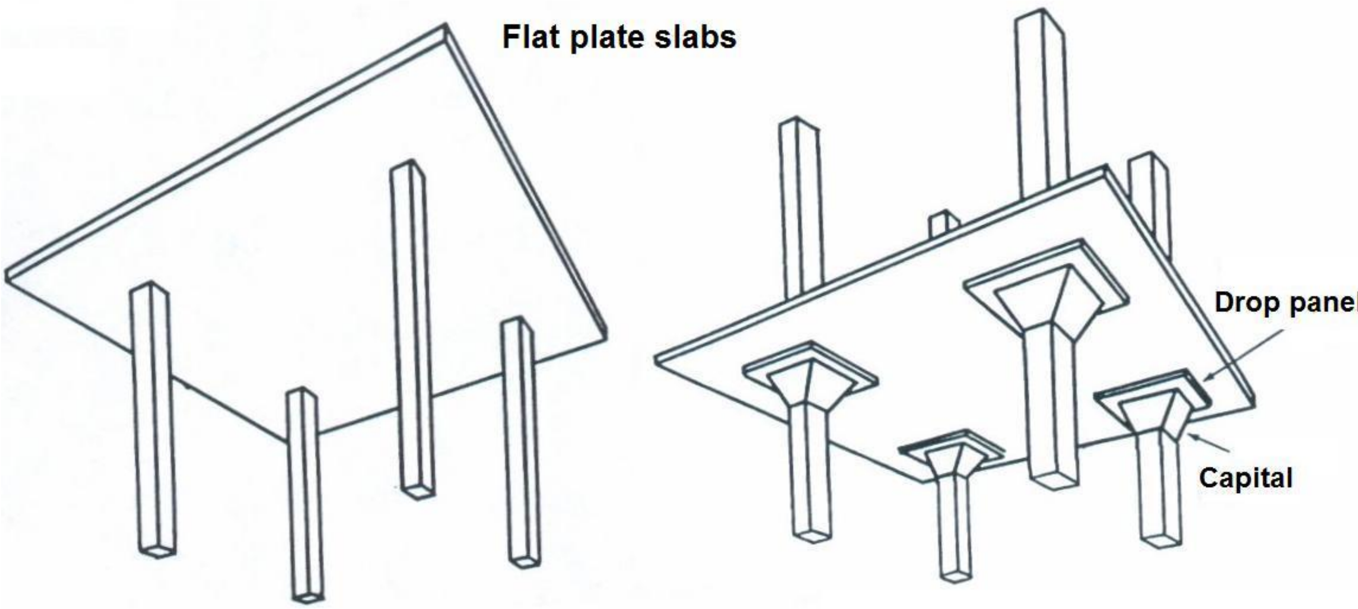
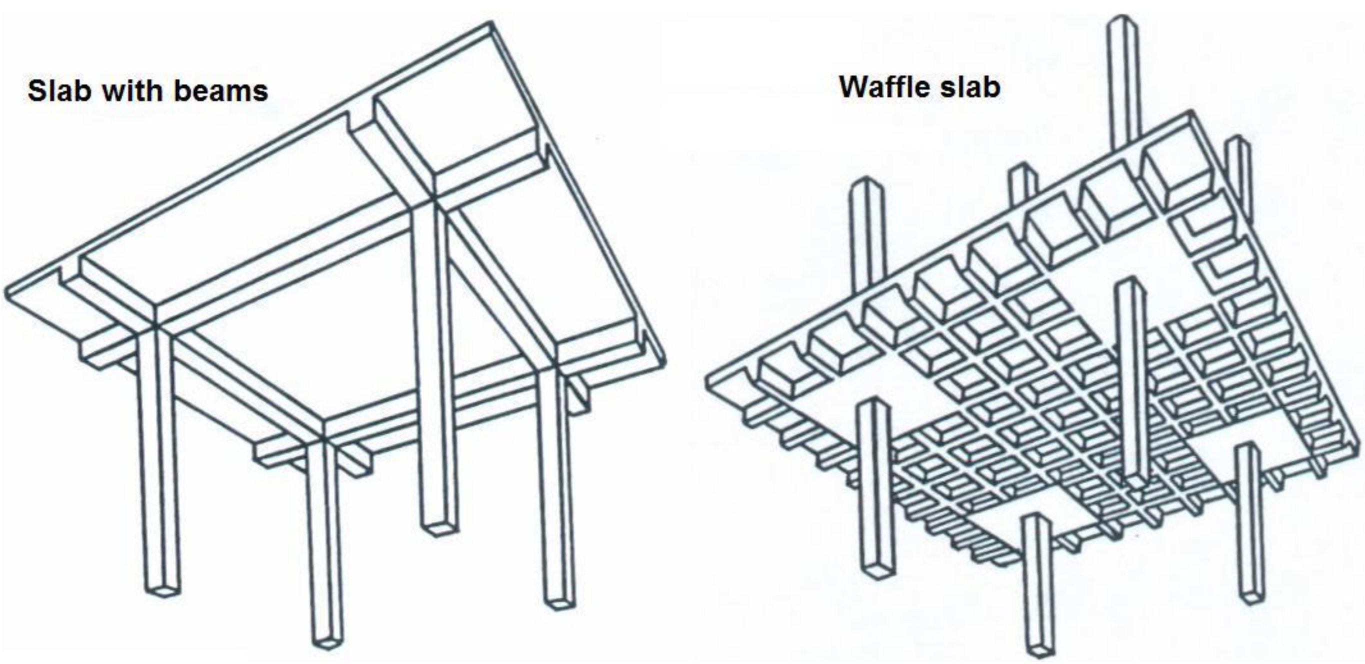
Two-way slabs

Two-way slab behavior is described by plate bending theory which is a complex extension of beam bending. Codes of practice allow use of simplified methods for analysis and design of two-way slabs.

This chapter will first cover the flat plate which is a slab supported directly by columns (without beams). Slabs with beams will be studied after. Columns in a flat plate may have drop panels or / and capitals.



Flat plate with or without drop panels / capitals



Flat Plate: Span not exceeding 6.0 to 7.5 m and live load not exceeding 3.5 to 4.5 kN/m^2

Advantages

- Low cost formwork
- Exposed flat ceilings
- Fast

Disadvantages

- Low shear capacity
- Low Stiffness (notable deflection)
- Need of special formwork for drop panels and capitals

Waffle Slab: Span up to 14 m and live load up to 7.5 kN/m^2

Advantages

- Carries heavy loads
- Attractive exposed ceilings

Disadvantages

- Formwork with panels is expensive

Slab with beams: Span up to 10 m

Advantages

- Versatile
- Framing of beams with columns

Disadvantages

- Visibility of drop beams in ceilings

Methods of analysis of slabs

Direct Design Method DDM (object of the course)

Equivalent Frame Method EFM

Yield Line Method YLM

Finite Element Method FEM (most powerful)

Part A: Flat plate

Thickness of two way slabs

Table 13.1: Minimum thickness for slabs without interior beams

f_y (MPa)	Without drop panels			With drop panels		
	Exterior panel		Internal panel	Exterior panel		Internal panel
	No edge beams	With edge beams		No edge beams	With edge beams	
300	L_n / 33	L_n / 36	L_n / 36	L_n / 36	L_n / 40	L_n / 40
420	L_n / 30	L_n / 33	L_n / 33	L_n / 33	L_n / 36	L_n / 36
520	L_n / 28	L_n / 31	L_n / 31	L_n / 31	L_n / 34	L_n / 34

L_n is the maximum clear length of the panel.

Linear interpolation must be performed for intermediate values of steel grade f_y.

The thickness of slabs with interior beams is given by equations to be seen later.

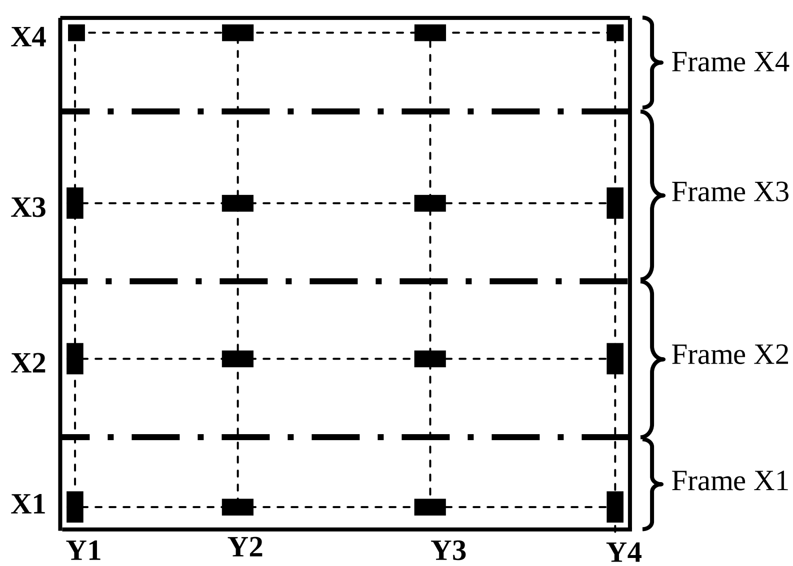
ACI, SBC and other codes of practice allow the use of simplified methods for the analysis and design of two way slabs. Among these is the Direct Design Method.

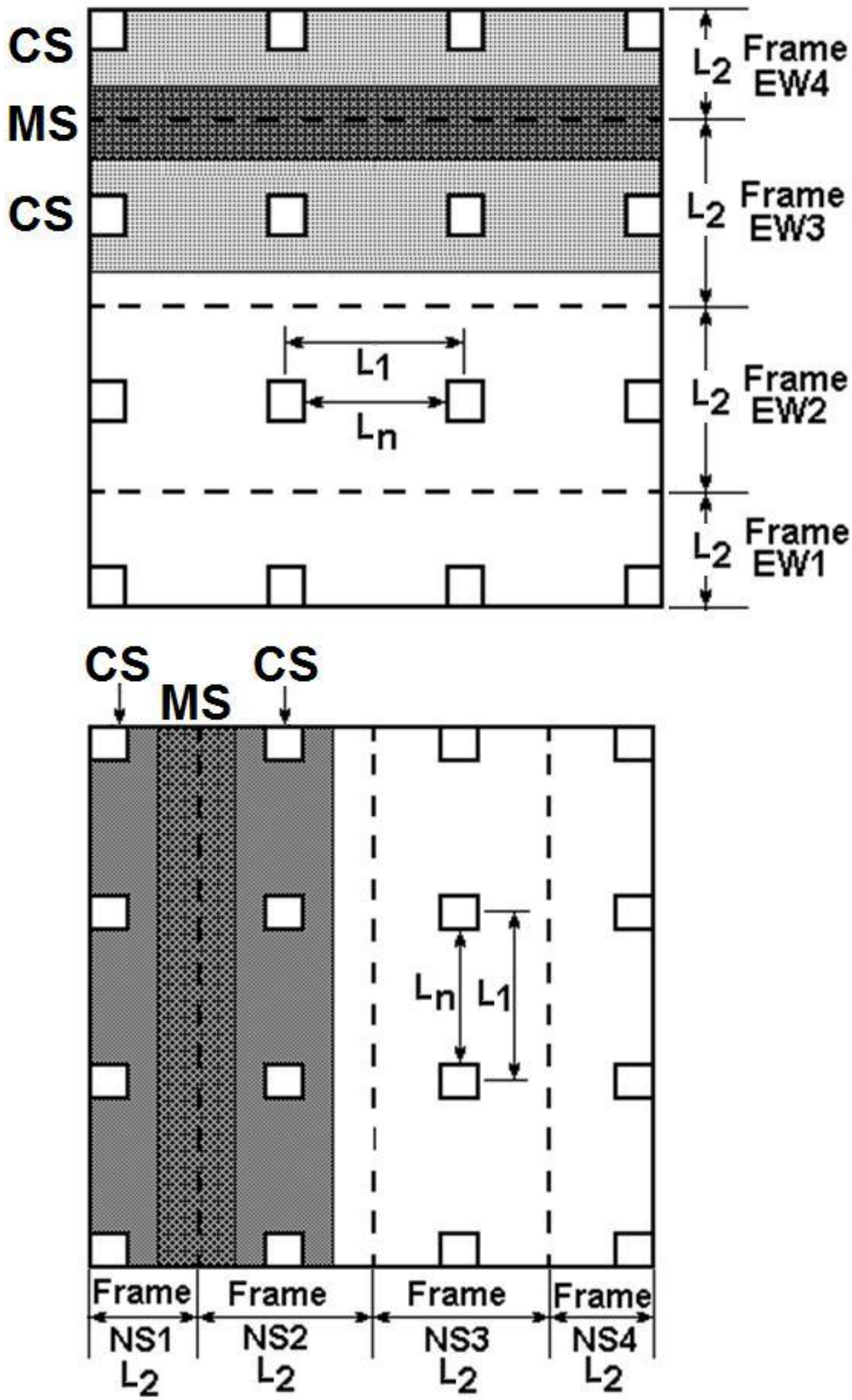
Direct Design Method DDM

The procedure consists first in dividing the slab in each direction into frames using mid-lines, then compute in each span, the static moment, as well as positive and negative moments using appropriate coefficients. These moments are then distributed across the frame width, based on geometry and stiffness.

The various parts (column strips, middle strips and possible beams) are then designed.

The next figure shows the frames in X-direction. The frames in Y-direction are obtained similarly.





Subdivision in frames in both directions

Panel dimensions, Column strip CS and middle strip MS

L_1 is the panel dimension in the studied direction.

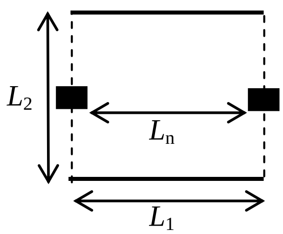
L_2 is the dimension in the other direction (transverse width of the frame).

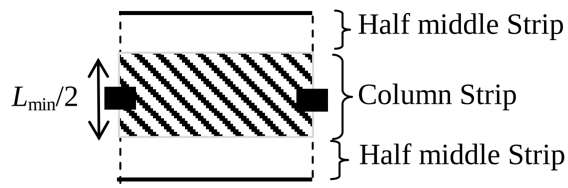
L_n is the clear length in the studied direction.

Moments vary across the width of the frame in each span.

Each span of the frame is composed of a **column strip** (containing the column line) and a **middle strip**

The column strip width is equal to half the minimum length.





$L_{min} = \text{Min}(L_1\ ,\ L_2)$

When computing the clear length L_n with respect to a circular column, the latter is replaced by an equivalent square section with the same area.

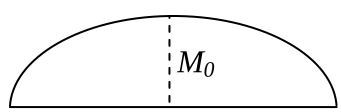
The moment distribution along the frame is deduced from the values of the static moments.

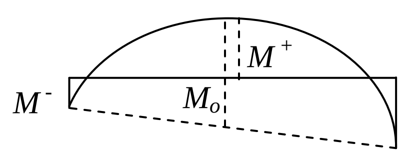
For each span, the positive and negative moments are function of the static moment equal to:

$M_0 = \dfrac{wL_n^2}{8}$ where w is the uniform line load (kN/m)

If w_s is the slab area load (kN/m^2), the beam load would be: $w = w_s L_2$

The static moment for the panel is therefore: $M_0 = w_s L_2 \dfrac{L_n^2}{8}$





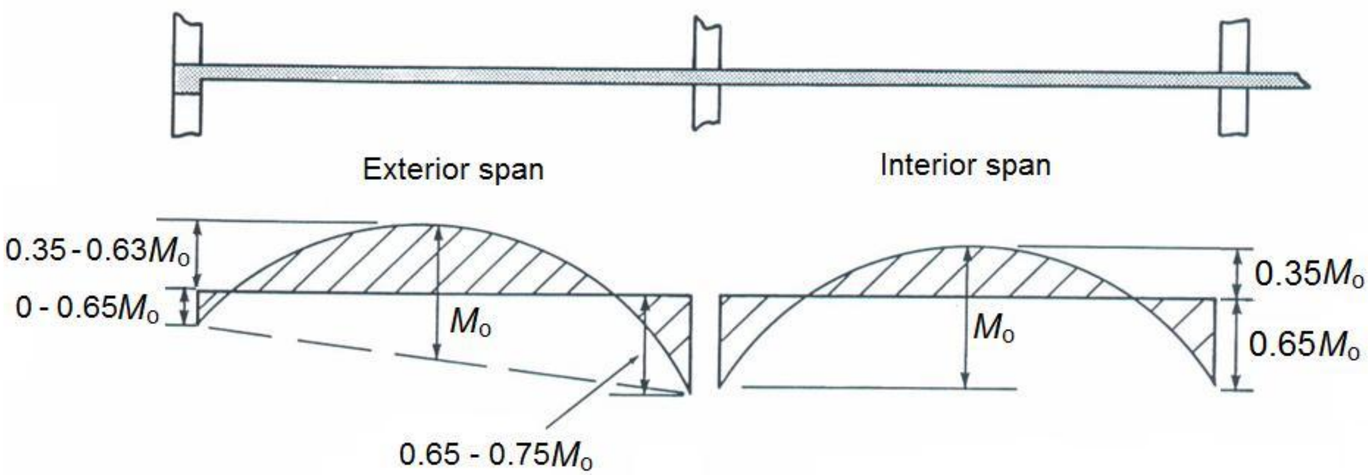
Positive and negative moments in each span are deduced according to the following table.

Table 13.2: Distribution of factored static moment *M₀*

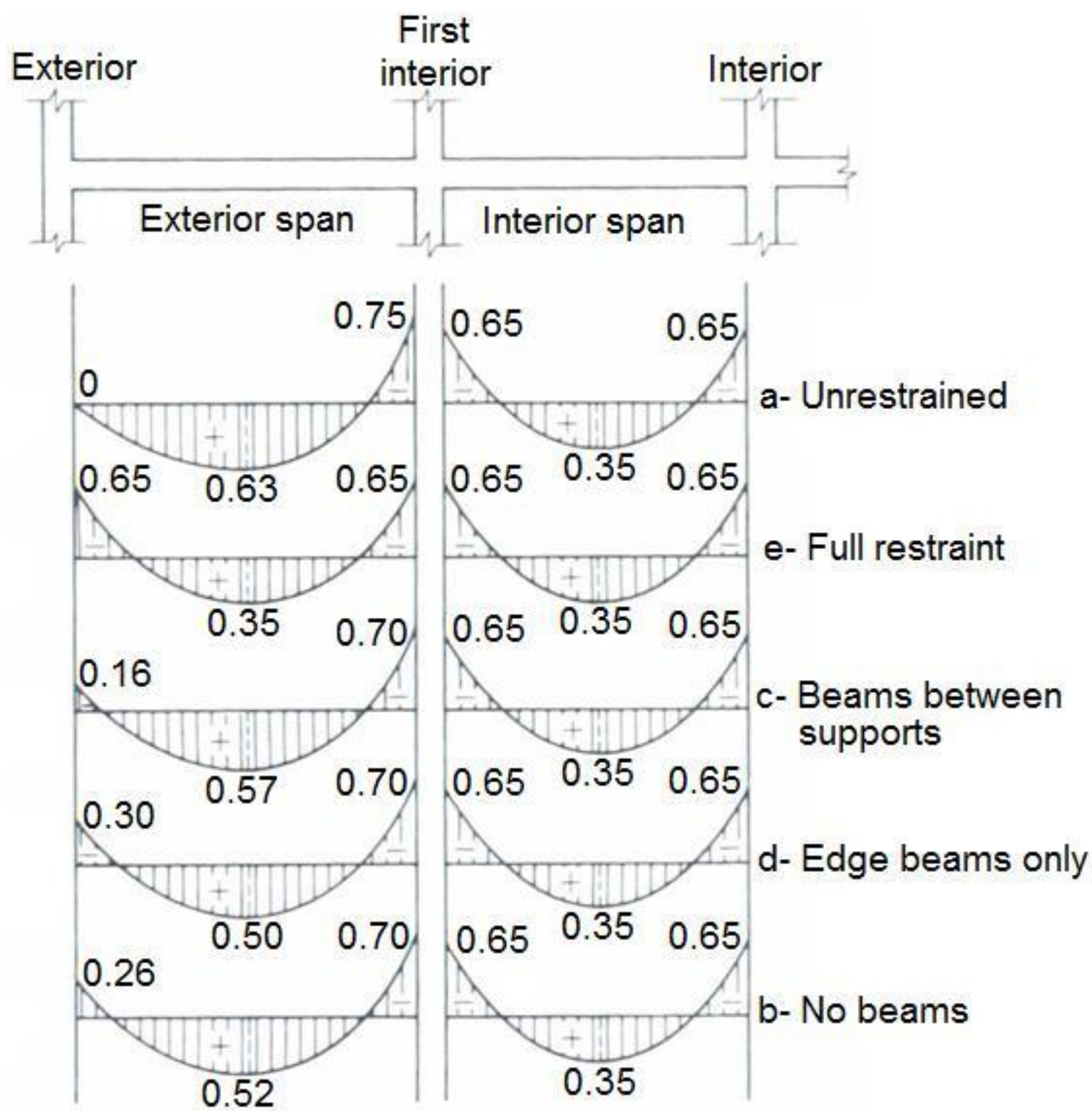
	(a)	**(b)**	**(c)**	**(d)**	**(e)**
Int. negative M	0.75	0.70	0.70	0.70	0.65
Positive M	0.63	0.57	0.52	0.50	0.35
Ext. negative M	0	0.16	0.26	0.30	0.65

(a): Exterior edge unrestrained **(b):** Slab with beams between all supports
(c): Slab with no beams at all **(d):** Slab with edge beams only
(e): Slab with exterior edge fully restrained (interior span)

In general: $\dfrac{\left|M_L^-\right| + \left|M_R^-\right|}{2} + M^+ \approx M_0 = w_s L_2 \dfrac{L_n^2}{8}$



For a typical interior panel, the total static moment is divided into positive moment 0.35M_o and negative moment of 0.65M_o.



