جامعة الملك سعود
King Saud University

CSC 220: Computer Organization

Unit 1
Number Systems

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Overview

- Common Number Systems
- Conversion Among Bases
- Binary Coded Decimal (BCD)

Chapter-1

M. Morris Mano, Charles R. Kime and Tom Martin, **Logic and Computer Design Fundamentals**, Global (5^{th}) Edition, Pearson Education Limited, 2016. ISBN: 9781292096124

Decimal review

- Decimal numbers consist of digits from 0 to 9, each with a weight.

1	6	2	.	3	7	5	digits
100	10	1		1/10	1/100	1/1000	weights

- Notice that the weights are all powers of the base, which is 10.

1	6	2	.	3	7	5	digits
10^2	10^1	10^0		10^{-1}	10^{-2}	10^{-3}	weights

- To find the decimal value of a number, you can multiply each digit by its weight and sum the products:

$$(1 \times 10^2) + (6 \times 10^1) + (2 \times 10^0) + (3 \times 10^{-1}) + (7 \times 10^{-2}) + (5 \times 10^{-3}) = 162.375$$

Common Number Systems

System	Base	Symbols	Used by humans?	Used in computers?
Decimal	10	0, 1, … 9	Yes	No
Binary	2	0, 1	No	Yes
Octal	8	0, 1, … 7	No	No
Hexa-decimal	16	0, 1, … 9, A, B, … F	No	No

Quantities/Counting (1 of 3)

Decimal	Binary	Octal	Hexa-decimal
0	0	0	0
1	1	1	1
2	10	2	2
3	11	3	3
4	100	4	4
5	101	5	5
6	110	6	6
7	111	7	7

Quantities/Counting (2 of 3)

Decimal	Binary	Octal	Hexa-decimal
8	1000	10	8
9	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

Quantities/Counting (3 of 3)

Decimal	Binary	Octal	Hexa-decimal
16	10000	20	10
17	10001	21	11
18	10010	22	12
19	10011	23	13
20	10100	24	14
21	10101	25	15
22	10110	26	16
23	10111	27	17

Etc.

Binary Numbers

- Binary, or base 2, numbers consist of only the digits 0 and 1. The weights are now powers of 2.
- For example, consider the binary number 1101.01:

1	1	0	1	.	0	1	binary digits, or bits
2^3	2^2	2^1	2^0		2^{-1}	2^{-2}	weights in decimal

- The decimal value of 1101.01 is computed just like before:

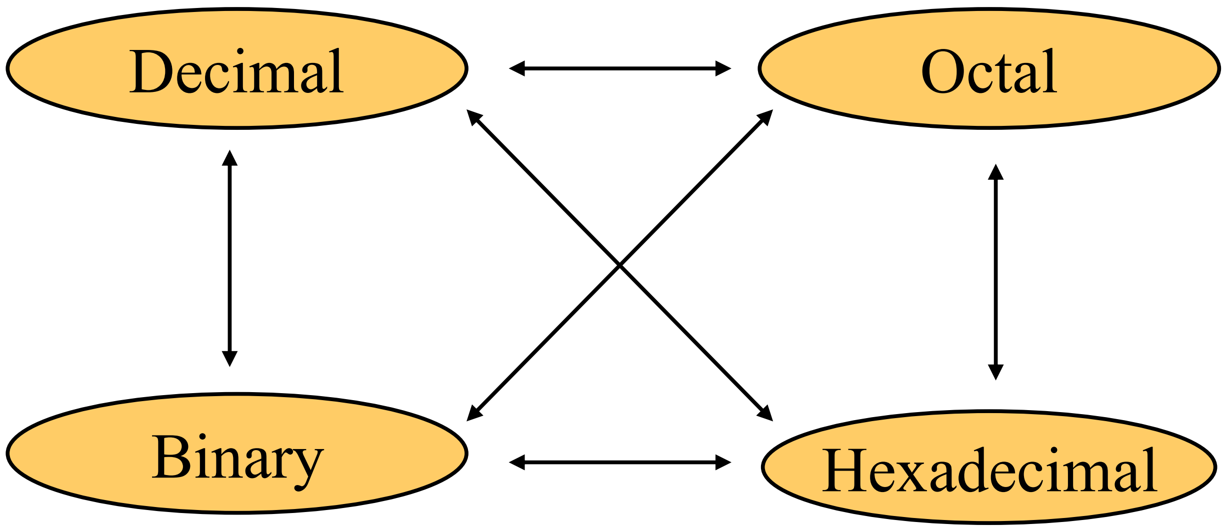
$$(1 \times 2^3) + (1 \times 2^2) + (0 \times 2^1) + (1 \times 2^0) + (0 \times 2^{-1}) + (1 \times 2^{-2}) =$$
$$8 + 4 + 0 + 1 + 0 + 0.25 = 13.25$$

