GE 403
Engineering Economy

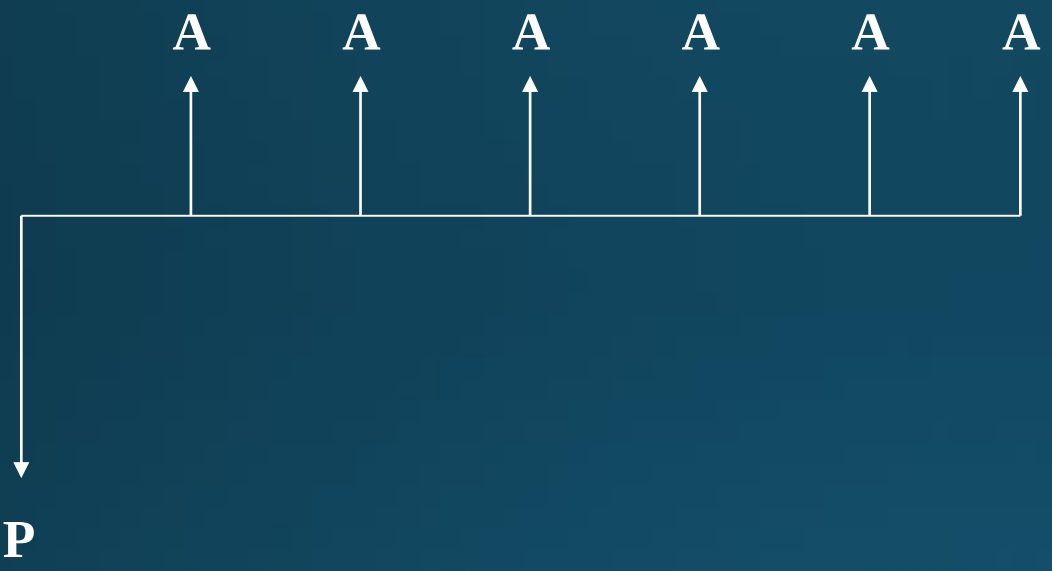
First Semester 1444 H

Eng. Howaidi Alotaibi

Civil Engineering Department

E-mail halshaibani@ksu.edu.sa

DCF Uniform Series Formulas



P occurs 1 period before first A

$$P = \frac{A[(1+i)^n-1]}{i(1+i)^n}$$

$$P = A(P|A\ i\%, n)$$

$$A = \frac{Pi(1+i)^n}{(1+i)^n-1}$$

$$A = P(A|P\ i\%, n)$$

Annual payments: every year/quarter/month/..etc

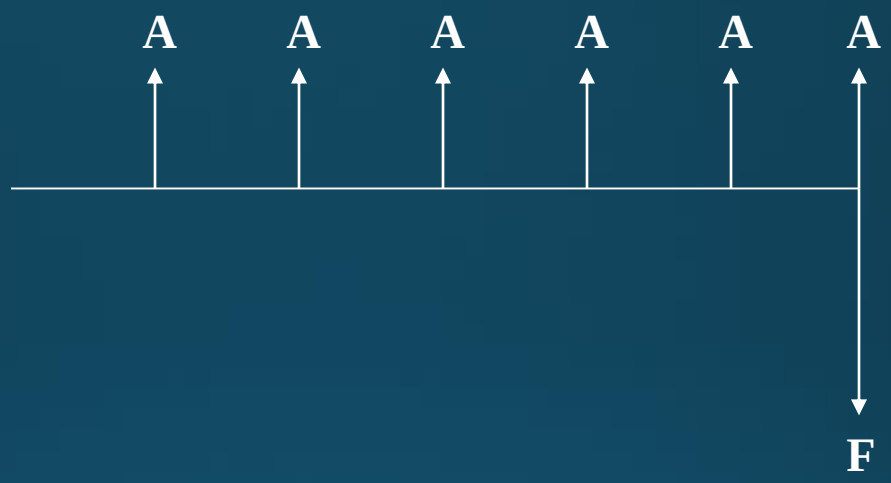
sized and spaced
one interval after P

$$F = \frac{A[(1+i)^n-1]}{i}$$

$$F = A(F|A\ i\%, n)$$

$$A = \frac{Fi}{(1+i)^n-1}$$

$$A = F(A|F\ i\%, n)$$



F occurs at the same time as last A

Note: Assume annual worth starts from 1 if not mentioned

Conclusion: Uniform Series Formulas

$$P = A(P|A\ i\%,n) = A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right] \qquad (2.22)$$

$$A = P(A|P\ i\%,n) = P\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right] \qquad (2.25)$$