Distribution of moments over column strips and middle strips

The following table shows the portions (%) of moments carried by column strips and middle strips.

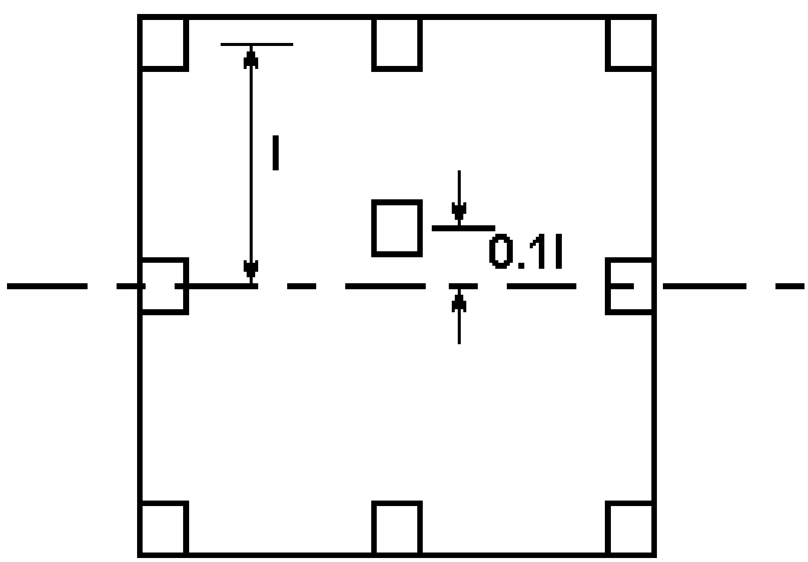
Moments in column strip and middle strip (%)

	Interior negative moment	Positive moment	Exterior negative moment
Column strip	**75**	**60**	**100**
Middle strip	**25**	**40**	**0**

Conditions of the DDM

1. Minimum of three spans in each direction. Minimum of 3 x 3 = 9 panels.
2. Rectangular panels with aspect ratio between 0.5 and 2.0 $\quad 0.5 \le \frac{L_{xi}}{L_{yi}} \le 2.0$
3. Successive spans in each direction must not differ by more than one third of the largest span. $|L_i - L_{i-1}| \le \frac{1}{3} Max(L_i, L_{i-1})$
4. Column offset from basic rectangular grid must not exceed 10 % of span in offset direction.
5. Gravity loading
6. Live load less or equal to twice the dead load $\quad LL \le 2DL$
7. For slabs with beams, the relative beam stiffness α must be such: $0.2 \le \frac{\alpha_1 (L_2)^2}{\alpha_2 (L_1)^2} \le 5.0$

Beam relative stiffness α to be defined later.

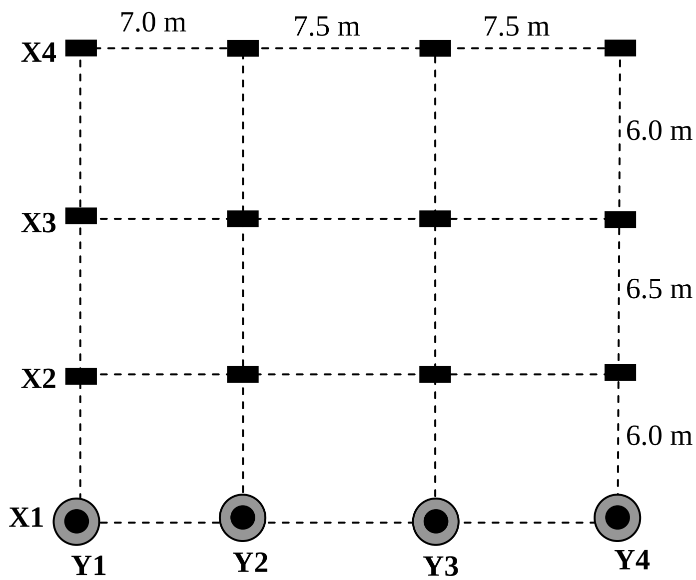


Column offset limited to 10%

Example 1: Thickness check and computation of static moments

A 250 mm slab is subjected to a live load of 4.5 kN/m^2 and a super imposed dead load of 1.0 kN/m^2.

The rectangular column section is 600 x 300 mm. The 300 mm diameter circular columns of line X1 have 600 mm diameter capitals. Check the slab thickness and determine the static moment for span X2Y3-X2Y4 of frame X2 in X-direction and for span Y3X1-Y3X2 for frame Y3 in Y-direction.



The ultimate slab load is $w_s = 1.4(24\,x\,0.250 + 1.0) + 1.7\,x\,4.5 = 17.45\,kN/m^2$

For columns with capitals, the clear length (used for minimum thickness and static moment) must be computed between capital faces and by replacing the circular capital by an equivalent square one with the same area.

$\pi \dfrac{D^2}{4} = c^2$ Thus $c = D\dfrac{\sqrt{\pi}}{2} = 0.886D = 0.866\,x\,600 = 532\,mm$

Thickness check

The minimum thickness is $\dfrac{L_n}{30}$ for the eight external panels and $\dfrac{L_n}{33}$ for the internal panel.

For each panel, L_n is the maximum clear length.

The maximum clear length for the external panels with rectangular columns is:

$L_n = 7.5 - \dfrac{0.6}{2} - \dfrac{0.6}{2} = 6.9\,m = 6900\,mm$ This gives a minimum thickness of $\dfrac{6900}{30} = 230\,mm$

The maximum clear length for the external panels with equivalent square capitals is:

$L_n = 7.5 - \dfrac{0.532}{2} - \dfrac{0.532}{2} = 6.968\,m = 6968\,mm$ This gives a minimum thickness of $\dfrac{6968}{30} = 232.27\,mm$

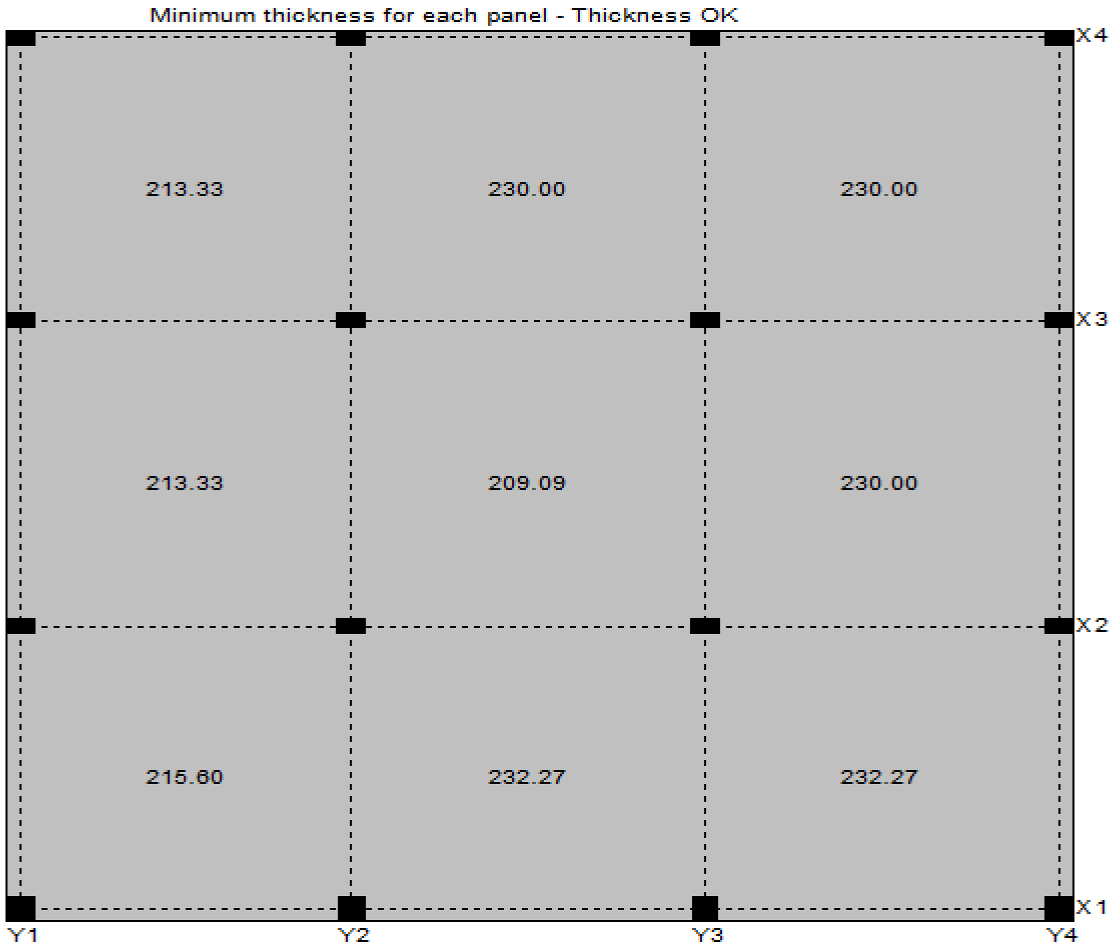
The internal panel has a maximum clear length of $L_n = 7.5 - \frac{0.6}{2} - \frac{0.6}{2} = 6.9\,m = 6900\,mm$

The minimum thickness is $\frac{6900}{33} = 209.09\,mm$

The slab minimum thickness is therefore 232.27 mm.

The actual thickness of 250 mm is thus OK.

The next figure shows the minimum thickness check using RC-SLAB2 software. The value of the minimum thickness is shown for each panel.

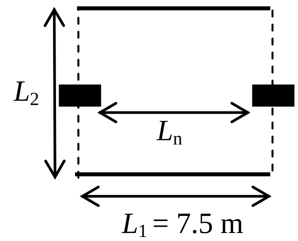


Minimum thickness check using RC-SLAB2 software

showing minimum thickness for each panel

Static moment for span X2Y3-X2Y4

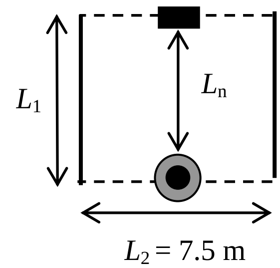
Panel dimensions and static moment are:



$$L_2 = \frac{6}{2} + \frac{6.5}{2} = 6.25\,m \qquad L_n = 7.5 - \frac{0.6}{2} - \frac{0.6}{2} = 6.9\,m$$

$$M_0 = w_s L_2 \frac{L_n^2}{8} = 17.45\,x\,6.25\frac{6.9^2}{8} = 649.06\,kN.m$$

Static moment for span Y3X1-Y3X2



$L_1 = 6.0\,m \qquad L_2 = 7.5\,m$

The clear length must be computed by replacing the circular capital by an equivalent square one with the same area.

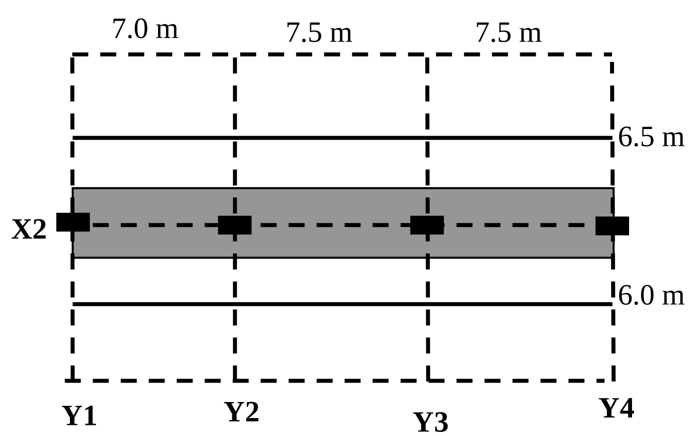
$$\pi\frac{D^2}{4} = c^2 \qquad \text{Thus} \qquad c = D\frac{\sqrt{\pi}}{2} = 0.886D = 0.866\,x\,600 = 532\,mm$$

$$L_n = 6.0 - \frac{0.3}{2} - \frac{0.532}{2} = 5.584\,m \qquad M_0 = w_s L_2 \frac{L_n^2}{8} = 17.45\,x\,7.5\frac{5.584^2}{8} = 510.10\,kN.m$$

Example 2: Moment distribution over column strip and middle strip

Determine moments along strips of frame X2 of the previous example.

The figure shows frame X2 and with the column strips shaded.



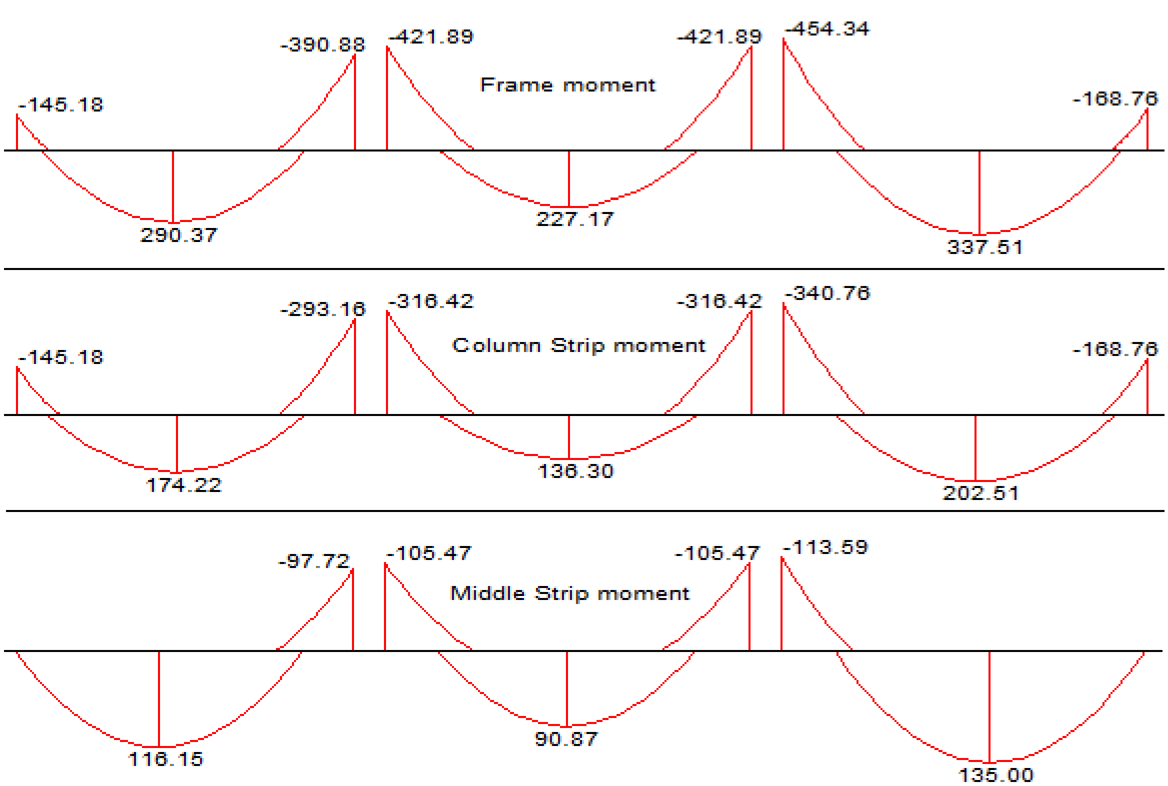
The following table sums all the results including the panel dimensions, static moment, column strip width, positive and negative moments, as well as CS and MS moments.

	Span X2Y1- X2Y 2			Span X2Y2- X2Y3			Span X2Y3- X2Y4		
L_1 (m)	7.0			7.5			7.5		
L_2 (m)	6.25			6.25			6.25		
L_n (m)	6.4			6.9			6.9		
$M_0 = w_s L_2 \frac{L_n^2}{8}$ ($kN.m$)	558.4			649.06			649.06		
L_{min} (m)	6.25			6.25			6.25		
CS width (m) 0.5L_{min}	3.125			3.125			3.125		
Moment coefficients	-0.26	0.52	-0.70	-0.65	0.35	-0.65	-0.70	0.52	-0.26
-ve and +ve moments	-145.2	290.4	-390.9	-421.9	227.2	-421.9	-454.3	337.5	-168.8
CS moment (%)	100	60	75	75	60	75	75	60	100
CS moments ($kN.m$)	-145.2	174.2	-293.2	-316.4	136.3	-316.4	-340.7	202.5	-168.8
MS moment (%)	0	40	25	25	40	25	25	40	0
MS moments ($kN.m$)	0	116.2	-97.7	-105.5	90.9	-105.5	-113.6	135.0	0

The next figure shows RC-SLAB2 output in tabular and graphical forms.

Analysis of Frame X2 Slab ultimate load (kN/m2) = 17.450

Span	X2Y1 - X2Y2			X2Y2 - X2Y3			X2Y3 - X2Y4		
L1 (m)		7.0000			7.5000			7.5000	
L2 (m)		6.2500			6.2500			6.2500	
Ln (m)		6.4000			6.9000			6.9000	
Mo (kN.m)		558.400			649.058			649.058	
CS (m)		3.12500			3.12500			3.12500	
M-Coef	-0.26	0.52	-0.70	-0.65	0.35	-0.65	-0.70	0.52	-0.26
M (kN.m)	-145.18	290.37	-390.88	-421.89	227.17	-421.89	-454.34	337.51	-168.76
CS-M %	100.00	60.00	75.00	75.00	60.00	75.00	75.00	60.00	100.00
CS-M	-145.18	174.22	-293.16	-316.42	136.30	-316.42	-340.76	202.51	-168.76
MS-M	0.00	116.15	-97.72	-105.47	90.87	-105.47	-113.59	135.00	0.00



Analysis of frame X2 using RC-SLAB2 software

(Tabular and graphical output)

RC design of strips

In DDM, strips have known widths. The column and middle strips are designed using the actual strip width *b*. RC design must therefore deliver the total required bar number and not the bar spacing as in one way slabs. Both minimum steel and maximum spacing requirements must be met.

Minimum steel in slabs: $$A_{s\min} = \begin{cases} 0.020\,bh & \text{if } f_y = 300 \text{ to } 350\,MPa \\ 0.0018\,bh & \text{if } f_y = 420\,MPa \\ 0.0018\,bh\dfrac{420}{f_y} & \text{if } f_y > 420\,MPa \end{cases}$$

Maximum spacing in slabs: $S_{\max} = Min(2h_s\ ,\ 300)$.

For a known bar diameter d_b with bar area $A_b = \pi \dfrac{d_b^2}{4}$, maximum spacing is equivalent to another minimum steel area given by: $A_{s\min 2} = \dfrac{A_b b}{S_{\max}}$

The actual reinforcement must be greater than or equal to for both values of minimum steel.

The minimum bar number is therefore $N_{b\min} = \dfrac{Max\left(A_{s\min}\ ,\ A_{s\min 2}\right)}{A_b}$

RC design of the column strip under maximum negative moment:

The maximum internal negative moment in the column strip is 340.7 kN.m

The section dimensions for the column strip are b = 3125 mm and h = 250 mm

Using 18-mm diameter bars, the bar area is $A_b = \pi \dfrac{18^2}{4} = 254.47\,mm^2$

Steel depth is computed as: $d = h - \text{cover} - \dfrac{d_b}{2} = 250 - 20 - \dfrac{18}{2} = 221\,mm$

Assuming $f_c^{'} = 25\,MPa$, $f_y = 420\,MPa$, the required steel area is $A_s = 4349.0\,mm^2$

Minimum steel is $A_{s\min} = 0.0018bh = 0.0018\,x\,3125\,x\,250 = 1406.25\,mm^2$

The maximum spacing is $S_{\max} = Min(2h, 300) = 300\,mm$.

Maximum spacing is equivalent to another minimum steel area:

$$A_{s\min 2} = \frac{A_b b}{S_{\max}} = \frac{254.47\,x\,3125}{300} = 2650.73\,mm^2$$

The minimum bar number is $N_{b\min} = \dfrac{Max\left(A_{s\min}\ ,\ A_{s\min 2}\right)}{A_b} = \dfrac{2650.73}{254.47} = 10.42$ Thus $N_{bmin} = 11$

The actual reinforcement must be greater than or equal to for both values of minimum steel.