Some powers of 2

$2^0 = 1$	$2^4 = 16$	$2^8 = 256$
$2^1 = 2$	$2^5 = 32$	$2^9 = 512$
$2^2 = 4$	$2^6 = 64$	$2^{10} = 1024$
$2^3 = 8$	$2^7 = 128$	

Conversion Among Bases

- The possibilities:



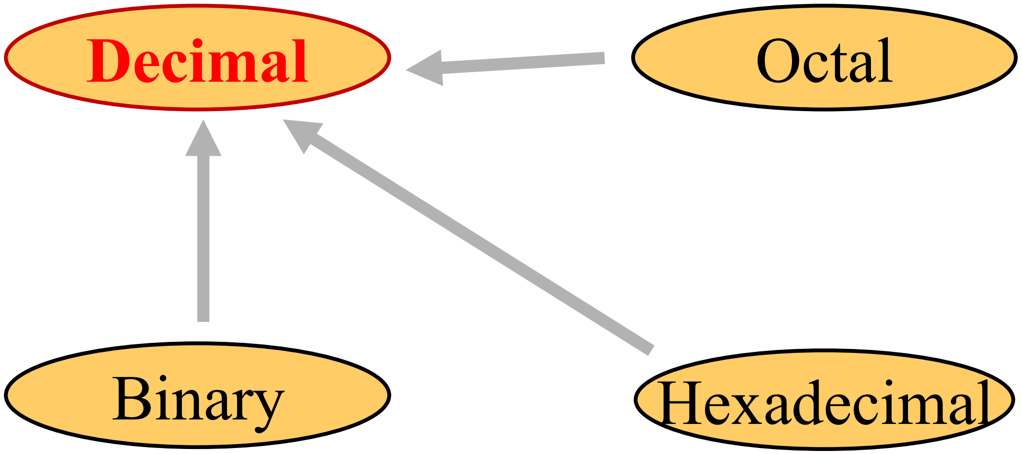
Quick Example

$$25_{10} = 11001_2 = 31_8 = 19_{16}$$

Base

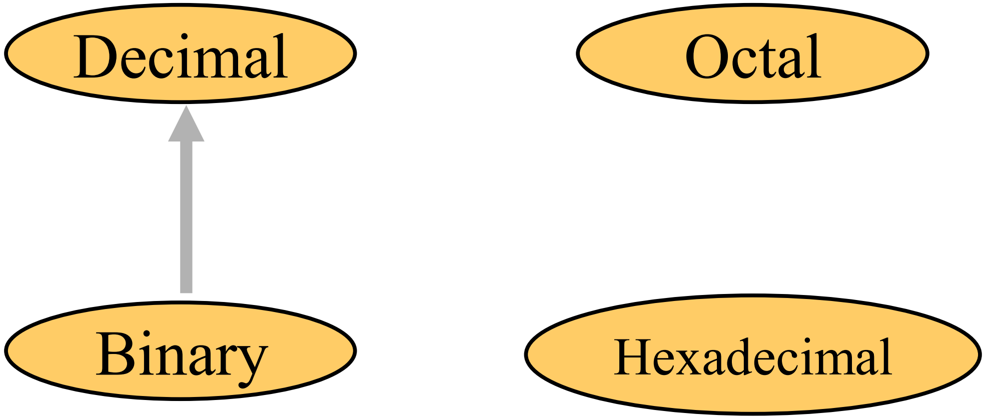
Group 1: To Decimal

- Technique
  - Multiply each bit by b^n, where b is the “base”
  - n is the position of the bit, starting from 0 on the right
  - Add the results



Binary to Decimal

- Technique
  - Multiply each bit by 2^n, where 2^n is the "weight" of the bit
  - n is the position of the bit, starting from 0 on the right
  - Add the results



Example

Bit "0"

$$
\begin{aligned}
101011_2 \Rightarrow\quad & 1 \times 2^0 = 1 \\
& 1 \times 2^1 = 2 \\
& 0 \times 2^2 = 0 \\
& 1 \times 2^3 = 8 \\
& 0 \times 2^4 = 0 \\
& 1 \times 2^5 = 32 \\
\hline
& \qquad\quad 43_{10}
\end{aligned}
$$