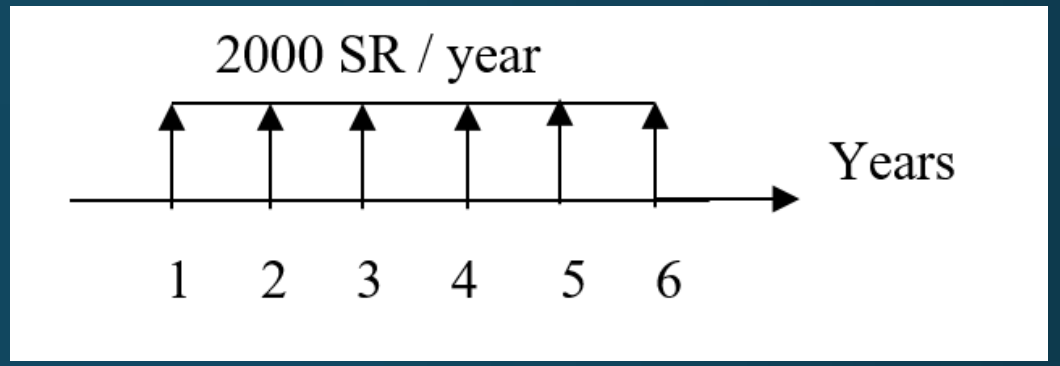
P occurs one period before the first A

$$F = A(F|A\ i\%,n) = A\left[\frac{(1+i)^n - 1}{i}\right] \qquad (2.28)$$

$$A = F(A|F\ i\%,n) = F\left[\frac{i}{(1+i)^n - 1}\right] \qquad (2.30)$$

F occurs at the same time as the last A

Ex. Find the present worth and future worth for this series of cash flow if i= 8% compounded annually.



Solution

P = A(P/A i%, n) = 2000(P/A 8%, 6) = 2000(4.62288) = 9245.76 SR P at t =0 ⟹ Pw

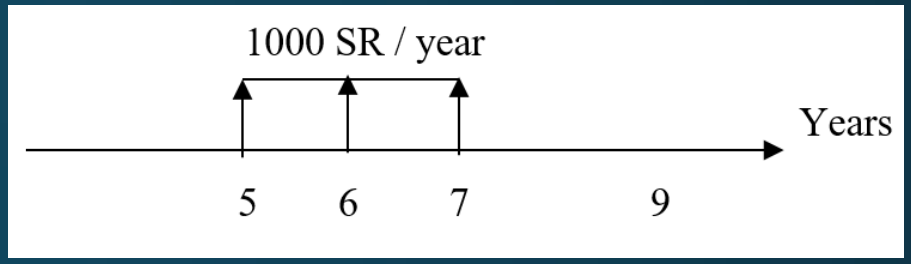
F = A(F/A i%, n) = 2000(F/A 8%, 6) = 2000(7.33593) = 14671.86 SR ⟹ Fw

OR **F= P(F/P i%, n) =9245.76 (F/P 8%, 6) =9245.76 (1.58687) = 14671.82 SR ⟹ Fw**