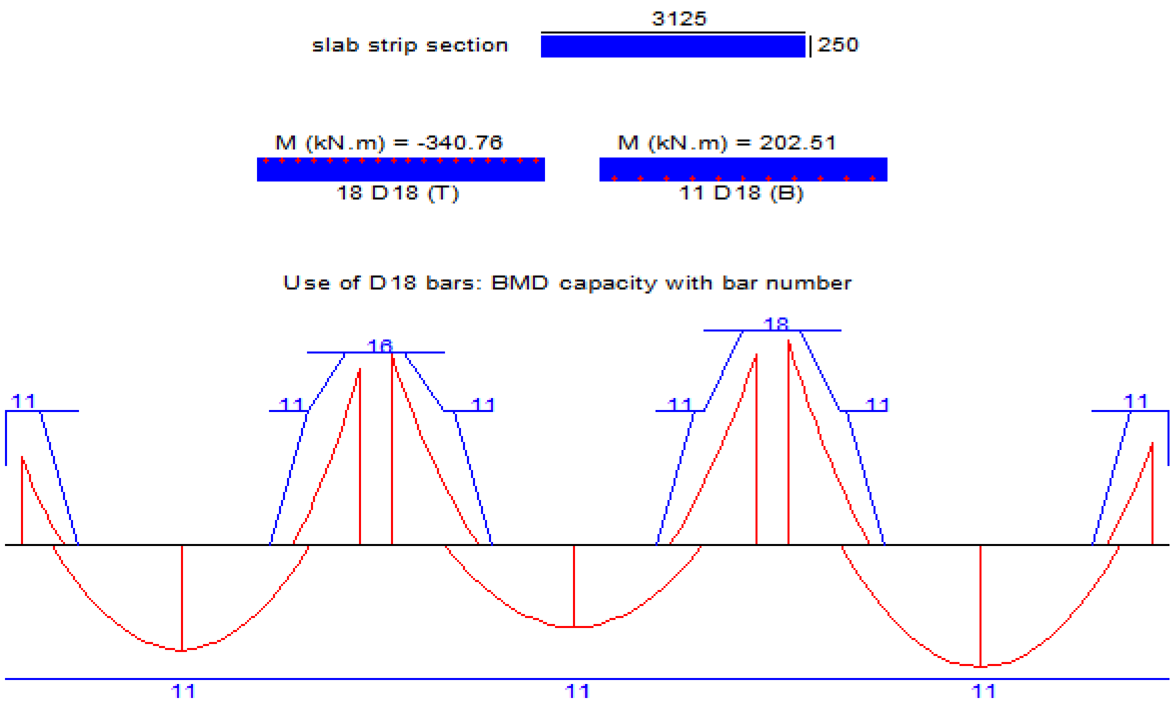
We thus adopt a steel area $A_s = 4349.0\,mm^2$

This gives a number of bars equal to $N_b = \dfrac{As}{A_b} = \dfrac{4349.0}{254.47} = 17.1$

18 bars of 18-mm diameter are therefore required at the maximum internal negative moment.

This is equivalent to a spacing $S = \dfrac{b}{N_b} = 18 = 173.61\,mm$

The next figure shows RC-SLAB2 output for design of the column strip X2.



RC design of the column strip X2 using RC-SLAB2 software

showing capacity moment diagram with bar cutoff

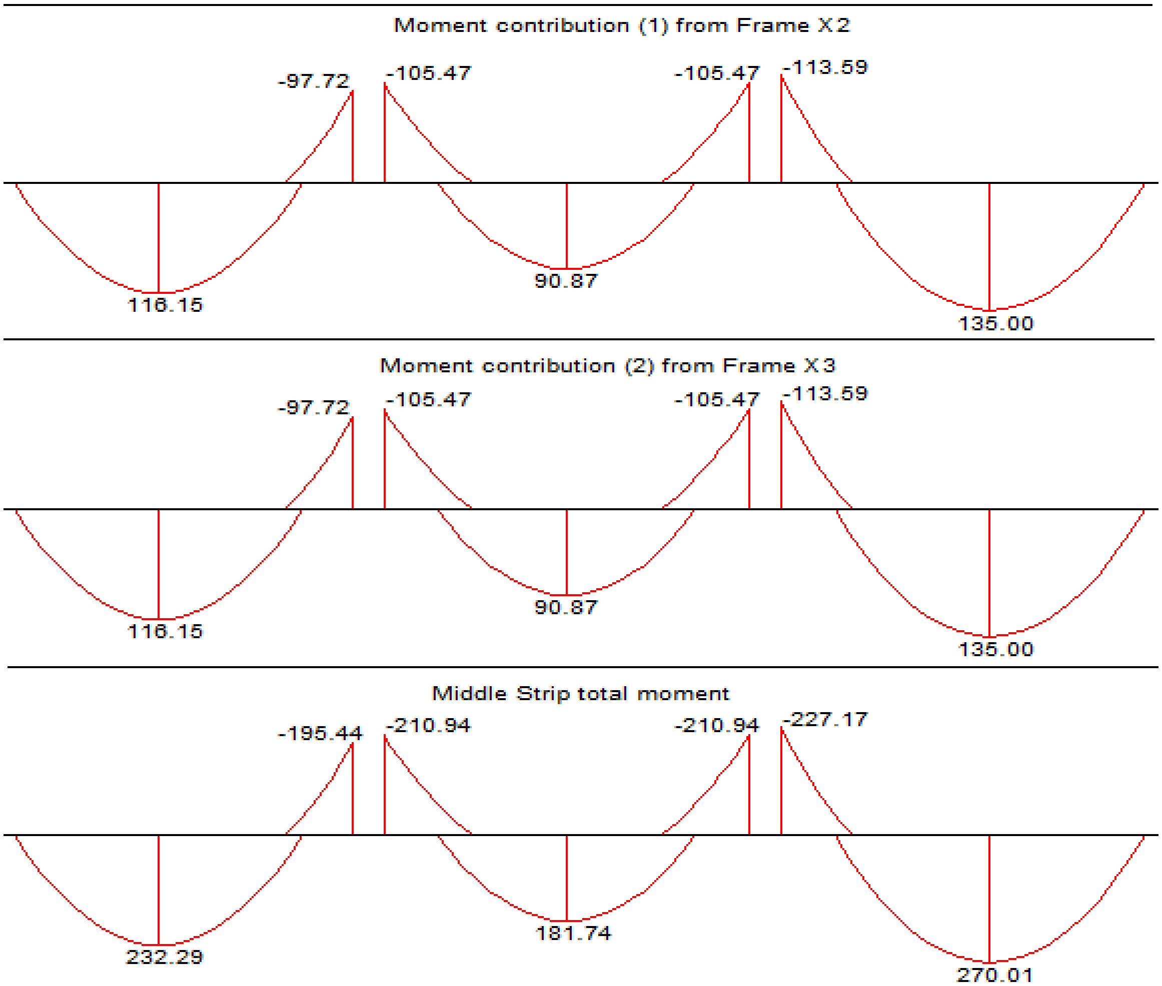
RC design of middle strips

As seen previously, a middle strip has two parts belonging to two adjacent frames. Design of a middle strip requires therefore the analysis of the two adjacent frames and summing both contributions to find the total middle strip moments and widths in all spans.

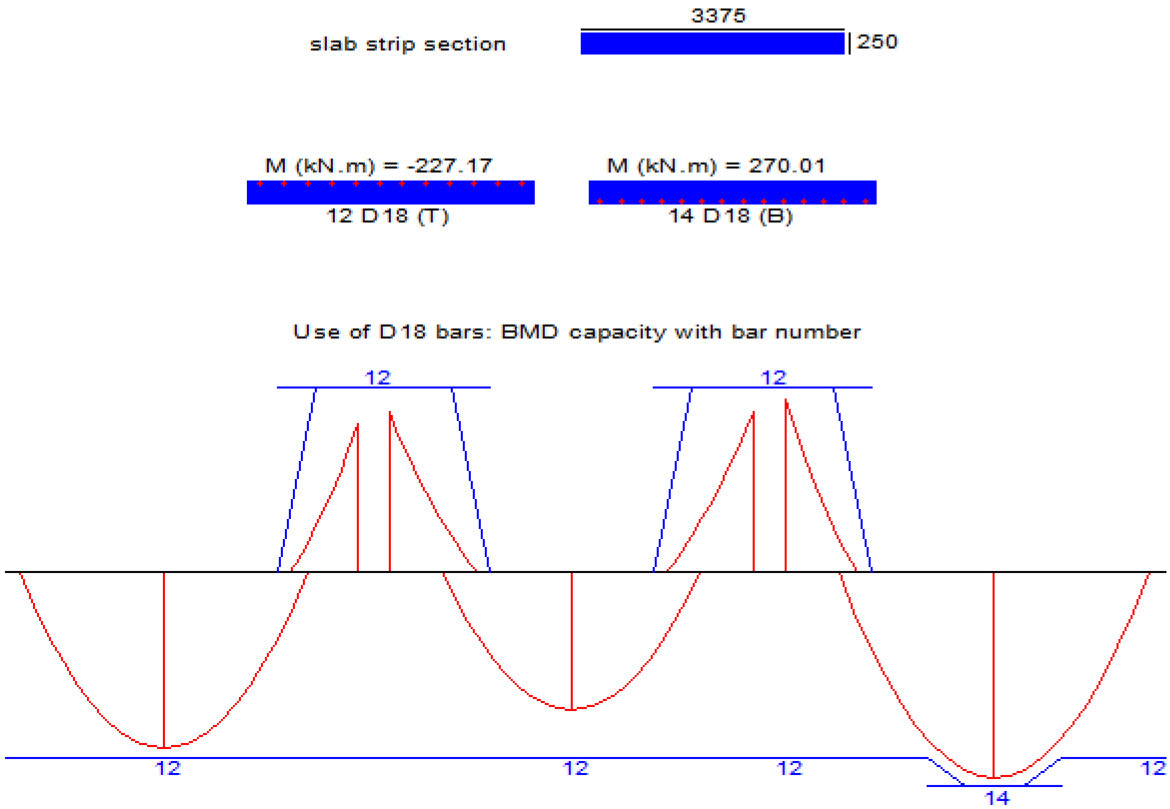
Middle strip X2 in X-direction takes contributions from frames X2 and X3.

Analysis of Middle Strip of Span X2 with two adjacent contributions:
(1): From Frame X2 (2): From Frame X3 (Moments in KN.m)

Span		Y1 - Y2			Y2 - Y3			Y3 - Y4	
MS-1 (m)		1.68750			1.68750			1.68750	
MS-2 (m)		1.68750			1.68750			1.68750	
MS-Tot (m)		3.37500			3.37500			3.37500	
MS-M1	0.00	116.15	-97.72	-105.47	90.87	-105.47	-113.59	135.00	0.00
MS-M2	0.00	116.15	-97.72	-105.47	90.87	-105.47	-113.59	135.00	0.00
MS-Mtot	0.00	232.29	-195.44	-210.94	181.74	-210.94	-227.17	270.01	0.00



Analysis of middle strip X2 using RC-SLAB2 software



RC design of the middle strip X2 using RC-SLAB2 software

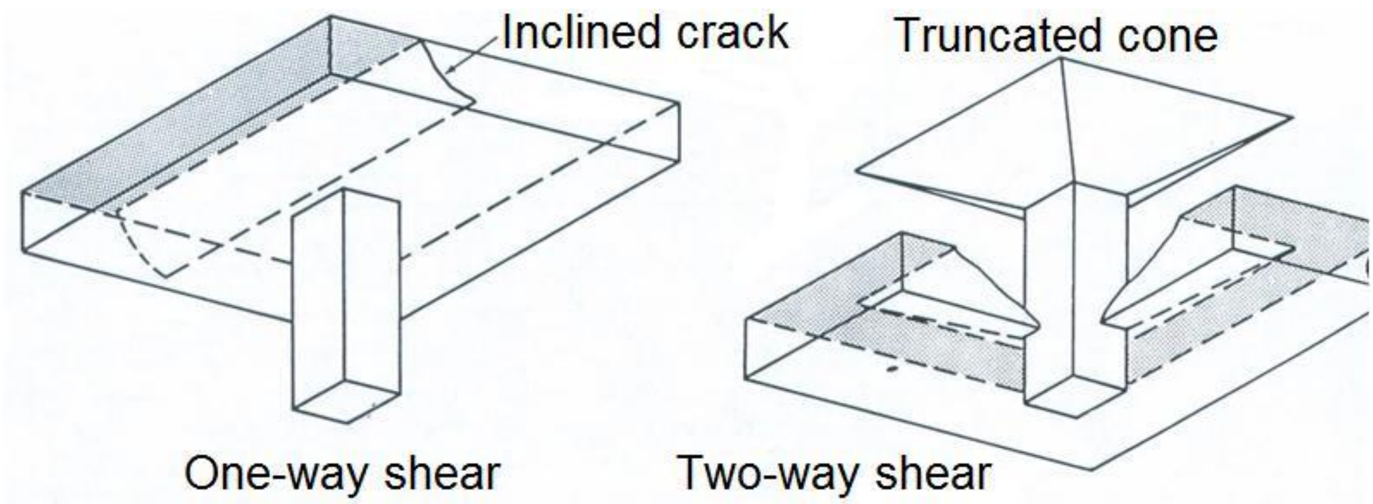
showing capacity moment diagram with bar cutoff

The same steps must be performed for the Y-direction.

Shear strength of two way slabs (flat plate)

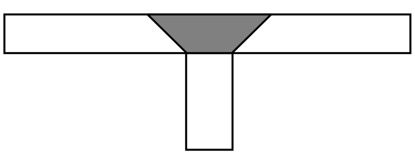
There are two possible shear failure mechanisms:

- One-way shear or beam shear at distance d from the column
- Two-way or punching shear which occurs along a truncated cone at distance $d/2$ from the column.

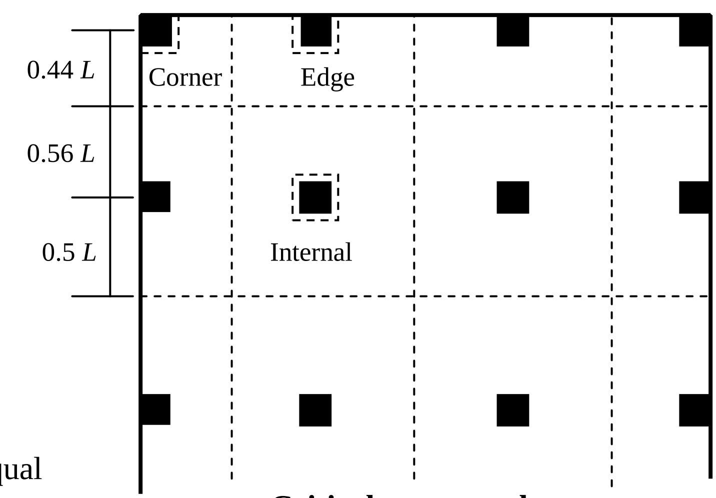


Two-way shear fails along a truncated cone or pyramid around the column. The critical section is located $d/2$ from the column face, column capital, or drop panel.

Forces are transferred between the slab and the column through the conic region as shown.



The tributary areas for shear in a flat plate are shown in the figure with the critical transfer perimeter shown in broken lines for internal, edge and corner columns.

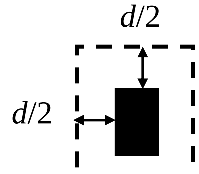


Critical two-way shear zones

The shear panel dimensions are obtained by dividing the first (and last) span in two unequal parts (0.44L and 0.56L), The other spans are halved.

For two-way shear mechanism, the critical perimeter is located at a distance $d/2$ from the column faces, where d is the average steel depth in the slab.

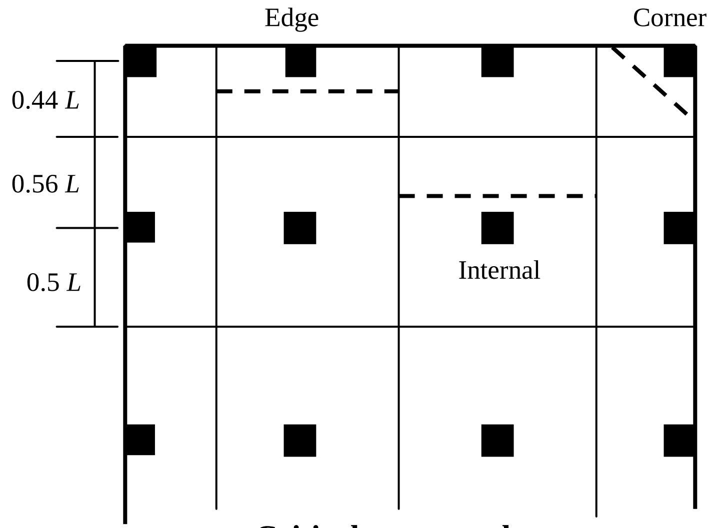
For an internal column the critical perimeter is as shown.



For one-way shear the critical distance is *d*.

Usually two-way shear is most critical for internal and edge columns and one-way shear controls corner columns. It is convenient to check both two-way and one-way shear.

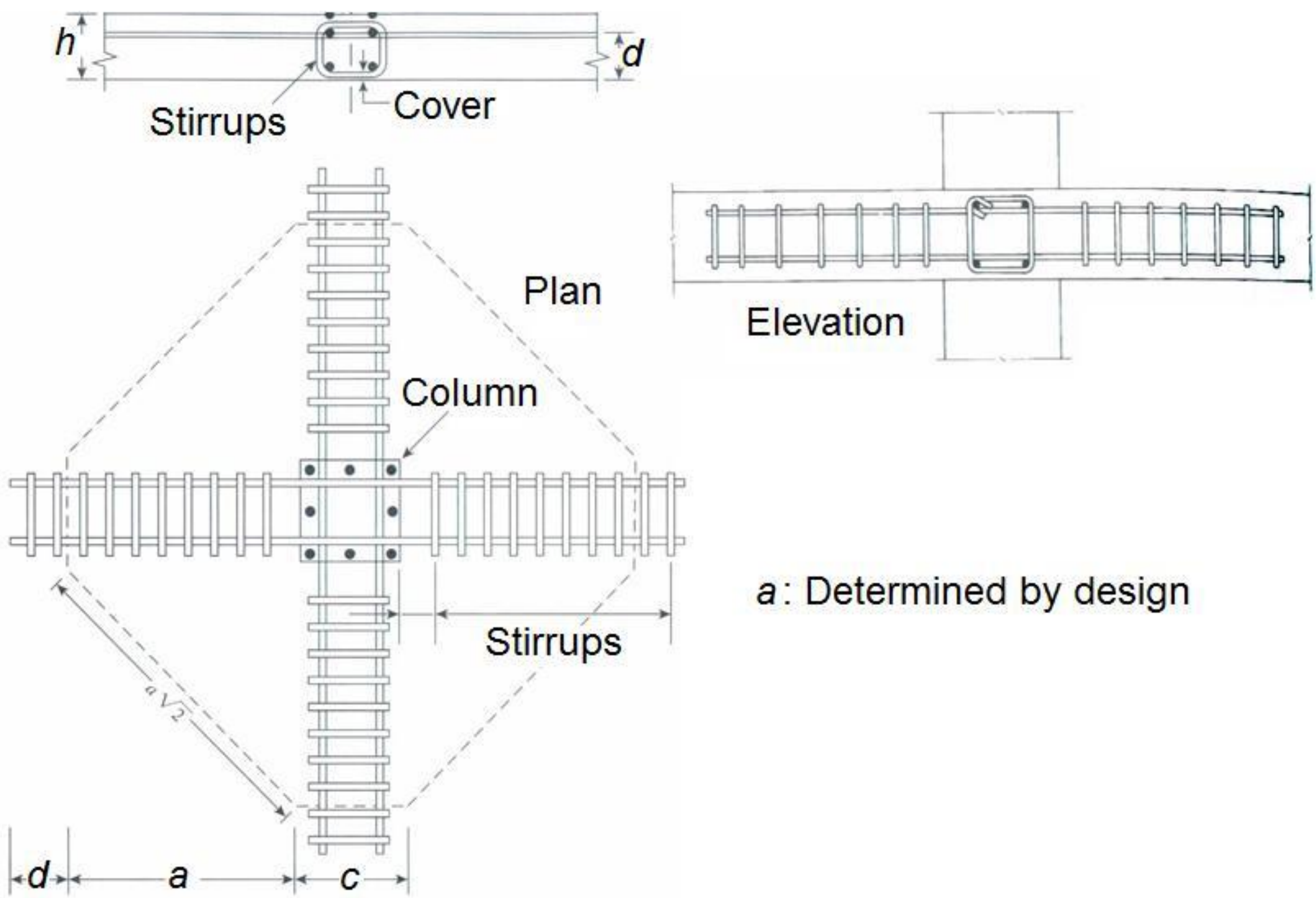
We must check in both cases that: $\phi V_c \geq V_u$



Critical one-way shear zones

If shear is not OK, either we increase the slab thickness or provide drop panels / capitals.

Special shear reinforcement may also be provided, such the stirrup cages shown.



Stirrup cages for slab shear reinforcement

Concrete shear strength in two-way slabs

For two-way shear, the concrete nominal shear strength is given by:

$$V_c = Min\begin{cases} \left(1+\frac{2}{\beta_c}\right)\frac{\sqrt{f_c^{'}}}{6}b_0 d & (1) \quad d: \text{average steel depth}, \ b_0: \text{length of critical perimeter} \\ \left(2+\frac{\alpha_s d}{b_0}\right)\frac{\sqrt{f_c^{'}}}{12}b_0 d & (2) \quad \beta_c: \text{column aspect ratio (long side / short side)} \\ \frac{\sqrt{f_c^{'}}}{3}b_0 d & (3) \quad \alpha_s: \text{column location factor (40 : Internal, 30 : Edge, 20 : Corner)} \end{cases}$$

The ultimate shear is computed over the loaded area defined as the total shear panel area minus the critical area: $V_u = w_u(A_0 - A_c) = w_u(l_1 l_2 - p_1 p_2)$

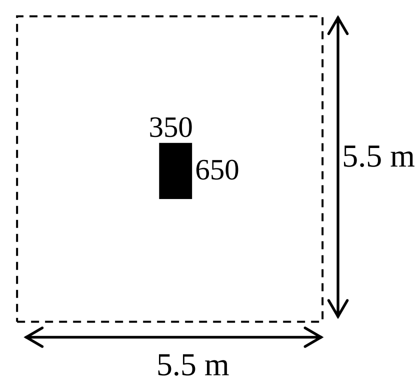
l_1 and l_2 are the shear panel dimensions and p_1 and p_2 are the critical perimeter dimensions

If steel data is not available, the average steel depth may be estimated by: $d = h - 40$

For one-way shear, concrete nominal shear strength is as in beams given by: $V_c = \frac{\sqrt{f_c^{'}}}{6}b_w d \quad (4)$

Example 3: Shear check in a flat plate

A 150 mm flat plate slab with a concrete grade of $f_c^{'} = 25\,MPa$, is subjected to a live load of 3.0 kN/m^2 and a super imposed dead load of 1.0 kN/m^2.



The steel depths in both directions are $d_1 = 125\,mm \quad d_2 = 115\,mm$

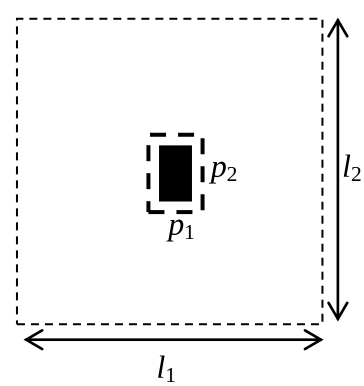
Check shear for the shown internal shear panel.

The ultimate slab load is:

$$w_u = 1.4(24\,x\,0.150 + 1.0) + 1.7\,x\,3.0 = 11.54\,kN/m^2$$

a) Two-way shear

The shear panel dimensions are: $l_1 = l_2 = 5.5\,m$



The column dimensions are:

$c_1 = 350\,mm = 0.35\,m \qquad c_2 = 650\,mm = 0.65\,m$

The average steel depth is: $d = \frac{d_1 + d_2}{2} = 120.0\,mm$

The critical perimeter dimensions are thus:

$p_1 = c_1 + d = 350 + 120 = 470\,mm = 0.47\,m$

$p_2 = c_2 + d = 650 + 120 = 770\,mm = 0.77\,m$

The ultimate shear is obtained from the loaded area:

$$V_u = w_u(A_0 - A_c) = w_u(l_1 l_2 - p_1 p_2) = 11.54(5.5^2 - 0.77\,x\,0.47) = 344.9\,kN$$

For two-way shear, the concrete nominal shear strength is given by the minimum of equations (1) to (3).

$$V_c = Min\begin{cases}\left(1+\dfrac{2}{\beta_c}\right)\dfrac{\sqrt{f_c^{'}}}{6}b_0 d & (1) \quad d = 120\,mm \quad b_0 = 2(p_1 + p_2) = 2(470 + 770) = 2480\,mm \\ \left(2+\dfrac{\alpha_s d}{b_0}\right)\dfrac{\sqrt{f_c^{'}}}{12}b_0 d & (2) \quad \beta_c = \dfrac{650}{350} = 1.857 \\ \dfrac{\sqrt{f_c^{'}}}{3}b_0 d & (3) \quad \alpha_s = 40 \text{ (Internal column)}\end{cases}$$

$$V_c = Min\begin{cases}\left(1+\dfrac{2}{1.857}\right)\dfrac{\sqrt{25}}{6}2480\,x120 = 515097.5\,N = 515.1\,kN \\ \left(2+\dfrac{40\,x120}{2480}\right)\dfrac{\sqrt{25}}{12}2480\,x120 = 488000\,N = 488.0\,kN \\ \dfrac{\sqrt{25}}{3}2480\,x120 = 496000\,N = 496.0\,kN\end{cases} \qquad V_c = 488.0\,kN$$

$\phi V_c = 0.75\,x\,488.0 = 366.0\,kN$ this is greater than V_u. Two-way shear is thus OK.

Two-way shear ratio is: $\dfrac{V_u}{\phi V_c} = \dfrac{344.9}{366.0} = 0.94$

b) One-way shear

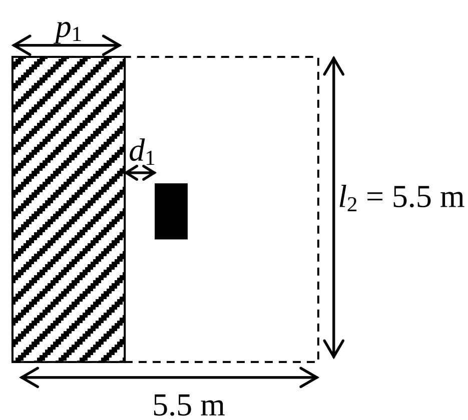
For one-way shear, there are two possibilities:

Case 1:

$$p_1 = \frac{l_1}{2} - \frac{c_1}{2} - d_1 = \frac{5.5}{2} - \frac{0.35}{2} - 0.125 = 2.45\,m$$

The ultimate shear given by the shaded loaded area is:

$$V_{u1} = w_u p_1 l_2 = 11.54\,x\,2.45\,x\,5.5 = 155.5\,kN$$

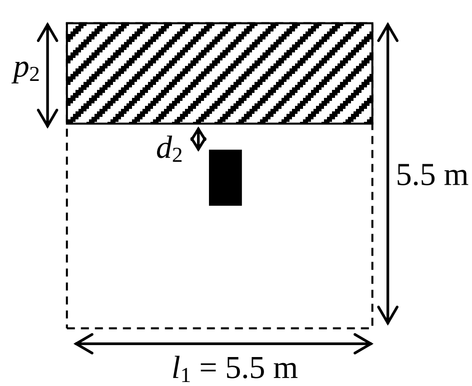


Case 2:

$$p_2 = \frac{l_2}{2} - \frac{c_2}{2} - d_2 = \frac{5.5}{2} - \frac{0.65}{2} - 0.115 = 2.31\,m$$

The ultimate shear given by the shaded loaded area is:

$$V_{u2} = w_u p_2 l_1 = 11.54\,x\,2.31\,x\,5.5 = 146.6\,kN$$



The first case is the controlling one as it gives the highest ultimate shear force.

The nominal concrete shear strength given by equation (4) is:

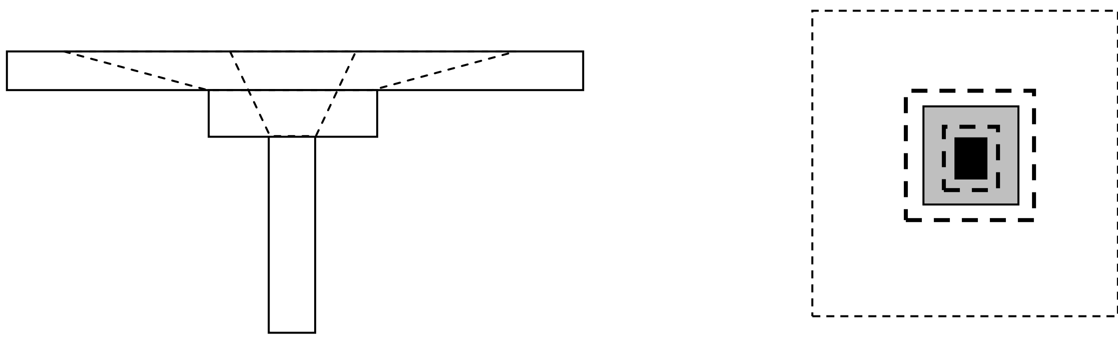
$$V_c = \frac{\sqrt{f_c^{'}}}{6} b_w d = \frac{\sqrt{f_c^{'}}}{6} l_1 d_1 = \frac{\sqrt{25}}{6} 5500\,x125 = 572917\;N = 572.9\,kN$$

$\phi V_c = 0.75\,x\,572.9 = 429.7\,kN \geq V_{u1} = 155.5\,kN$ One way shear is also OK.

One-way shear ratio is: $\frac{V_u}{\phi V_c} = \frac{155.5}{429.7} = 0.36$

Two-way shear (with a greater ratio) is more critical for this internal column.

It must be pointed out that in case of presence of drop panels, punching shear must be checked around the drop panel using the slab thickness and around the column using the drop panel thickness.

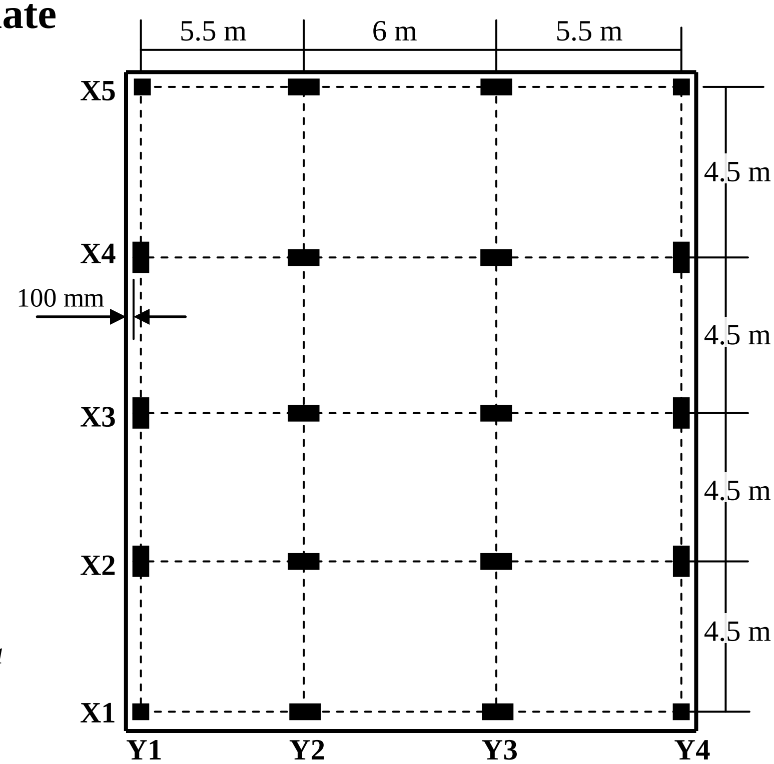


Steps for analysis and design of two-way slabs:

It is usually preferable to check the slab thickness for shear before performing analysis and design. The recommended order is therefore:

1. Slab thickness
2. Shear check
3. Slab loading
4. Static moment M_0
5. Positive and negative moments M^+ and M^-
6. Distribution of moments over column strip and middle strip
7. Design of strips
8. Detailing

Two way slab example 4: Flat plate



The figure shows a flat plate floor.
The slab extends 100 mm offset past the exterior column face in both directions.
The slab thickness is 185 mm.
Corner columns have a square section 300 x 300 mm whereas other columns have the same rectangular section 500 x 300 mm with the orientations as shown.
Superimposed dead load SDL = 1.2 kN/m^2
Live load LL = 2.0 kN/m^2
Materials: $f_c^{'} = 25\,MPa$ $\quad f_y = 420\,MPa$

$\gamma_c = 24\,kN/m^3$

(1) Check if conditions of DDM are satisfied.

1. Minimum of three spans in each direction: OK
2. Fore each panel, ratio of longer span to shorter span less than 2: OK
3. Successive spans differ by not more than one third of longer span: OK
4. Column offsets up to 10%: OK
5. Uniform gravity load: OK
6. Live load not exceeding twice dead load. LL = 2.0 DL = 24 x 0.185 + 1.2 = 5.64 : OK
7. There are no beams. The condition on relative beam stiffness does not apply

All conditions are therefore satisfied.

(2) Check the slab thickness for deflection control

Check all panels using the minimum thickness Table 13.1 and take the maximum.

There are no beams and no drop panels. For the steel grade of 420 MPa, in all corner and edge panels, the minimum thickness is l_n/30 and for the interior panel it is l_n/33. l_n is the longer clear length of the panel.

Corner panel: $l_{n,\max} = 5500 - (150 + 250) = 5100$ $\qquad h_{\min} = \dfrac{l_n}{30} = \dfrac{5100}{30} = 170.0\,mm$

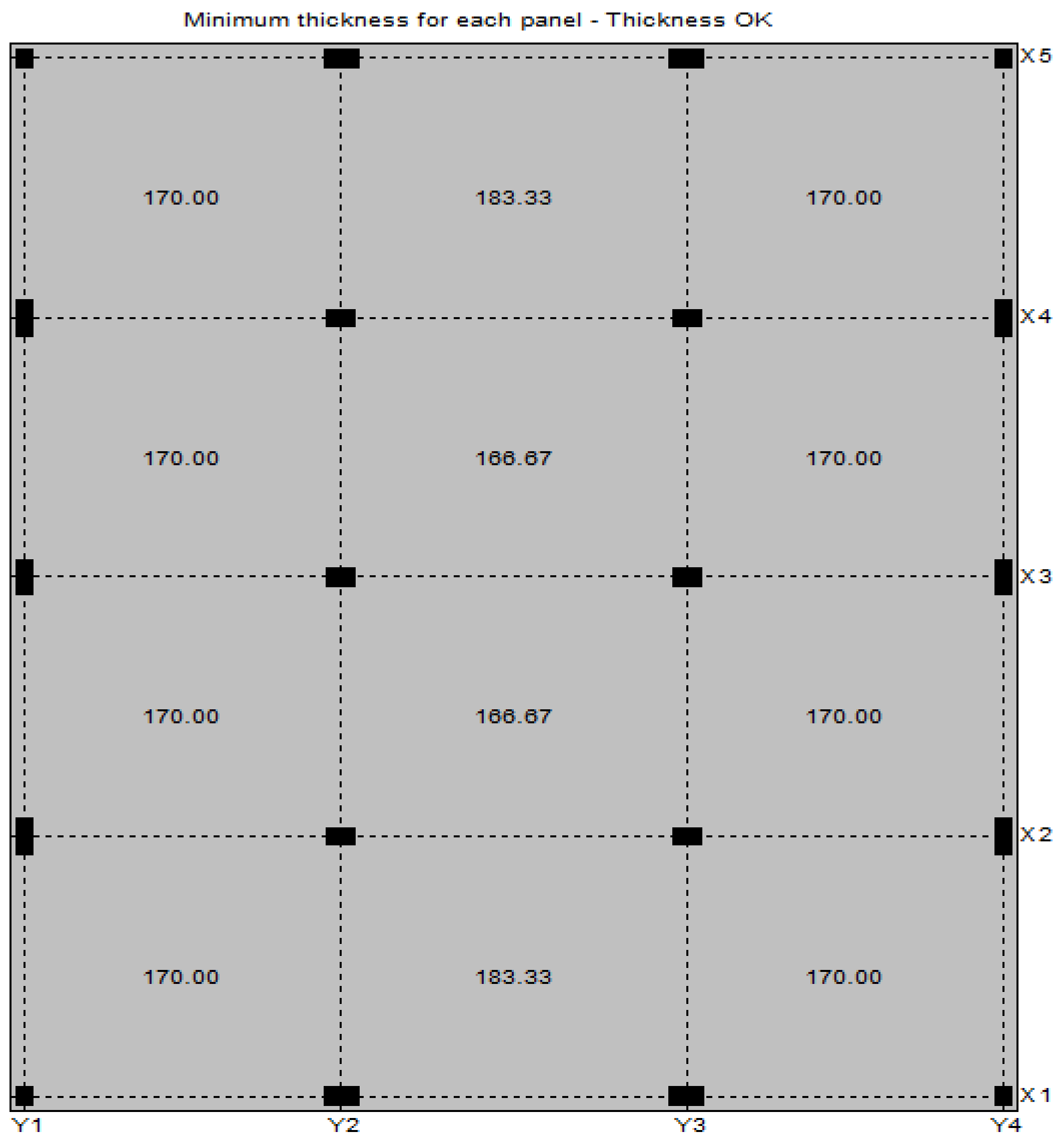
Edge panel: $l_{n,\max} = 6000 - (250 + 250) = 5500$ $\qquad h_{\min} = \dfrac{l_n}{30} = \dfrac{5500}{30} = 183.33\,mm$

Interior panel: $l_{n,\max} = 6000 - (250 + 250) = 5500$ $\qquad h_{\min} = \dfrac{l_n}{33} = \dfrac{5500}{33} = 166.67\,mm$

The minimum thickness is therefore 183.33 mm and is just less than the actual thickness of 185 mm: OK

Table 13.1: Minimum thickness for slabs without interior beams

f_y (MPa)	Without drop panels			With drop panels		
	Exterior panel		Int. panel	Exterior panel		Int. panel
	No edge beams	With edge beams		No edge beams	With edge beams	
300	L_n / 33	L_n / 36	L_n / 36	L_n / 36	L_n / 40	L_n / 40
420	L_n / 30	L_n / 33	L_n / 33	L_n / 33	L_n / 36	L_n / 36
520	L_n / 28	L_n / 31	L_n / 31	L_n / 31	L_n / 34	L_n / 34



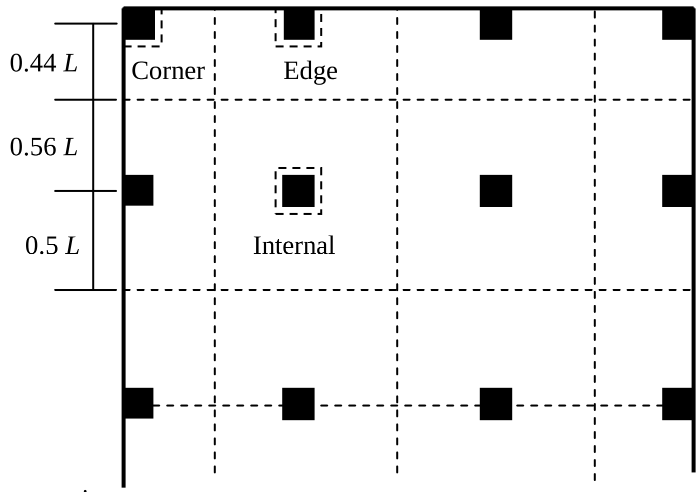
Minimum thickness check using RC-SLAB2 software

(3) Check the thickness for two-way shear at edge / internal columns

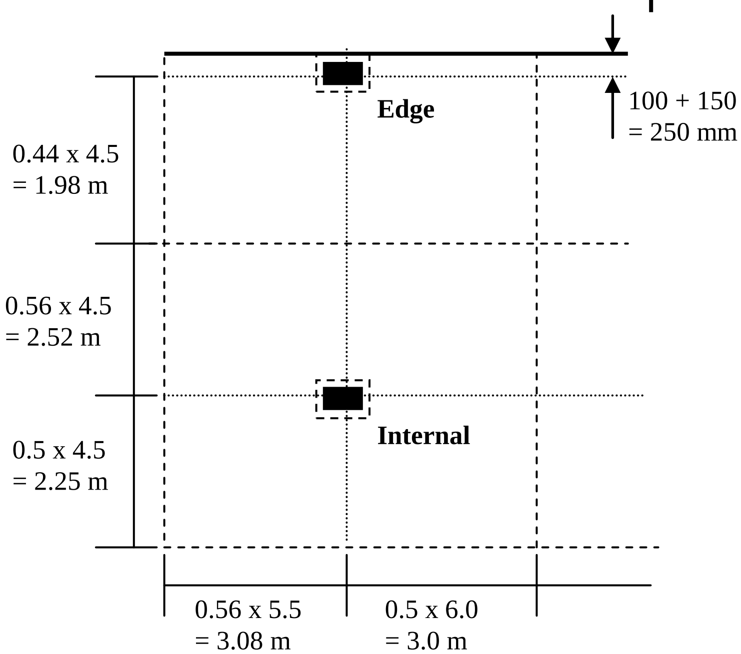
The factored slab load is: $w_u = 1.4(24\,x\,0.185 + 1.2) + 1.7\,x\,2.0 = 11.3\,kN/m^2$

The tributary areas for shear in a flat plate are shown in the next figure:

Any offset must be added to the corresponding span.