TABLE A-a-14

n	Single Sums: To Find F Given P $(F\|P\ i\%,n)$	Single Sums: To Find P Given F $(P\|F\ i\%,n)$	Uniform Series: To Find F Given A $(F\|A\ i\%,n)$	Uniform Series: To Find A Given F $(A\|F\ i\%,n)$	Uniform Series: To Find P Given A $(P\|A\ i\%,n)$	Uniform Series: To Find A Given P $(A\|P\ i\%,n)$	Gradient Series: To Find P Given G $(P\|G\ i\%,n)$	Gradient Series: To Find A Given G $(A\|G\ i\%,n)$
1	1.08000	0.92593	1.00000	1.00000	0.92593	1.08000	0.00000	0.00000
2	1.16640	0.85734	2.08000	0.48077	1.78326	0.56077	0.85734	0.48077
3	1.25971	0.79383	3.24640	0.30803	2.57710	0.38803	2.44500	0.94874
4	1.36049	0.73503	4.50611	0.22192	3.31213	0.30192	4.65009	1.40396
5	1.46933	0.68058	5.86660	0.17046	3.99271	0.25046	7.37243	1.84647
6	1.58687	0.63017	7.33593	0.13632	4.62288	0.21632	10.52327	2.27635
7	1.71382	0.58349	8.92280	0.11207	5.20637	0.19207	14.02422	2.69366
8	1.85093	0.54027	10.63663	0.09401	5.74664	0.17401	17.80610	3.09852
9	1.99900	0.50025	12.48756	0.08008	6.24689	0.16008	21.80809	3.49103
10	2.15892	0.46319	14.48656	0.06903	6.71008	0.14903	25.97683	3.87131
11	2.33164	0.42888	16.64549	0.06008	7.13896	0.14008	30.26566	4.23950
12	2.51817	0.39711	18.97713	0.05270	7.53608	0.13270	34.63391	4.59575
13	2.71962	0.36770	21.49530	0.04652	7.90378	0.12652	39.04629	4.94021
14	2.93719	0.34046	24.21492	0.04130	8.24424	0.12130	43.47228	5.27305
15	3.17217	0.31524	27.15211	0.03683	8.55948	0.11683	47.88566	5.59446
16	3.42594	0.29189	30.32428	0.03298	8.85137	0.11298	52.26402	5.90463
17	3.70002	0.27027	33.75023	0.02963	9.12164	0.10963	56.58832	6.20375
18	3.99602	0.25025	37.45024	0.02670	9.37189	0.10670	60.84256	6.49203
19	4.31570	0.23171	41.44626	0.02413	9.60360	0.10413	65.01337	6.76969
20	4.66096	0.21455	45.76196	0.02185	9.81815	0.10185	69.08979	7.03695
21	5.03383	0.19866	50.42292	0.01983	10.01680	0.09983	73.06291	7.29403
22	5.43654	0.18394	55.45676	0.01803	10.20074	0.09803	76.92566	7.54118

Ex.2 Find the present worth for this series if i= 10% compounded annually, and also find the future worth at the end of the ninth year.



Solution

P = A(P/A i%, n)= 1000(P/A 10%, 3) = 1000(2.48685)= 2486.85 SR. P at t=4

P = F(P/F i%, n)= 2486.85 (P/F 10%, 4) = 2486.85 (0.68301)= 1698.54 SR ⟹ Pw

F = A(F/A i%, n)= 1000 (F/A 10%, 3) = 1000(3.31000)= 3310 SR. F at t=7

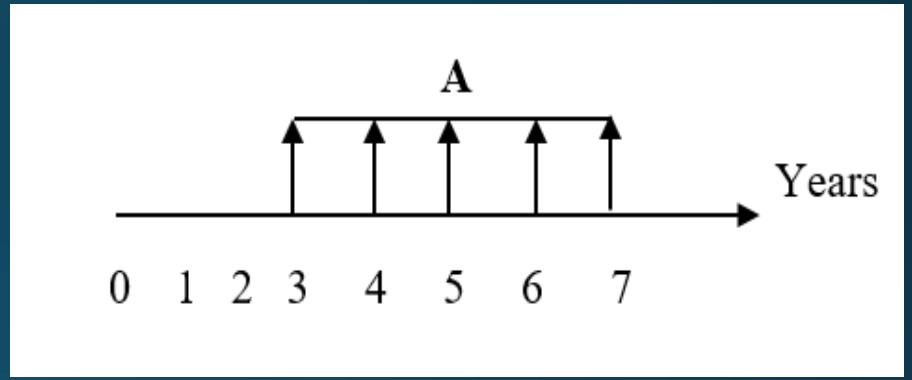
F = P(F/P i%, n)= 3310 (F/P 10%, 2) = 3310(1.21000)= 4005.1 SR ⟹ Fw