For the two columns, shear areas are as shown



The concrete nominal shear strength is given by:

$$V_c = Min\begin{cases} \left(1+\dfrac{2}{\beta_c}\right)\dfrac{\sqrt{f_c^{'}}}{6}b_0 d & (1) \quad d: \text{average steel depth, } b_0: \text{length of critical perimeter} \\ \left(2+\dfrac{\alpha_s d}{b_0}\right)\dfrac{\sqrt{f_c^{'}}}{12}b_0 d & (2) \quad \beta_c: \text{column aspect ratio (long side / short side)} \\ \dfrac{\sqrt{f_c^{'}}}{3}b_0 d & (3) \quad \alpha_s: \text{column location factor (40 : Internal, 30 : Edge, 20 : Corner)} \end{cases}$$

The ultimate shear is computed over the loaded area defined as the total panel area minus the critical area:

$$V_u = w_u(A_0 - A_c) = w_u(l_1 l_2 - p_1 p_2)$$

The average steel depth is estimated by: $d = h - 40 = 185 - 40 = 145\,mm$

Internal column:

Critical perimeter dimensions are: $p_1 = c_1 + d = 500 + 145 = 645\,mm \quad p_2 = c_2 + d = 300 + 145 = 445\,mm$

The perimeter length is therefore $b_0 = 2(645 + 445) = 2180\,mm$

The column coefficients are $\beta_c = \dfrac{500}{300} = 1.667$ and $\alpha_s = 40$ (internal column)

V_c as determined by (1) to (3) is: V_c = Min (579.52 , 613.83 , 526.83) = 526.83 kN

With shear strength reduction factor $\phi = 0.75$, we find: $\phi V_c = 395.125\,kN$

The ultimate shear is: $V_u = w_u((3.08+3.0)\,x\,(2.52+2.25) - 0.645\,x\,0.445) = 324.5\,kN$

$\phi V_c > V_u$ The thickness is therefore OK Two-way shear ratio = 0.82

Edge column:

Critical perimeter: $p_1 = c_1 + d = 500 + 145 = 645\,mm \quad p_2 = c_2 + 0.5d + 100 = 300 + 72.5 + 100 = 472.5\,mm$

The perimeter length with three sides is therefore $b_0 = 2p_2 + p_1 = 2\,x\,472.5 + 645 = 1590\,mm$

The column coefficients are $\beta_c = \dfrac{500}{300} = 1.667$ and $\alpha_s = 30$ (edge column)

V_c is determined in the same manner as for a beam. We find that $\phi V_c = 288.2\,kN$

The ultimate shear is obtained as: $V_u = w_u((3.08+3.0)\,x\,(1.98+0.25) - 0.645\,x\,0.4725) = 149.8\,kN$

$\phi V_c > V_u$ The thickness is therefore OK Two-way shear ratio = 0.52

RC-SLAB2 output for shear check:
RC-SLAB2 software performs one-way and two-way shear checks for all columns and delivers the ratios of demand to capacity. $\text{Ratio} = \dfrac{V_u}{\phi V_c}$

A safe shear corresponds to a ratio less than or equal to unity.
It also delivers detailed calculations for any user-chosen column.
The following listing gives shear ratios for all columns.

```
One and Two way shear check with following data
Ultimate slab load (kN/m2) =   11.2960
Steel depth in X-direction dx (mm) =  155.0
Steel depth in Y-direction dy (mm) =  135.0
Average steel depth d (mm) =  145.0
Concrete strength f'c (MPa) =  25.00

One / Two way shear ratios for column X1-Y1 = 0.4778   0.3779
One / Two way shear ratios for column X2-Y1 = 0.2466   0.4873
One / Two way shear ratios for column X3-Y1 = 0.2466   0.4590
One / Two way shear ratios for column X4-Y1 = 0.2466   0.4873
One / Two way shear ratios for column X5-Y1 = 0.4778   0.3779
One / Two way shear ratios for column X1-Y2 = 0.2269   0.5195
One / Two way shear ratios for column X2-Y2 = 0.2992   0.8209
One / Two way shear ratios for column X3-Y2 = 0.3119   0.7740
One / Two way shear ratios for column X4-Y2 = 0.2992   0.8209
One / Two way shear ratios for column X5-Y2 = 0.2269   0.5195
One / Two way shear ratios for column X1-Y3 = 0.2269   0.5195
One / Two way shear ratios for column X2-Y3 = 0.2992   0.8209
One / Two way shear ratios for column X3-Y3 = 0.3119   0.7740
One / Two way shear ratios for column X4-Y3 = 0.2992   0.8209
One / Two way shear ratios for column X5-Y3 = 0.2269   0.5195
One / Two way shear ratios for column X1-Y4 = 0.4778   0.3779
One / Two way shear ratios for column X2-Y4 = 0.2466   0.4873
One / Two way shear ratios for column X3-Y4 = 0.2466   0.4590
One / Two way shear ratios for column X4-Y4 = 0.2466   0.4873
One / Two way shear ratios for column X5-Y4 = 0.4778   0.3779

Maximum shear ratio is: 0.821   Shear is OK
```

It can be noted that for corner columns, one way-shear is more important.

The following listing includes details for the two studied columns

```
One / Two way shear for column X4-Y2
Column section dimensions (mm)   Cx = 500.0  Cy = 300.0
Shear panel dimensions (m)   Lx =  6.0800    Ly =  4.7700
Critical perimeter dimensions (mm)  Px = 645.0  Py = 445.0
Critical perimeter length (mm)  b0 = 2180.0
Ultimate two way shear force (kN) Vu = 324.3598
Nominal concrete shear strength = Min of following 3 equations
Two way equations (1) to (3) =   579.5167   613.8333   526.8333
Nominal two way shear strength =  526.8333
Design two way shear strength =  395.1250
Two way shear ratio = 0.8209
One way shear loaded area dimensions (m) = 2.2350   6.0800
Ultimate one way shear force (kN) Vu =  153.4991
Nominal one way shear strength (kN) Vc = 684.0000
Design one way shear strength (kN) =  513.0000
One way shear ratio = 0.2992

One / Two way shear for column X5-Y2
Column section dimensions (mm)   Cx = 500.0  Cy = 300.0
Shear panel dimensions (m)   Lx =  6.0800    Ly =  2.2300
Critical perimeter dimensions (mm)  Px = 645.0  Py = 472.5
Critical perimeter length (mm)  b0 = 1590.0
Ultimate two way shear force (kN) Vu = 149.7131
Nominal concrete shear strength = Min of following 3 equations
Two way equations (1) to (3) =   422.6750  454.9375  384.2500
Nominal two way shear strength =  384.2500
Design two way shear strength =  288.1875
Two way shear ratio = 0.5195
One way shear loaded area dimensions (m) = 6.0800  1.6950
Ultimate one way shear force (kN) Vu = 116.4121
Nominal one way shear strength (kN) Vc = 684.0000
Design one way shear strength (kN) = 513.0000
One way shear ratio = 0.2269
```

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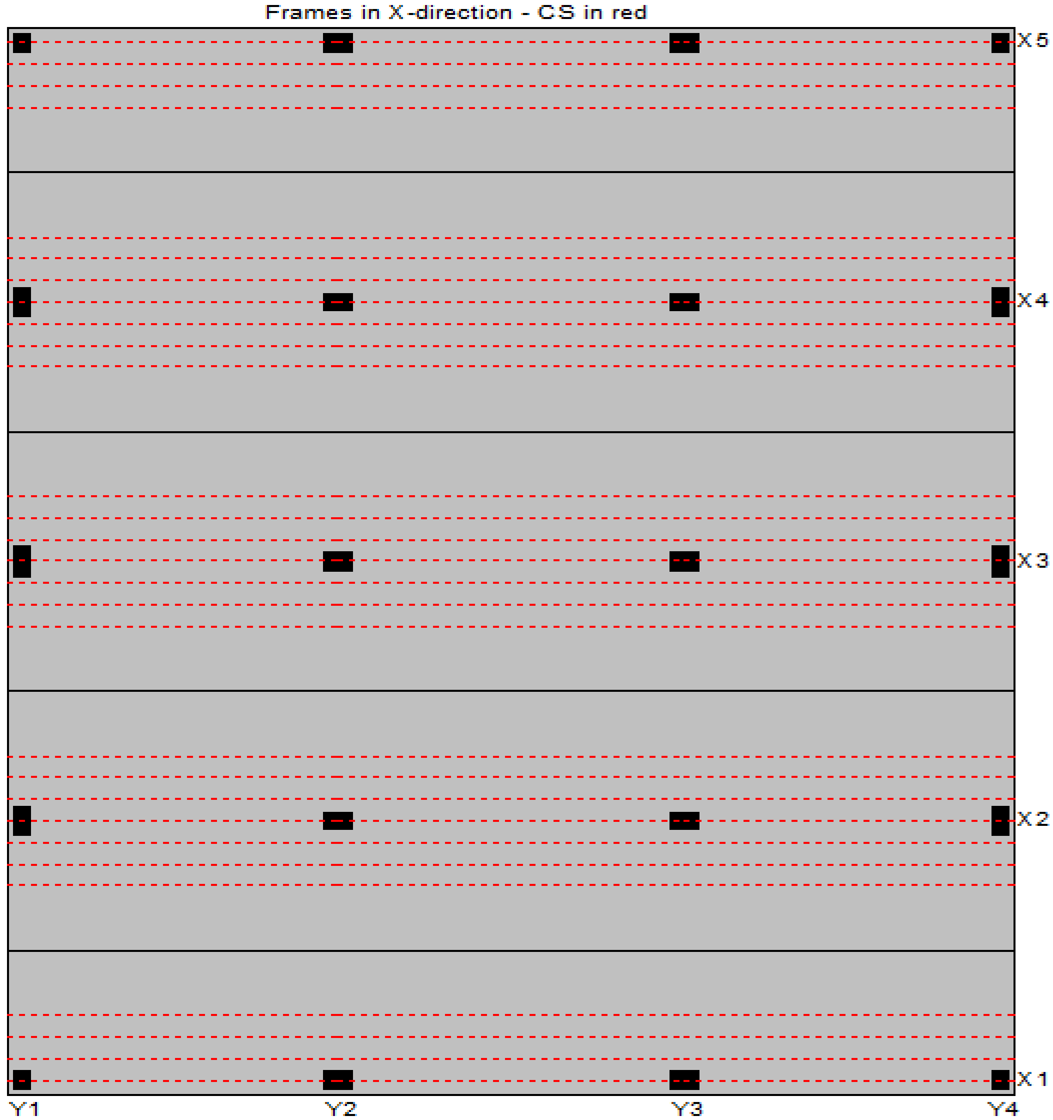
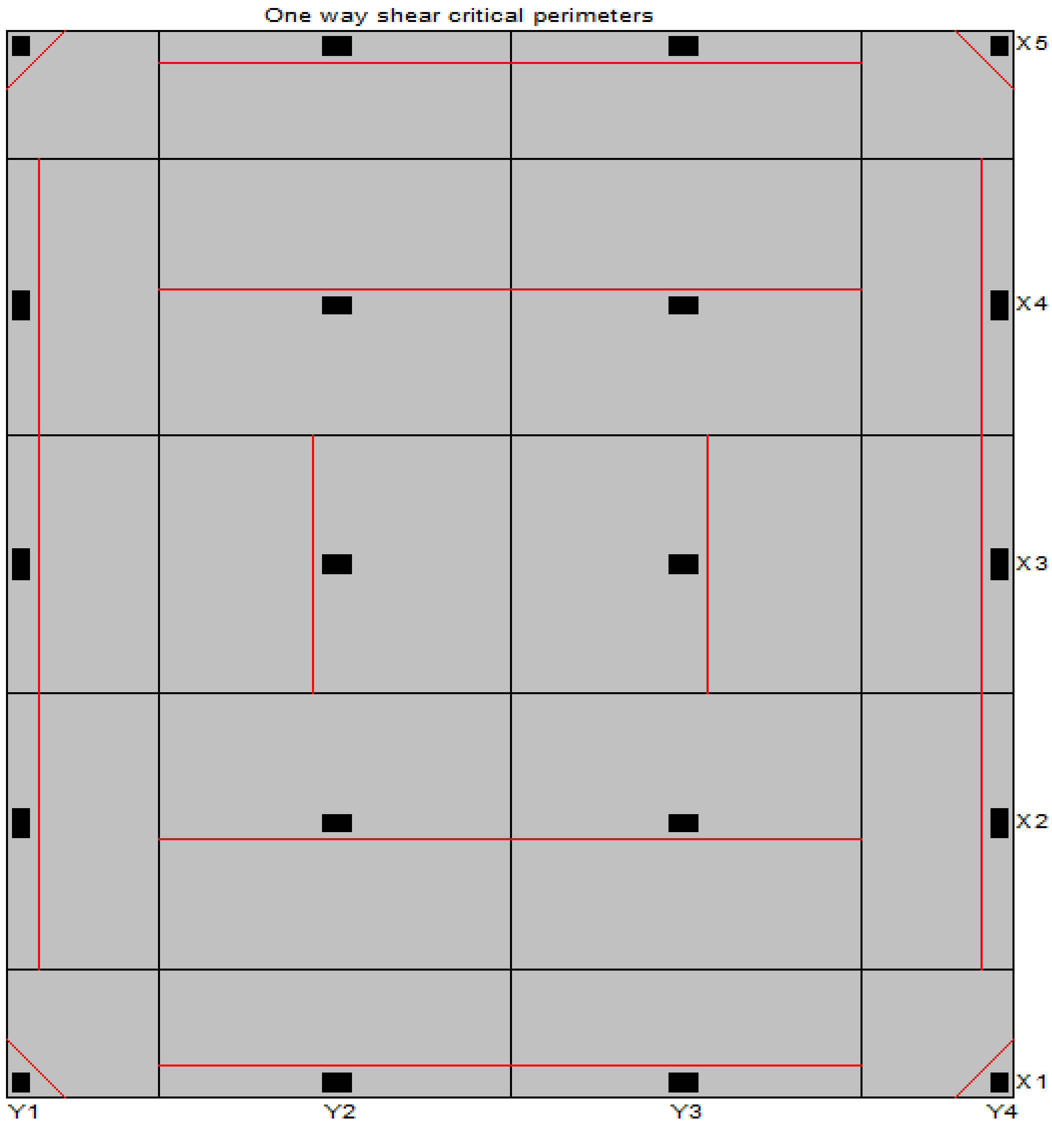
The results are identical to those obtained previously.

These numerical results are similar to the previous analytical ones.

RC-SLAB2 also displays frames in both directions highlighting column and middle strips. It also displays critical shear perimeters in one-way and two-way shear.



RC-SLAB2 display of X-frames with column strips highlighted



RC-SLAB2 display of one-way shear critical zones

(4) Compute the moments in the slab strips along column line X2 (Frame X2)

Frame X2 (in X-direction) has three spans and includes four columns (supports). The panel layout is determined by lines mid-way between column lines. L_1 is the length of the panel (X-direction) and L_2 is its width (Y-direction).

L_n is the clear length. The static moment is each span is given by: $M_0 = w_u L_2 \dfrac{L_n^2}{8}$.

Negative and positive moments in each span as well as portions of moments in column strips, are deduced using appropriate coefficients. The following Table gives all the results.

	Span X2Y1- X2Y2			**Span X2Y2- X2Y3**			**Span X2Y3- X2Y4**		
L_1 (m)	5.5			6.0			5.5		
L_2 (m)	4.5			4.5			4.5		
L_n (m)	5.1			5.5			5.1		
M_0 $(kN.m)$	165.3			192.2			165.3		
CS width (m) $0.5L_{min}$	2.25			2.25			2.25		
Moment coefficients	-0.26	0.52	-0.70	-0.65	0.35	-0.65	-0.70	0.52	-0.26
-ve and +ve moments	-43.0	86.0	-115.7	-124.9	67.3	-124.9	-115.7	86.0	-43.0
CS moment (%)	100	60	75	75	60	75	75	60	100
CS moments $(kN.m)$	-43.0	51.6	-86.8	-93.7	40.4	-93.7	-86.8	51.6	-43.0
MS moments $(kN.m)$	0	34.4	-28.9	-31.2	26.9	-31.2	-28.9	34.4	0

Distribution of factored static moment

	(a)	**(b)**	**(c)**	**(d)**	**(e)**
Int. negative M	0.75	0.70	0.70	0.70	0.65
Positive M	0.63	0.57	0.52	0.50	0.35
Ext. negative M	0	0.16	0.26	0.30	0.65

(a): Exterior edge unrestrained **(b):** Slab with beams between all supports

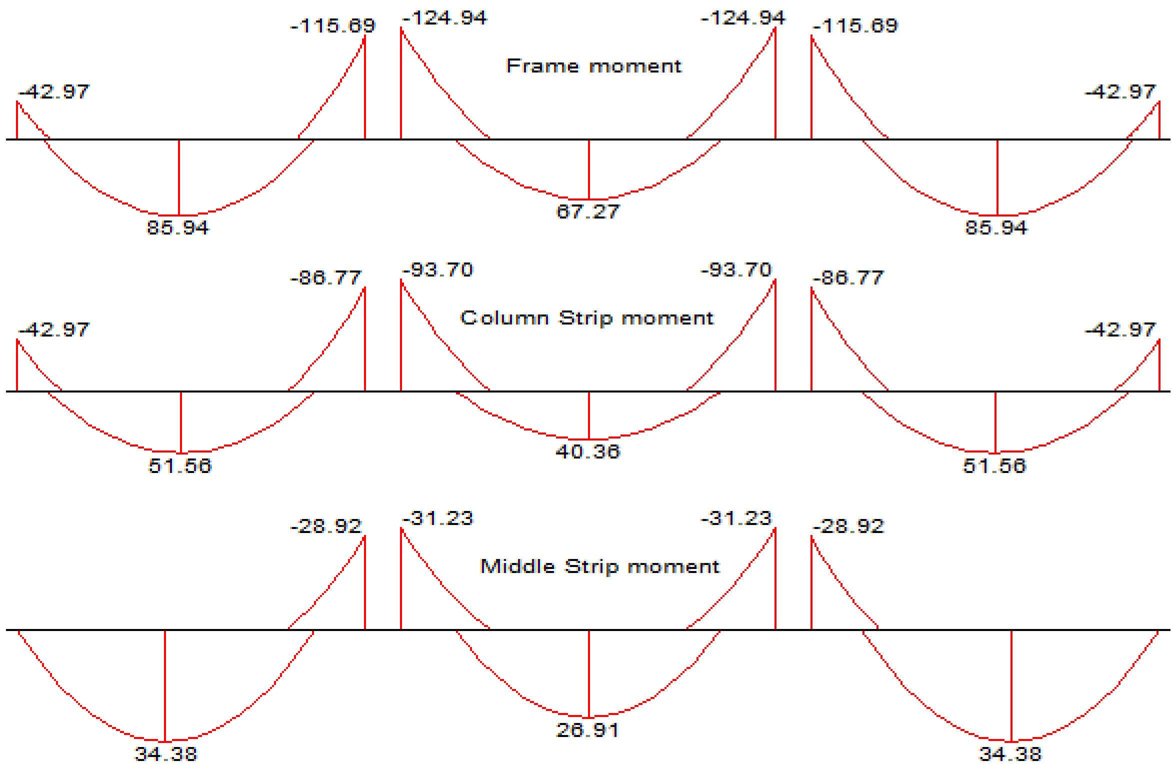
(c): Slab with no beams at all **(d):** Slab with edge beams only

(e): Slab with exterior edge fully restrained (interior span)

Column (c) was used for the external spans and column (e) was used for the internal span.

Analysis of Frame X2   Slab ultimate load (kN/m2) = 11.296

Span	X2Y1 - X2Y2			X2Y2 - X2Y3			X2Y3 - X2Y4		
L1 (m)	5.5000			6.0000			5.5000		
L2 (m)	4.5000			4.5000			4.5000		
Ln (m)	5.1000			5.5000			5.1000		
Mo (kN.m)	165.268			192.208			165.268		
CS (m)	2.25000			2.25000			2.25000		
M-Coef	-0.26	0.52	-0.70	-0.65	0.35	-0.65	-0.70	0.52	-0.26
M (kN.m)	-42.97	85.94	-115.69	-124.94	67.27	-124.94	-115.69	85.94	-42.97
CS-M %	100.00	60.00	75.00	75.00	60.00	75.00	75.00	60.00	100.00
CS-M	-42.97	51.56	-86.77	-93.70	40.36	-93.70	-86.77	51.56	-42.97
MS-M	0.00	34.38	-28.92	-31.23	26.91	-31.23	-28.92	34.38	0.00



RC-SLAB2 analysis output

(5) Design column strip for the maximum interior negative moment

The moment is 93.75 $kN.m$ and the section dimensions are: b = 2250 mm, and h = 185 mm

Using 14-mm bars, bar area and steel depth are:

$$A_b = \pi \frac{14^2}{4} = 153.9\, mm^2 \qquad d = h - \text{cover} - (d_b/2) = 185 - 20 - 7 = 158\ mm$$

We find that the required steel area is A_s = 1645 mm^2

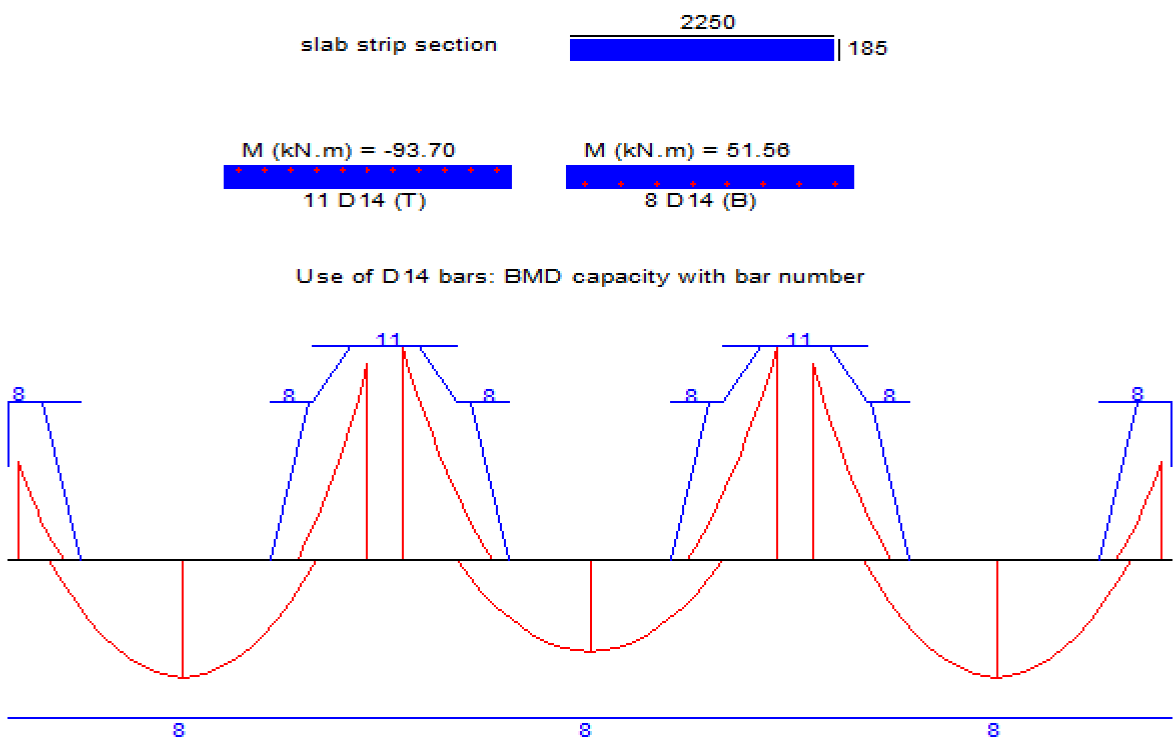
It is greater than the minimum steel area given by: A_{smin} = 0.0018 bh = 0.0018 x 2250 x 185 = 749.3 mm^2

Maximum spacing is $S_{\max} = Min(2h\ ,\ 300) = 300\, mm$

The corresponding minimum steel is: $A_{s\min 2} = \dfrac{A_b b}{S_{\max}} = \dfrac{153.9\, x\, 2250}{300} = 1154.25\, mm^2$.