Or Fw=Pw(F/P i%, n)= 1698.54 (F/P 10%, 9) = 1698.54 (2.35795)= 4005.07 SR

TABLE A-a-16

n	Single Sums		Uniform Series				Gradient Series	
	To Find F Given P $(F\|P\ i\%,n)$	To Find P Given F $(P\|F\ i\%,n)$	To Find F Given A $(F\|A\ i\%,n)$	To Find A Given F $(A\|F\ i\%,n)$	To Find P Given A $(P\|A\ i\%,n)$	To Find A Given P $(A\|P\ i\%,n)$	To Find P Given G $(P\|G\ i\%,n)$	To Find A Given G $(A\|G\ i\%,n)$
1	1.10000	0.90909	1.00000	1.00000	0.90909	1.10000	0.00000	0.00000
2	1.21000	0.82645	2.10000	0.47619	1.73554	0.57619	0.82645	0.47619
3	1.33100	0.75131	3.31000	0.30211	2.48685	0.40211	2.32908	0.93656
4	1.46410	0.68301	4.64100	0.21547	3.16987	0.31547	4.37812	1.38117
5	1.61051	0.62092	6.10510	0.16380	3.79079	0.26380	6.86180	1.81013
6	1.77156	0.56447	7.71561	0.12961	4.35526	0.22961	9.68417	2.22356
7	1.94872	0.51316	9.48717	0.10541	4.86842	0.20541	12.76312	2.62162
8	2.14359	0.46651	11.43589	0.08744	5.33493	0.18744	16.02867	3.00448
9	2.35795	0.42410	13.57948	0.07364	5.75902	0.17364	19.42145	3.37235
10	2.59374	0.38554	15.93742	0.06275	6.14457	0.16275	22.89134	3.72546
11	2.85312	0.35049	18.53117	0.05396	6.49506	0.15396	26.39628	4.06405
12	3.13843	0.31863	21.38428	0.04676	6.81369	0.14676	29.90122	4.38840
13	3.45227	0.28966	24.52271	0.04078	7.10336	0.14078	33.37719	4.69879
14	3.79750	0.26333	27.97498	0.03575	7.36669	0.13575	36.80050	4.99553
15	4.17725	0.23939	31.77248	0.03147	7.60608	0.13147	40.15199	5.27893
16	4.59497	0.21763	35.94973	0.02782	7.82371	0.12782	43.41642	5.54934
17	5.05447	0.19784	40.54470	0.02466	8.02155	0.12466	46.58194	5.80710
18	5.55992	0.17986	45.59917	0.02193	8.20141	0.12193	49.63954	6.05256

Ex.3 Ali borrowed $10,000 at 10% compounded annually, he will pay off the loan over a 5- year period with uniform annual payments. The first payment will start from the third year. How much will be these payments?

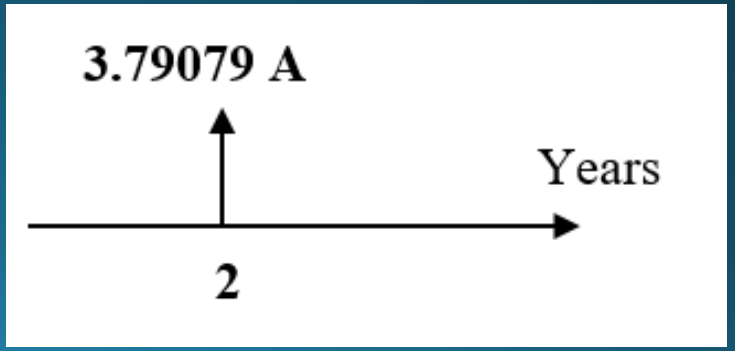


First Solution

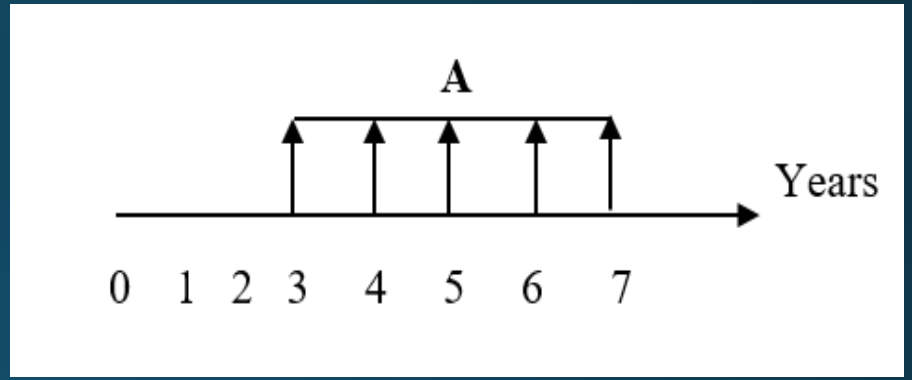
P= A(P/A 10%, 5) = A(3.79079) ⟹ P at t =2

P= F(P/F 10%, 2) = 3.79079 A (0.82645) = 3.1329A

Pw= 3.1329A=10,000 ⟹ A= $ 3,191.9



Ex.3 Ali borrowed $10,000 at 10% compounded annually, he will pay off the loan over a 5- year period with uniform annual payments. The first payment will start from the third year. How much will be these payments?



Second Solution

F= P(F/P 10%, 2)= 10,000 (1.21)= $ 12,100

P= A(P/A 10%, 5) OR A=P(A/P 10%, 5)

12,100=A (3.79079) ⟹ A= $ 3,191.9