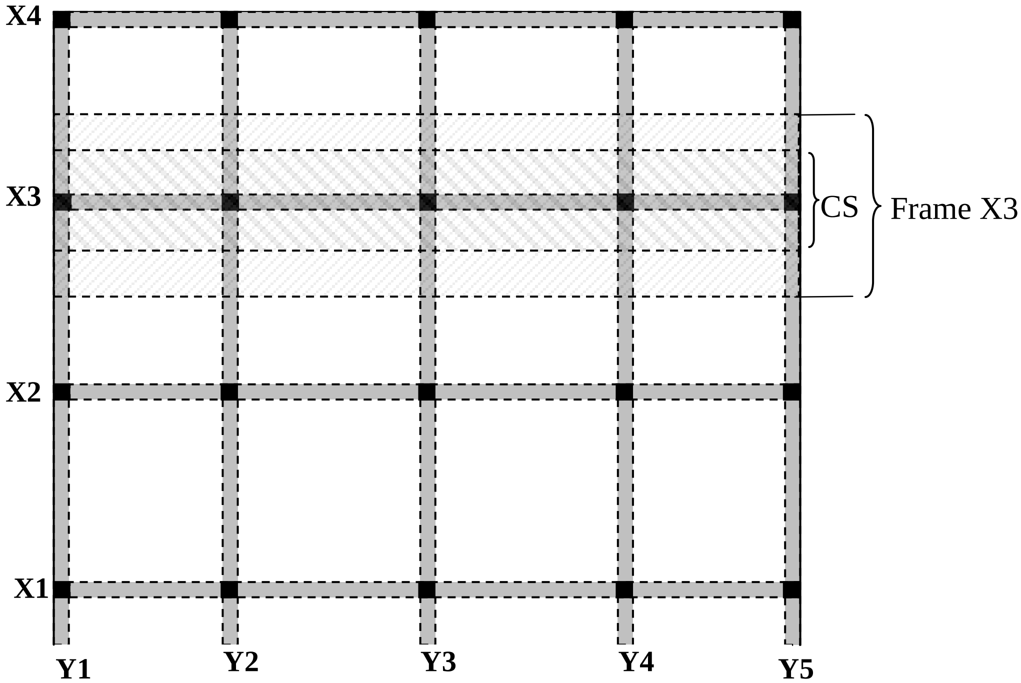
Minimum bar number $N_{b\min} = \dfrac{Max(A_{s\min}\ ,\ A_{s\min 2})}{A_b} = \dfrac{1154.25}{153.9} = 7.5$ That is N_{bmin} = 8 bars

Required bar number $N_b = \dfrac{A_s}{A_b} = \dfrac{1645.0}{153.9} = 10.7$ That is N_b = 11 bars



RC-SLAB2 design output

Part B: Two way slabs with beams

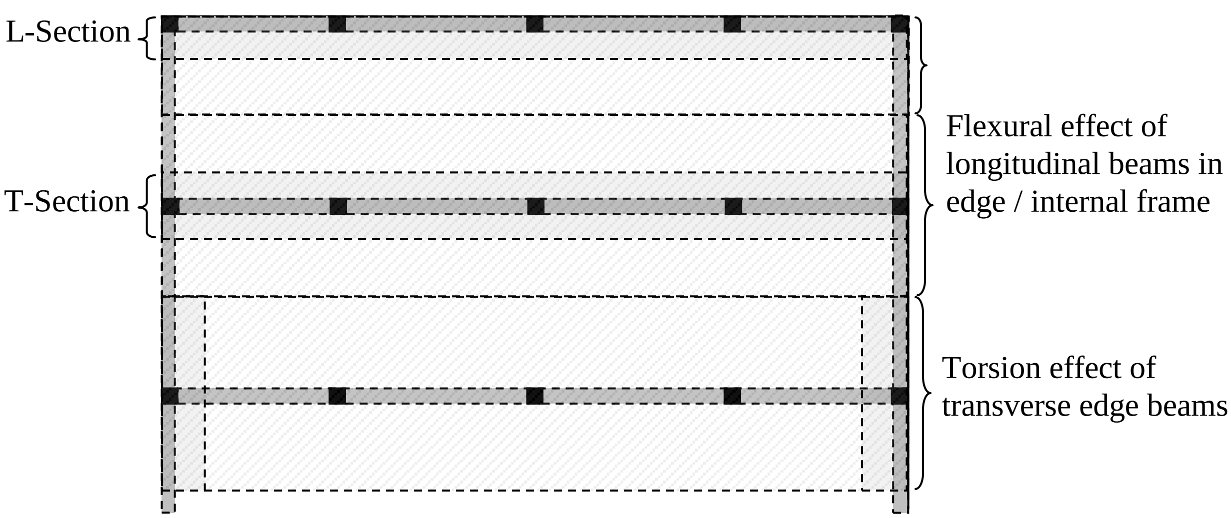


The direct design method can again be used provided its conditions are satisfied. Moment distribution is affected by beam presence.

In each frame, the longitudinal beam is part of the column strip and contributes to the flexural rigidity.

The frame is also affected by the torsion rigidity of the transverse edge beams.

The effective beam section is a T-section for internal beams and L-section for edge beams.



Relative flexural stiffness of longitudinal beams

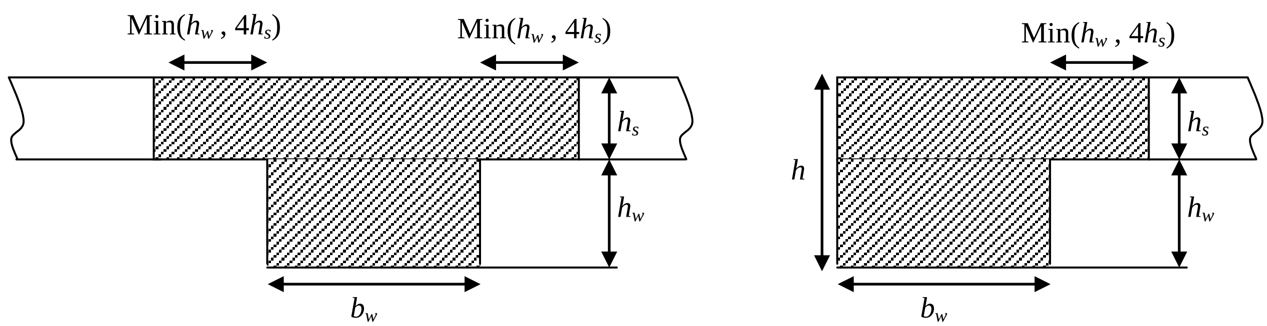
The flexural effect of the beam is related to its relative beam stiffness α compared to slab stiffness.

The relative beam stiffness is defined as: $$\alpha = \frac{E_b I_b}{E_s I_s}$$

Using the same concrete for beams and slabs leads to: $$\alpha = \frac{I_b}{I_s}$$

The beam moment of inertia is computed by considering the effective beam section which is a T-section for internal beams and L-section for edge beams. The slab moment of inertia is computed by considering a rectangular section defined by the panel (frame) width and slab thickness.

For the beam section, the extra distance to be added (on one side for the L-section and on both sides of the T-section), is equal to Min (h_w , $4h_s$).

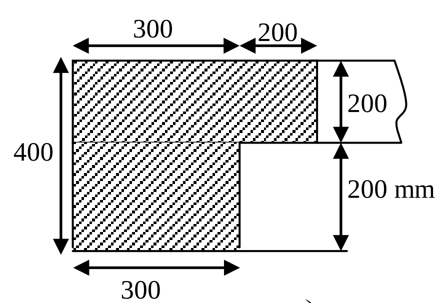


Example 5: Compute α for the edge beam shown.

Beam section and inertia

The extra distance to be added to the L-section is equal to Min (h_w , $4h_s$) = Min (200 , 800) = 200 mm

The L-section dimensions are therefore as shown.



The centroid Y-coordinate (from bottom base) is:

$$Y_g = \frac{A_1 Y_1 + A_2 Y_2}{A_1 + A_2} = \frac{60000\, x 100 + 100000\, x 300}{60000 + 100000} = 225.0\, mm$$

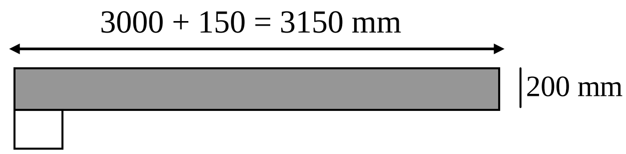
The centroid moment of inertia is therefore:

$$I_b = I_{b1} + I_{b2} = \left(\frac{300\, x 200^3}{12} + 60000\, x (225 - 100)^2 \right) + \left(\frac{500\, x 200^3}{12} + 100000\, x (300 - 225)^2 \right)$$

$$I_b = I_{b1} + I_{b2} = 11375\, x 10^5 + 8958.333\, x 10^5 = 20.333\, x 10^8\ mm^4$$

Slab section and inertia

The slab panel (frame) width is equal to half distance between the beams plus the offset (half beam section):



The slab inertia is therefore:

$$I_s = \frac{3150\,x\,200^3}{12} = 21.0\,x10^8\ mm^4$$

The beam relative stiffness is: $\alpha = \frac{I_b}{I_s} = \frac{20.333}{21.0} = 0.968$

The beam relative stiffness may also be obtained using the chart in Figure 13.21 of the Textbook.

It is given as: $\alpha = \frac{b}{l}\left(\frac{a}{h}\right)^3 f$ The chart gives f in function of $\frac{a}{h}$ and $\frac{b}{h}$

a = 400 mm (beam thickness) b = 300 mm (beam width)

h = 200 mm (slab thickness) l = 3150 mm (slab width)

$\frac{a}{h} = 2.0$ and $\frac{b}{h} = 1.5$ we read from the chart that f = 1.27

Thus $\alpha = \frac{300}{3150}\left(\frac{400}{200}\right)^3 1.27 = 0.968$ we find the same result

Relative torsion stiffness of transverse edge beams

The distribution of external negative moment (in external spans) depends also on the relative torsion stiffness of the transverse edge beams. This is defined as: $\beta_t = \frac{E_b C}{2 E_s I_s}$

Using the same concrete for beams and slabs leads to: $\beta_t = \frac{C}{2 I_s}$

C is the torsional constant of the edge beam, roughly equal to the polar moment of inertia. It is determined by dividing the cross section (L-section) in rectangles as: $C = \sum\left[\left(1 - 0.63\frac{x}{y}\right)\frac{x^3 y}{3}\right]$

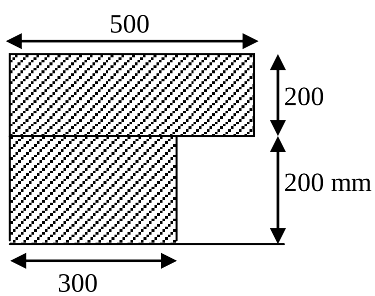
where x and y are the shorter and longer sides respectively of the rectangular section.

The subdivision leading to the largest value of C must be used.

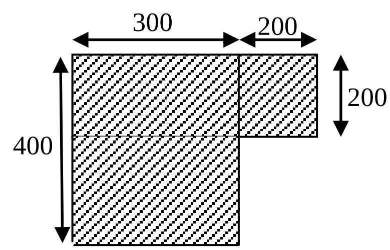
The slab moment of inertia I_s is computed as before.

Two possible subdivisions of L-section in rectangles for torsion constant C

Example: Compute torsion constant C for the previous edge beam



(a)



(b)

Subdivision (a): $C = \left[\left(1-0.63\frac{200}{300}\right)\frac{200^3 300}{3}\right] + \left[\left(1-0.63\frac{200}{500}\right)\frac{200^3 500}{3}\right] = 1461.333\,x10^6\,mm^4$

Subdivision (b): $C = \left[\left(1-0.63\frac{300}{400}\right)\frac{300^3 400}{3}\right] + \left[\left(1-0.63\frac{200}{200}\right)\frac{200^3 200}{3}\right] = 2096.333\,x10^6\,mm^4$

Therefore $C = 2096.333\,x10^6\,mm^4 = 0.002096333\,m^4$

Beam relative torsion stiffness is then obtained by: $\beta_t = \dfrac{C}{2I_s}$

A typical frame would have a constant flexural stiffness (α) in all its spans and two different torsion relative stiffnesses (β_t) at its two end spans (resulting from the two transverse edge beams).

An edge beam with a given torsion constant C generates different values of the relation torsion stiffness in the various frames it crosses.

Thickness of slabs with beams

The thickness for each slab panel depends on the average beam relative stiffness α_m which is the average of the values for the four beams of the panel. $\alpha_m = \frac{\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4}{4}$

The minimum thickness is determined as follows:

(a) $\alpha_m \le 0.2$: Use minimum thickness Table 13.1 for flat plate (and slabs without interior beams)

(b) $0.2 < \alpha_m < 2.0$:

$$h_{\min} = \frac{L_n \left[0.8 + \frac{f_y}{1500} \right]}{36 + 5\beta(\alpha_m - 0.2)} \qquad \text{Equation (13.10)}$$

$$h_{\min} \ge 120\, mm$$

(c) $\alpha_m \ge 2.0$:

$$h_{\min} = \frac{L_n \left[0.8 + \frac{f_y}{1500} \right]}{36 + 9\beta} \qquad \text{Equation (13.11)}$$

$$h_{\min} \ge 90\, mm$$

L_n is the maximum clear length of the panel and β is the clear length ratio (Max L_n / Min L_n)

$$L_n = Max(L_{n1}, L_{n2}) \qquad \beta = \frac{Max(L_{n1}, L_{n2})}{Min(L_{n1}, L_{n2})}$$

Column strip moments

With the presence of beams, the column strip portions of the moments change as compared to flat plates. The CS portion of moments depends on $\alpha_1 \, {}^{L_2}\!/_{L_1}$ and ${}^{L_2}\!/_{L_1}$.

L_1 is the panel length in the studied direction and L_2 is the panel length in the other direction.

α_1 is the value of α in direction of L_1.

For exterior negative moments, the distribution depends also on the torsion parameter β_t.

Portion (%) of column strip moment in slabs with beams

	Interior negative moment			Positive moment			Exterior negative moment			
L_2/L_1	0.5	1.0	2.0	0.5	1.0	2.0	0.5	1.0	2.0	
$\alpha_1 L_2/L_1 = 0$	75	75	75	60	60	60	100	100	100	$\beta_t = 0$
							75	75	75	$\beta_t \geq 2.5$
$\alpha_1 L_2/L_1 \geq 1.0$	90	75	45	90	75	45	100	100	100	$\beta_t = 0$
							90	75	45	$\beta_t \geq 2.5$

Linear interpolation must be performed at intermediate points

It can be noted the flat plate portions are retrieved if beam stiffness is equal to zero.

Distribution of column strip moments over the beam and slab

Column strip moments are divided between the beam and the slab according to the value of $\alpha_1 L_2/L_1$:

If $\alpha_1 L_2/L_1 \geq 1.0$ then the beam takes 85 % of the CS moment and the slab takes 15 %.

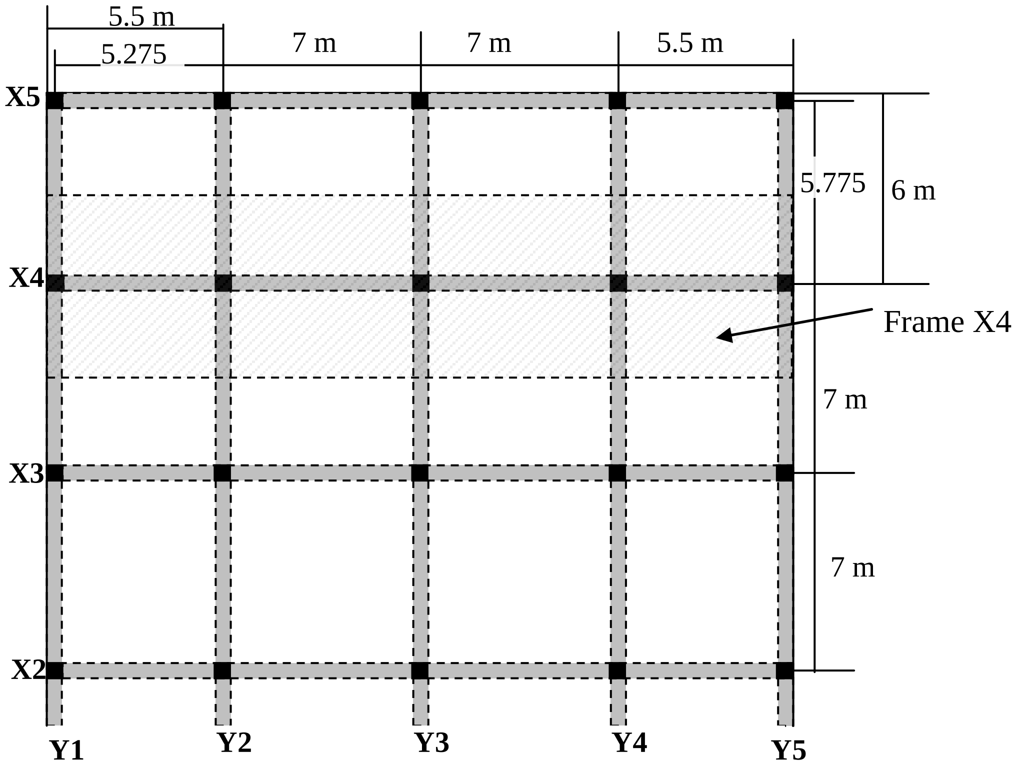
If $\alpha_1 L_2/L_1 < 1.0$ a linear interpolation is performed of (between 0 and 85 % for beam part).

Beam direct loading

It must be pointed out that the beam moments must be added to those caused by direct loading on beams. Beam loading causes moments in the beam only. These are determined using the same approach, by computing the static moment, then deducing positive and negative moments using the same coefficients. Beam direct loading includes the weight of the beam web (not considered in the slab load) and any possible wall loading.

It should lastly be signaled that beam presence cancels the risk of punching shear.

Two way solid slab example 6: Slab with beams



The figure shows a two-way slab with beams. The slab thickness is 170 mm.

All columns have the same square section 450 x 450 mm and the beams have the same section with a width of 450 mm and a total thickness of 450 mm.

Concrete: $f_c^{'} = 25\,MPa$ $\gamma_c = 24\,kN/m^3$ Steel: $f_y = 420\,MPa$

Superimposed dead load SDL = 1.4 kN/m^2 Live load LL = 3.8 kN/m^2

All edge beams support a wall with a weight of 4.5 kN/m

The span ratio (L_{max}/L_{min}) is less than two for all panels (two way action).

Factored slab load $w_u = 1.4(24\,x\,0.170 + 1.4) + 1.7\,x\,3.8 = 14.132\,kN/m^2$

The slab has a vertical axis of symmetry (column line Y3) so only half of the floor panels are considered.

1/ Check the slab thickness

The minimum slab thickness depends on the average beam relative stiffness α_m. For each of the four beams of a panel, α depends on the beam section (L-section for edge beam and T-section for internal beam) and on the slab panel width.

In all, there are four different panels to consider for the determination of the minimum slab thickness:

(a): Corner panel, with two edge beams and two internal beams

(b): Vertical edge panel, with one edge beam and three internal beams

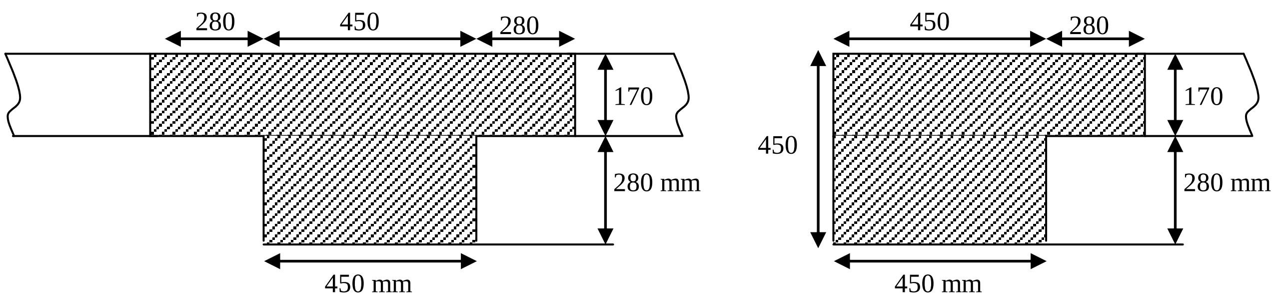
(c): Horizontal edge panel, with one edge beam and three internal beams

(d): Internal panel, with four internal beams

Effective sections of beams

The added offset from the basic rectangular shape is equal to $Min(h_w, 4h_s) = Min(280, 680) = 280\,mm$

The cross sections of the internal beam and edge beams are shown.



Internal beam Edge beam

The beam relative stiffness $\alpha = \frac{I_b}{I_s}$ is determined by computing the moments of inertia for both beam and slab sections (about centroid axis) or by using the chart in Fig. 13.21 of the Textbook. We find for beams:

Internal beam: $I_b = 4.9157\,x10^9\ mm^4$ **Edge beam:** $I_b = 4.2872\,x10^9\ mm^4$

The slab section inertia is $I_s = \frac{b_s h_s^3}{12}$ where b_s is the width of the panel and h_s = 170 mm

There are two different slab panel widths for the edge beams (lines Y1 and X5) and three slab panel widths for the internal beam (lines Y2, Y3 and X4, line X3 is similar to line Y3)

Slab panel width along edge line Y1: $b_s = \frac{5775}{2} + \frac{450}{2} = 3112.5\,mm$ thus $I_s = 1.2743\,x10^9\ mm^4$

This gives for edge beam along line Y1: $\alpha = 3.364$

Slab panel width along edge line X5: $b_s = \frac{5275}{2} + \frac{450}{2} = 2862.5\,mm$ thus $I_s = 1.1720\,x10^9\ mm^4$

This gives for edge beams along line X5: $\alpha = 3.658$

Slab panel width along internal line Y2: $b_s = \frac{5775}{2} + \frac{7000}{2} = 6387.5\,mm$ thus $I_s = 2.6151\,x10^9\ mm^4$

This gives for internal beams along line Y2: $\alpha = 1.880$

Slab panel width along internal lines Y3 and X3: b_s = 7000 mm thus $I_s = 2.8659\,x10^9\ mm^4$

This gives for internal beams along lines Y3 and X3: $\alpha = 1.715$

Slab panel width along internal line X4: $b_s = \frac{5275}{2} + \frac{7000}{2} = 6137.5\,mm$ thus $I_s = 2.5128\,x10^9\ mm^4$

This gives for internal beams along lines X4: $\alpha = 1.956$

For example, for the internal panel, the average beam relative stiffness is:

$$\alpha_m = \frac{1.880 + 1.715 + 1.715 + 1.956}{4} = 1.8165$$

The minimum thickness is defined as follows:

(a) $\alpha_m \le 0.2$: Use minimum thickness Table 13.1 for flat plate

(b) $0.2 < \alpha_m < 2.0$: $$h_{\min} = \frac{L_n\left[0.8 + \frac{f_y}{1500}\right]}{36 + 5\beta(\alpha_m - 0.2)}$$ Equation (13.10)

$$h_{\min} \ge 120\, mm$$

(c) $\alpha_m \ge 2.0$: $$h_{\min} = \frac{L_n\left[0.8 + \frac{f_y}{1500}\right]}{36 + 9\beta}$$ Equation (13.11)

$$h_{\min} \ge 90\, mm$$

L_n is the maximum clear length of the panel and β is the clear length ratio (Max L_n / Min L_n)

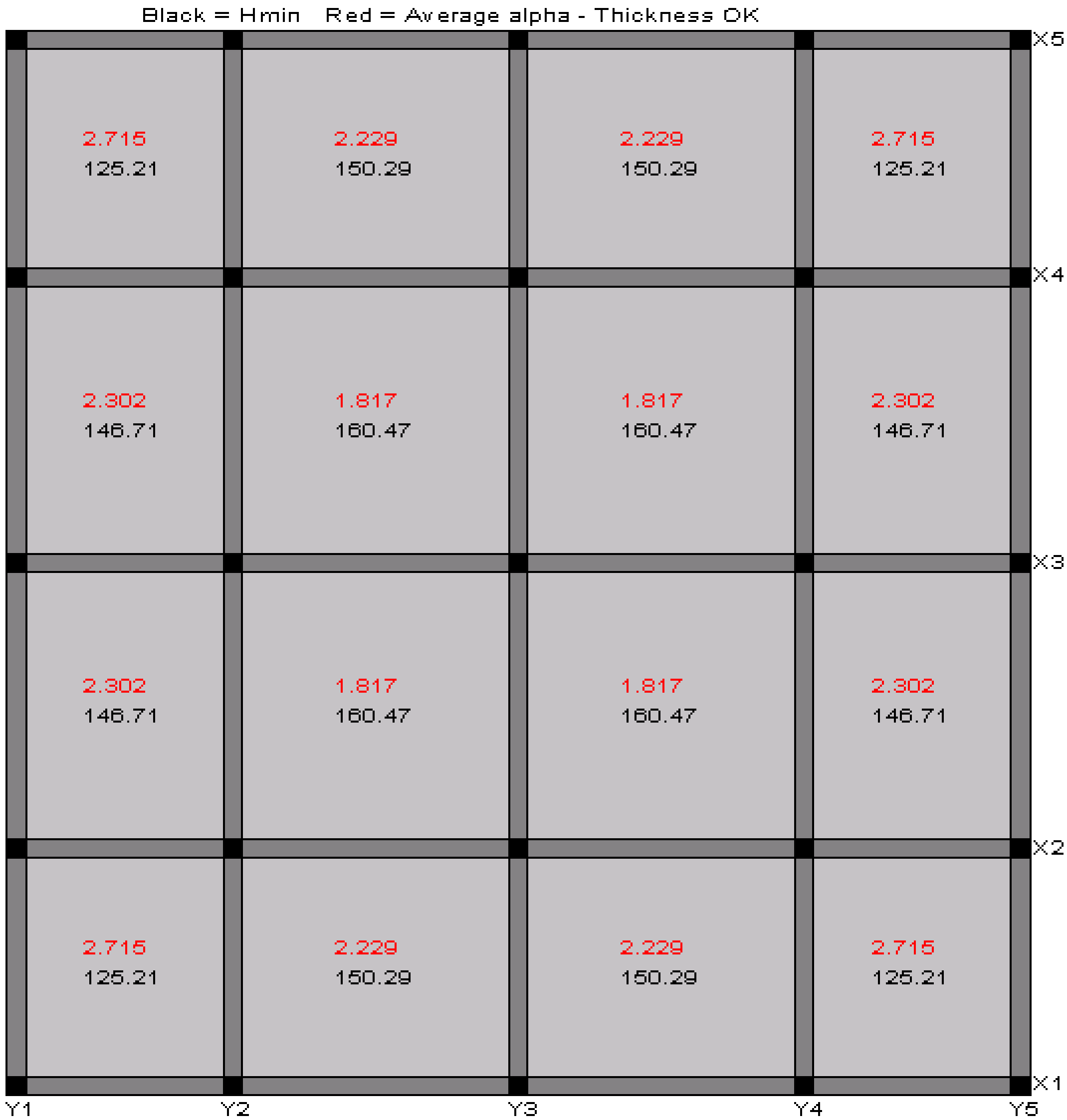
$$L_n = Max(L_{n1}, L_{n2}) \qquad \beta = \frac{Max(L_{n1}, L_{n2})}{Min(L_{n1}, L_{n2})}$$

The next Table gives all the calculations leading to minimum thickness for the four panels

Panel	**Corner**	**Horizontal edge**	**Vertical edge**	**Internal**
$\boldsymbol{L_{n1}}$ **(mm)**	5275-450 = 4825	7000-450 = 6550	5275-450 = 4825	7000-450 = 6550
$\boldsymbol{L_{n2}}$ **(mm)**	5775-450 = 5325	5775-450 = 5325	7000-450 = 6550	7000-450 = 6550
$\boldsymbol{L_n}$	5325	6550	6550	6550
β	1.104	1.230	1.358	1.000
α_m	2.715	2.229	2.302	1.817
$\boldsymbol{h_{min}}$ **equation**	**13.11, ACI 9.13**	**13.11, ACI 9.13**	**13.11, ACI 9.13**	**13.10, ACI 9.12**
$\boldsymbol{h_{min}}$ **value (mm)**	125.20	150.29	146.70	**160.46**

The minimum slab thickness is therefore 160.46 mm. The actual thickness of 170 mm is therefore OK.

The next figure shows the thickness check using RC-SLAB2 software. Minimum thickness and average beam relative stiffness, are displayed for each panel



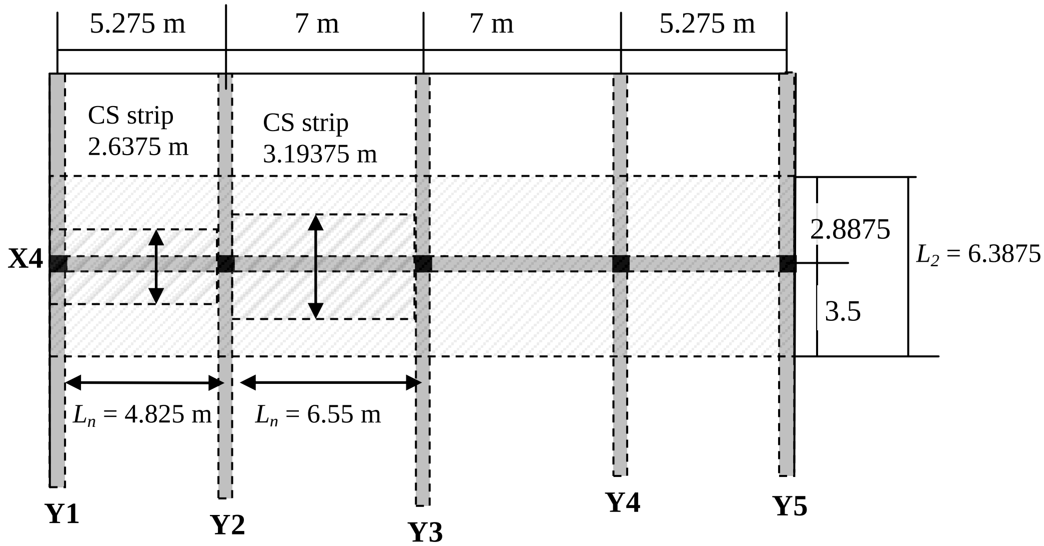
Thickness check using RC-SLAB2 software

RC-SLAB2 delivers also values of beam relative stiffnesses as well as torsion constants for edge beams and performs thickness check for beams, as shown in the next listing.

```
 Relative beam flexural stiffnes Alpha = Ib/Is
 where Ib and Is are the beam and slab moment of inertia
 The slab width is that of the frame.
 Edge beams have torsional constants C
 Each frame has 2 torsional relative parameters Beta-t = C/(2Is)
 C is the torsion constant for the 2 perpendicular edge beams

 X-Frames:        Alpha       Beta-t (left)   Beta-t (right)
Frame X1:         3.364           2.095           2.095
Frame X2:         1.880           1.021           1.021
Frame X3:         1.715           0.932           0.932
Frame X4:         1.880           1.021           1.021
Frame X5:         3.364           2.095           2.095

 Y-Frames:        Alpha       Beta-t (bot)    Beta-t (top)
Frame Y1:         3.658           2.278           2.278
Frame Y2:         1.956           1.063           1.063
Frame Y3:         1.715           0.932           0.932
Frame Y4:         1.956           1.063           1.063
Frame Y5:         3.658           2.278           2.278

 Edge  beam torsion constant C (m4)
 Bottom edge X-beam:     C =   0.00534059
 Top edge X-beam:        C =   0.00534059
 Left edge Y-beam:       C =   0.00534059
 Rigth edge Y-beam:      C =   0.00534059

 X-Beams:  Real thickness (mm) =  450.00
 Minimum thickness (mm) =       333.33

 Y-Beams:  Real thickness (mm) =  450.00
 Minimum thickness (mm) =       333.33

 All Beam thicknesses OK
```

2/ Analysis of internal frame X4

Because of symmetry (about column line Y3), only two spans are considered. The figure below shows the panel dimensions as well the column strip widths for panel X4Y1-X4Y2 and panel X4Y2-X4Y3



For each panel, the dimensions, the clear length and the static moment are computed.

L_1 is the length of the panel (parallel to line X4) and L_2 is its width. L_n is the clear length.

The static moment in each span is given by: $M_0 = w_u L_2 \dfrac{L_n^2}{8}$.

Negative and positive moments in each span as well as portions of moments in column strips, are deduced using appropriate coefficients. The CS moments must then be distributed between the beam and the slab. The following Table gives all the results.

Direct beam load includes the beam web self weight and any possible wall weight. For beam X4, there is no wall. The factored beam web weight is $w_{beam} = 1.4\gamma_c b h_w = 1.4 x 24 x 0.45 x 0.28 = 4.234 kN/m$

Beam loading causes moments in the beam only. These are determined using the same approach, by computing the static moment, then deducing positive and negative moments using the same moment coefficients of row R10. For each span, the beam static moment is computed as: $M_{0b} = w_{beam} \dfrac{L_n^2}{8}$

Moments along Frame X4

R1		**Span X3Y1-X3Y2**			**Span X3Y2-X3Y3**		
R2	L_1 (m)	5.275			7.0		
R3	L_2 (m)	6.3875			6.3875		
R4	L_n (m)	4.825			6.55		
R5	M_0 $(kN.m)$	262.7			484.0		
R6	CS width (m) $0.5L_{min}$	2.6375			3.19375		
R7	α_1	1.88			1.88		
R8	$\alpha_1 \, {}^{L_2}\!/\!{}_{L_1}$	2.27			1.71		
R9	β_t	1.02					
R10	Moment coefficients	-0.16	0.57	-0.70	-0.65	0.35	-0.65
R11	-ve and +ve moments	-42.0	149.7	-183.9	-314.6	169.4	-314.6
R12	CS moment (%)	87.2	68.7	68.7	77.6	77.6	77.6
R13	CS moments ($kN.m$)	-36.6	102.8	-126.3	-244.2	131.5	-244.2
R14	Slab portion (15 %)	-5.5	15.4	-18.9	-36.6	19.7	-36.6
R15	Beam portion (85 %)	-31.1	87.4	-107.4	-207.6	111.8	-207.6
R16	Beam static moment	12.3			22.7		
R17	-ve / +ve M_{add} in beams	-2.0	7.0	-8.6	-14.8	8.0	-14.8
R18	Total beam moments	-33.1	94.4	-116.0	-222.4	119.8	-222.4

The moment coefficients in row R10 are determined using static moment distribution Table:

Distribution of factored static moment

	(a)	**(b)**	**(c)**	**(d)**	**(e)**
Int. negative M	0.75	0.70	0.70	0.70	0.65
Positive M	0.63	0.57	0.52	0.50	0.35
Ext. negative M	0	0.16	0.26	0.30	0.65

(a): Exterior edge unrestrained **(b):** Slab with beams between all supports

(c): Slab with no beams at all **(d):** Slab with edge beams only

(e): Slab with exterior edge fully restrained (interior span)

For the first span, we use column (b) and for the internal span we use column (e).

The CS portion of moments is computed by interpolating using Tables below, according to $\alpha_1 L_2/L_1$ and L_2/L_1. For exterior negative moments, the distribution depends also on the torsion parameter β_t.

For frame X4, $\beta_t = 1.02$.

For the exterior moment for instance, the double interpolation gives a portion of 87.2 % for the CS moment.

The CS moments are divided between the beam and the slab according to $\alpha_1 L_2/L_1$. This value is greater than 1.0 in all cases. Therefore, 85 % of the CS moments are assigned to the beam and 15 % to the slab.

The total beam moments are obtained by adding the slab loading moments (R15) and the beam load moments (R17).

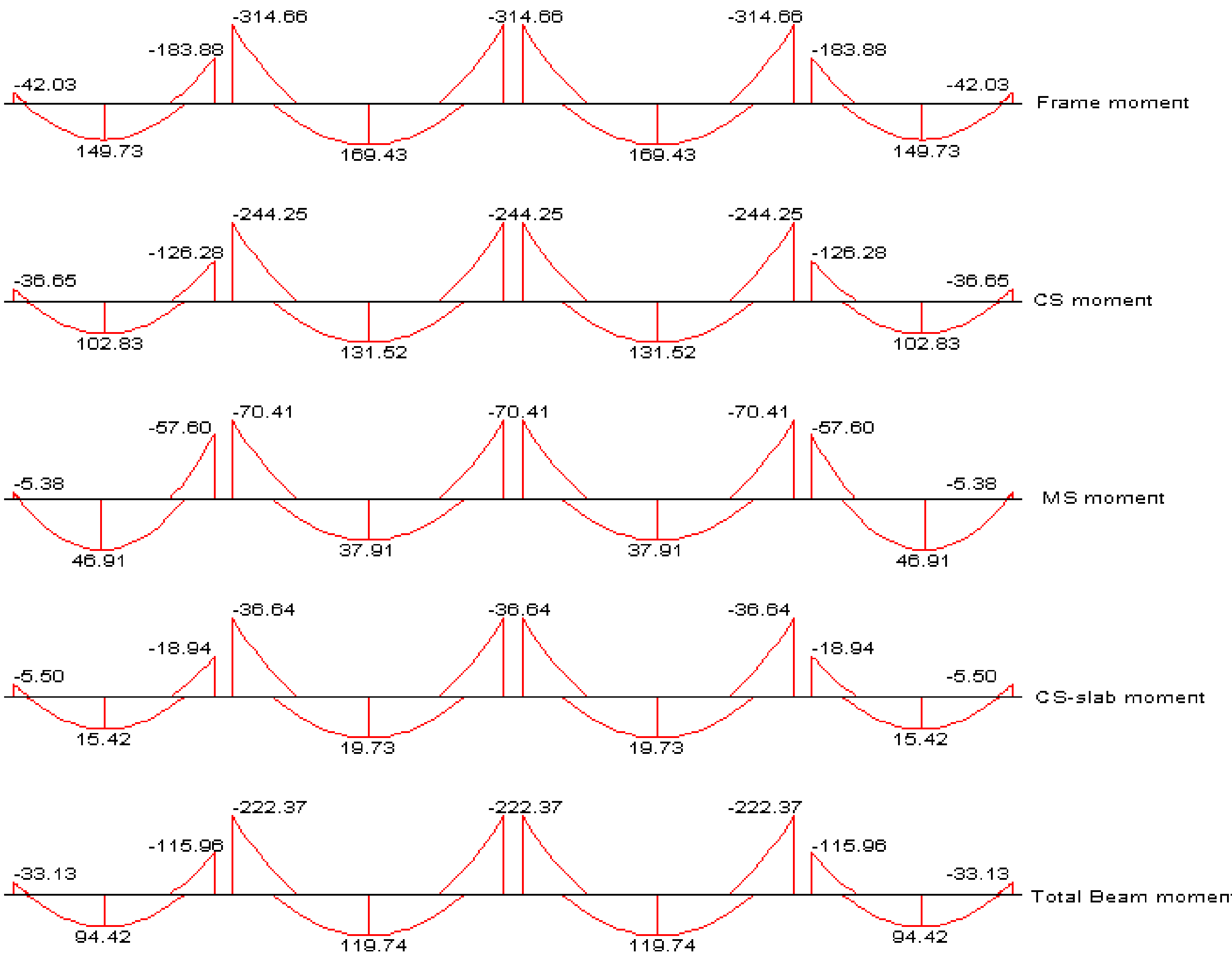
Portion (%) of column strip moment

<table>
<tr><th></th><th colspan="3">Interior negative moment</th><th colspan="3">Positive moment</th><th colspan="4">Exterior negative moment</th></tr>
<tr><td>L_2/L_1</td><td>0.5</td><td>1.0</td><td>2.0</td><td>0.5</td><td>1.0</td><td>2.0</td><td>0.5</td><td>1.0</td><td>2.0</td><td></td></tr>
<tr><td rowspan="2">$\alpha_1 L_2/L_1 = 0$</td><td rowspan="2">75</td><td rowspan="2">75</td><td rowspan="2">75</td><td rowspan="2">60</td><td rowspan="2">60</td><td rowspan="2">60</td><td>100</td><td>100</td><td>100</td><td>$\beta_t = 0$</td></tr>
<tr><td>75</td><td>75</td><td>75</td><td>$\beta_t \geq 2.5$</td></tr>
<tr><td rowspan="2">$\alpha_1 L_2/L_1 \geq 1.0$</td><td rowspan="2">90</td><td rowspan="2">75</td><td rowspan="2">45</td><td rowspan="2">90</td><td rowspan="2">75</td><td rowspan="2">45</td><td>100</td><td>100</td><td>100</td><td>$\beta_t = 0$</td></tr>
<tr><td>90</td><td>75</td><td>45</td><td>$\beta_t \geq 2.5$</td></tr>
</table>

Linear interpolation must be performed at intermediate points

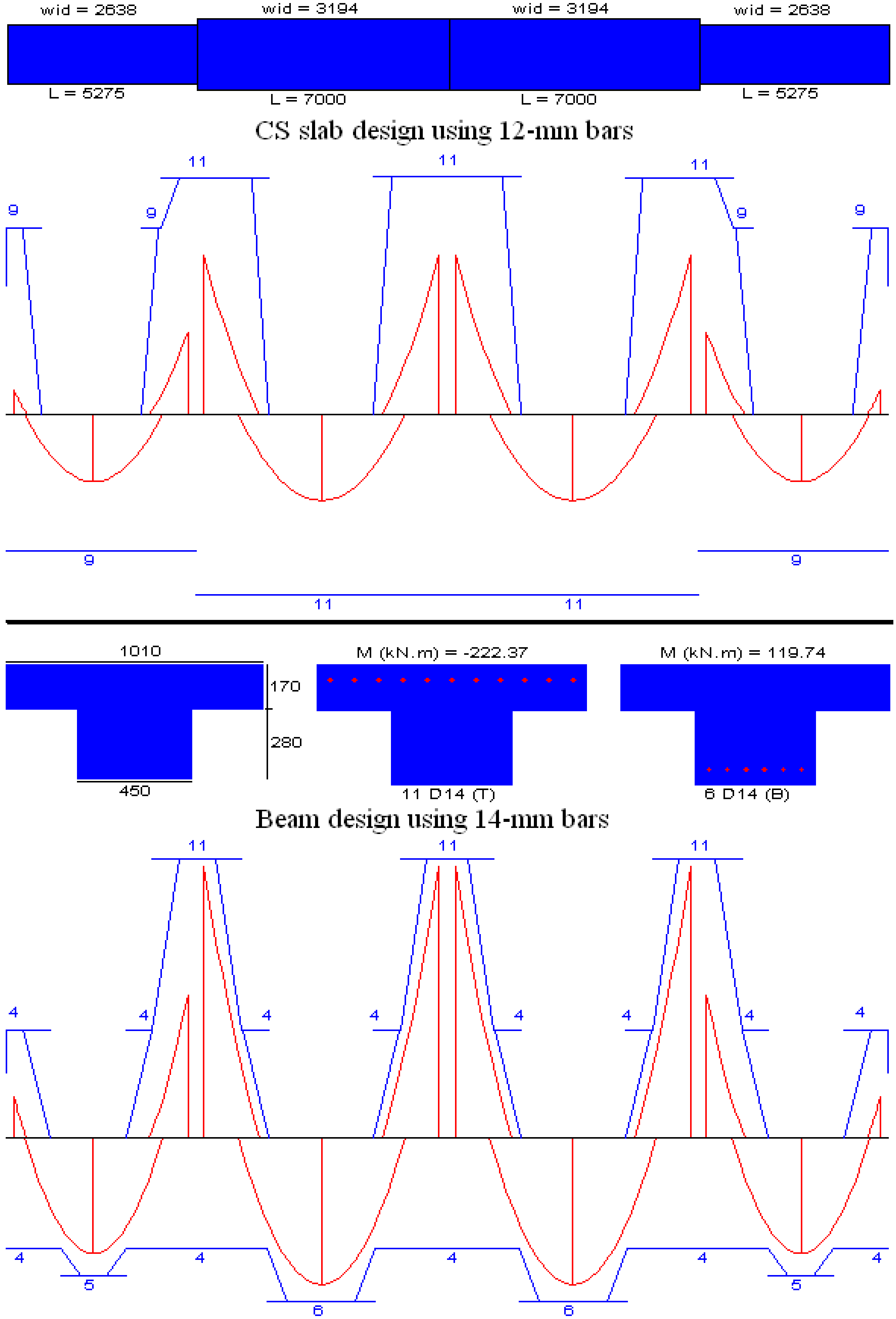
Analysis of Frame X4 Slab ultimate load (kN/m2) = 14.132 Ultimate beam load (kN/m) = 4.234

Span		X4Y1 - X4Y2			X4Y2 - X4Y3			X4Y3 - X4Y4			X4Y4 - X4Y5	
L1 (m)		5.2750			7.0000			7.0000			5.2750	
L2 (m)		6.3875			6.3875			6.3875			6.3875	
Ln (m)		4.8250			6.5500			6.5500			4.8250	
Mo (kN.m)		262.687			484.091			484.091			262.687	
CS (m)		2.63750			3.19375			3.19375			2.63750	
Alpha-1		1.880			1.880			1.880			1.880	
L2 / L1		1.211			0.912			0.912			1.211	
Alfa-1.L2/L1		2.276			1.715			1.715			2.276	
Beta-t		1.021									1.021	
M-Coef	-0.16	0.57	-0.70	-0.65	0.35	-0.65	-0.65	0.35	-0.65	-0.70	0.57	-0.16
M (kN.m)	-42.03	149.73	-183.88	-314.66	169.43	-314.66	-314.66	169.43	-314.66	-183.88	149.73	-42.03
CS-M %	87.20	68.67	68.67	77.62	77.62	77.62	77.62	77.62	77.62	68.67	68.67	87.20
CS-M	-36.65	102.83	-126.28	-244.25	131.52	-244.25	-244.25	131.52	-244.25	-126.28	102.83	-36.65
MS-M	-5.38	46.91	-57.60	-70.41	37.91	-70.41	-70.41	37.91	-70.41	-57.60	46.91	-5.38
Beam %		85.00			85.00			85.00			85.00	
Beam M	-31.15	87.40	-107.34	-207.62	111.79	-207.62	-207.62	111.79	-207.62	-107.34	87.40	-31.15
Slab M	-5.50	15.42	-18.94	-36.64	19.73	-36.64	-36.64	19.73	-36.64	-18.94	15.42	-5.50
Beam Mo		12.32			22.70			22.70			12.32	
Beam Mx	-1.97	7.02	-8.62	-14.76	7.95	-14.76	-14.76	7.95	-14.76	-8.62	7.02	-1.97
Beam Mtot	-33.13	94.42	-115.96	-222.37	119.74	-222.37	-222.37	119.74	-222.37	-115.96	94.42	-33.13



RC-SLAB2 analysis results of frame X4

The next figure shows RC design results of the slab part in the column strip using RC-SLAB2 software.

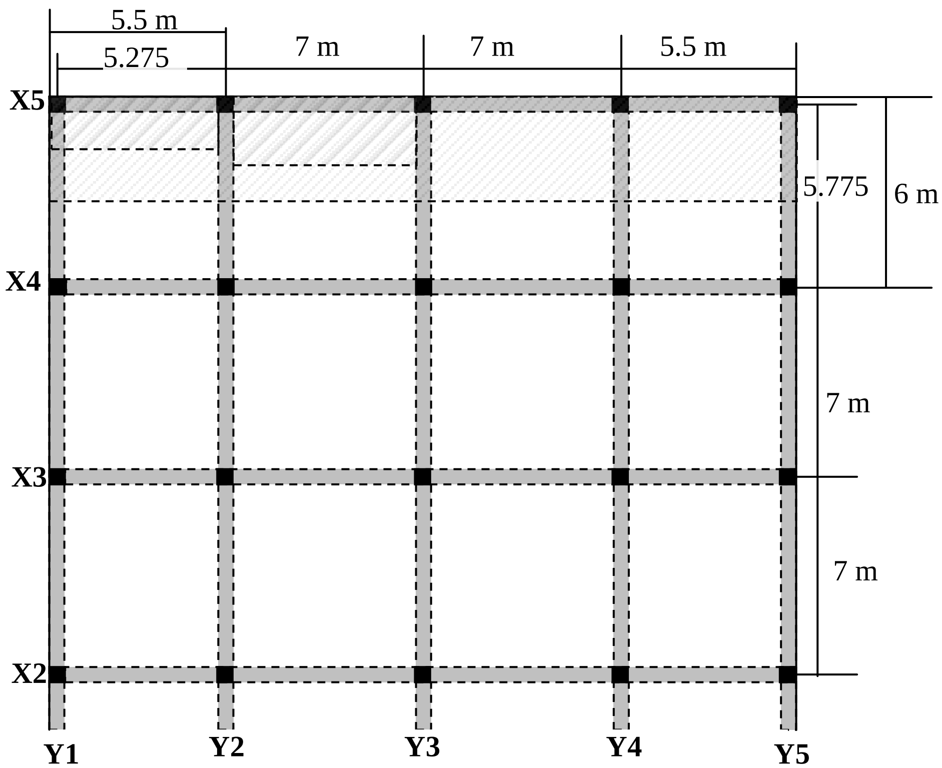


The software displays the variable slab model width as well as bar cutoff. The minimum bar number is also variable (9 in end spans and 11 in internal spans).

Middle strips are designed as in flat plates by considering the contributions from the two adjacent frames.

3/ Analysis of edge frame X5

A similar procedure is used for this edge frame.



The first particularity is that when computing ratios L_2/L_1 and $\alpha_1 L_2/L_1$, the panel width L_2 to be considered is equal to the full edge panel width, which is 5.775 mm.

The second particularity is that beam loading includes the web weight as well as the wall weight:

$$w_{beam} = 1.4(\gamma\, bh_w + w_{wall}) = 1.4(24\,x\,0.45\,x\,0.28 + 4.5) = 10.534\ kN/m$$

For frame X5, the torsion relative stiffness is: $\beta_t = 2.10$

Column strip width is equal to $\dfrac{L_{\min}}{4} + Offset$

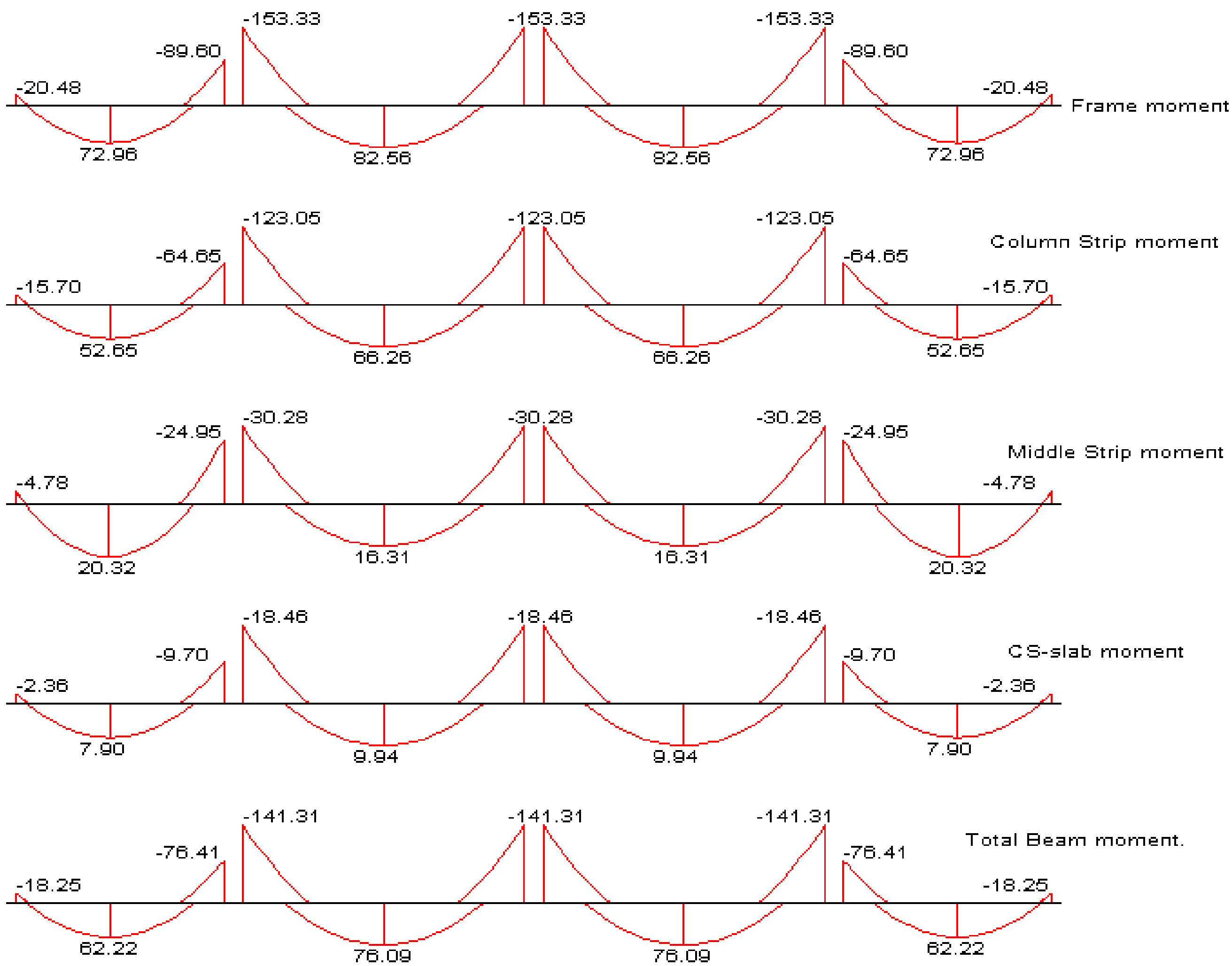
The same steps are used and the results are presented in a tabular form.

For middle strips, the moments are determined by analyzing the two bounding frames and summing the contributions from both frames.

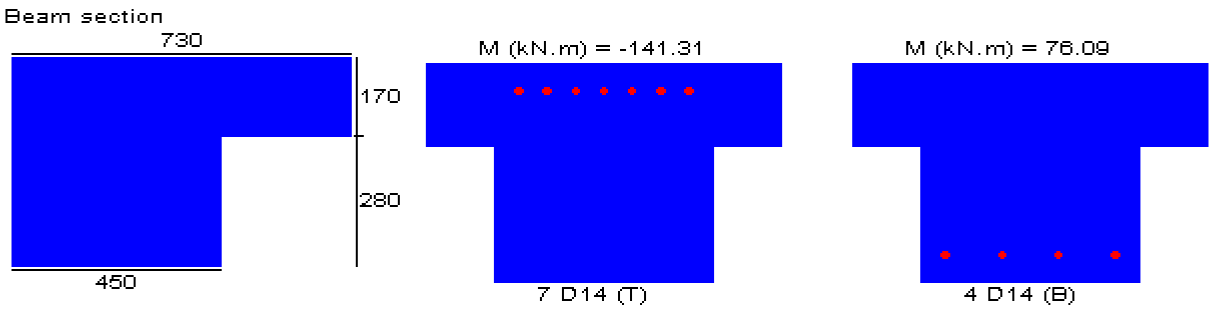
Moments in frame X5

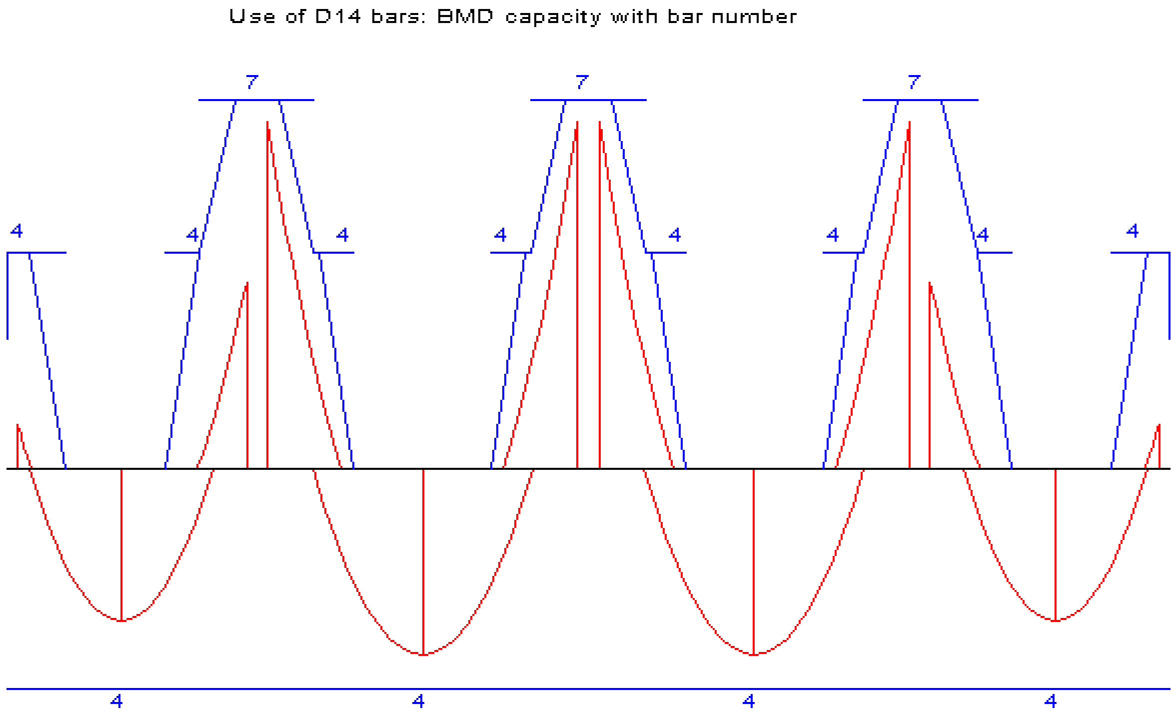
R1		**Span X5Y1-X5Y2**			**Span X5Y2-X5Y3**		
R2	L_1 (m)	5.275			7.0		
R3	L_2 (m)	3.1125			3.1125		
R4	$L_2^{'}$ (m) for L_2 / L_1	5.775			5.775		
R5	L_n (m)	4.825			6.55		
R6	M_0 $(kN.m)$	128.0			235.9		
R7	CS width (m)	1.54375			1.66875		
R8	α_1	3.364			3.364		
R9	$\alpha_1 L_2^{'} / L_1$	3.68			2.78		
R10	β_t	2.10			2.10		
R11	Moment coefficients	-0.16	0.57	-0.70	-0.65	0.35	-0.65
R12	-ve and +ve moments	-20.5	73.0	-89.6	-153.3	82.6	-153.3
R13	CS moment (%)	76.7	72.2	72.2	80.3	80.3	80.3
R14	CS moments $(kN.m)$	-15.7	52.7	-64.7	-123.1	66.3	-123.1
R15	Slab portion (15 %)	-2.4	7.9	-9.7	-18.5	9.9	-18.5
R16	Beam portion (85 %)	-13.3	44.8	-55.0	-104.6	56.4	-104.6
R17	Beam static moment	30.65			56.49		
R18	-ve / +ve M_{add} in beams	-4.9	17.5	-21.5	-36.7	19.8	-36.7
R19	Total beam moments	-18.2	62.3	-76.5	-141.3	76.2	-141.3

Span		X5Y1 - X5Y2			X5Y2 - X5Y3			X5Y3 - X5Y4			X5Y4 - X5Y5	
L1 (m)		5.2750			7.0000			7.0000			5.2750	
L2 (m)		3.1125			3.1125			3.1125			3.1125	
L2' (L2 / L1)		5.7750			5.7750			5.7750			5.7750	
Ln (m)		4.8250			6.5500			6.5500			4.8250	
Mo (kN.m)		128.002			235.888			235.888			128.002	
CS (m)		1.54375			1.66875			1.66875			1.54375	
Alpha-1		3.364			3.364			3.364			3.364	
L2 / L1		1.095			0.825			0.825			1.095	
Alfa-1.L2/L1		3.683			2.776			2.776			3.683	
Beta-t		2.095									2.095	
M-Coef	-0.16	0.57	-0.70	-0.65	0.35	-0.65	-0.65	0.35	-0.65	-0.70	0.57	-0.16
M (kN.m)	-20.48	72.96	-89.60	-153.33	82.56	-153.33	-153.33	82.56	-153.33	-89.60	72.96	-20.48
CS-M %	76.66	72.16	72.16	80.25	80.25	80.25	80.25	80.25	80.25	72.16	72.16	76.66
CS-M	-15.70	52.65	-64.65	-123.05	66.26	-123.05	-123.05	66.26	-123.05	-64.65	52.65	-15.70
MS-M	-4.78	20.32	-24.95	-30.28	16.31	-30.28	-30.28	16.31	-30.28	-24.95	20.32	-4.78
Beam %		85.00			85.00			85.00			85.00	
Beam M	-13.35	44.75	-54.96	-104.59	56.32	-104.59	-104.59	56.32	-104.59	-54.96	44.75	-13.35
Slab M	-2.36	7.90	-9.70	-18.46	9.94	-18.46	-18.46	9.94	-18.46	-9.70	7.90	-2.36
Beam Mo		30.65			56.49			56.49			30.65	
Beam Mx	-4.90	17.47	-21.46	-36.72	19.77	-36.72	-36.72	19.77	-36.72	-21.46	17.47	-4.90
Beam Mtot	-18.25	62.22	-76.41	-141.31	76.09	-141.31	-141.31	76.09	-141.31	-76.41	62.22	-18.25



Analysis of frame X5 using RC-SLAB2 software





RC design of beam X5 using RC-SLAB2 (L-section)