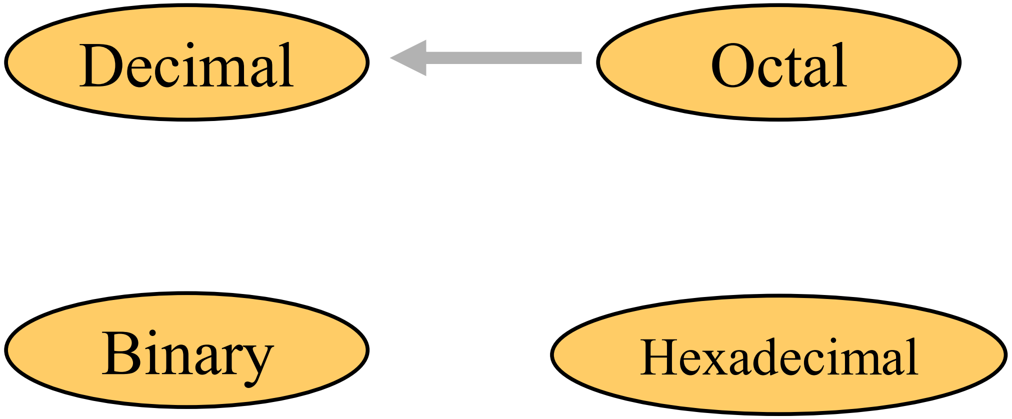
Converting Binary to Decimal

What is the decimal equivalent of the binary number 1101110?

$$
\begin{aligned}
& 1 \times 2^6 = 1 \times 64 = 64 \\
+ & 1 \times 2^5 = 1 \times 32 = 32 \\
+ & 0 \times 2^4 = 0 \times 16 = 0 \\
+ & 1 \times 2^3 = 1 \times 8 = 8 \\
+ & 1 \times 2^2 = 1 \times 4 = 4 \\
+ & 1 \times 2^1 = 1 \times 2 = 2 \\
+ & 0 \times 2^0 = 0 \times 1 = 0 \\
& = 110 \text{ in base } 10
\end{aligned}
$$

Octal to Decimal

- Technique
  - Multiply each bit by 8^n, where 8^n is the "weight" of the bit
  - n is the position of the bit, starting from 0 on the right
  - Add the results

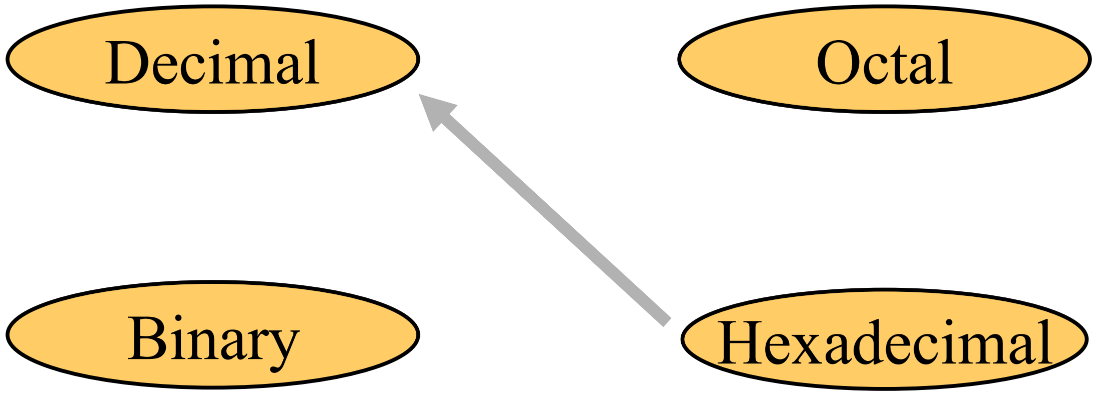


Example

$$
\begin{aligned}
724_8 \Rightarrow \quad & 4 \times 8^0 = 4 \\
& 2 \times 8^1 = 16 \\
& 7 \times 8^2 = 448 \\
& \overline{468_{10}}
\end{aligned}
$$

Hexadecimal to Decimal

- Technique
  - Multiply each bit by 16^n, where 16^n is the "weight" of the bit
  - n is the position of the bit, starting from 0 on the right
  - Add the results



Base 16 is useful too

- The hexadecimal system uses 16 digits:

  0 1 2 3 4 5 6 7 8 9 A B C D E F

- Hexadecimal is useful as a shorthand for binary numbers.
  - Since $16 = 2^4$, one hex digit is equivalent to four bits (including leading 0s).
  - It's often easier to work with numbers like "B4" instead of "10110100".
- Hex shows up in many different contexts.
  - IP addresses, such as "80.AE.05.27".
  - RGB color triplets, like "C0C0FF".
- You can convert between base 10 and base 16 using the same method as for converting from decimal to binary.

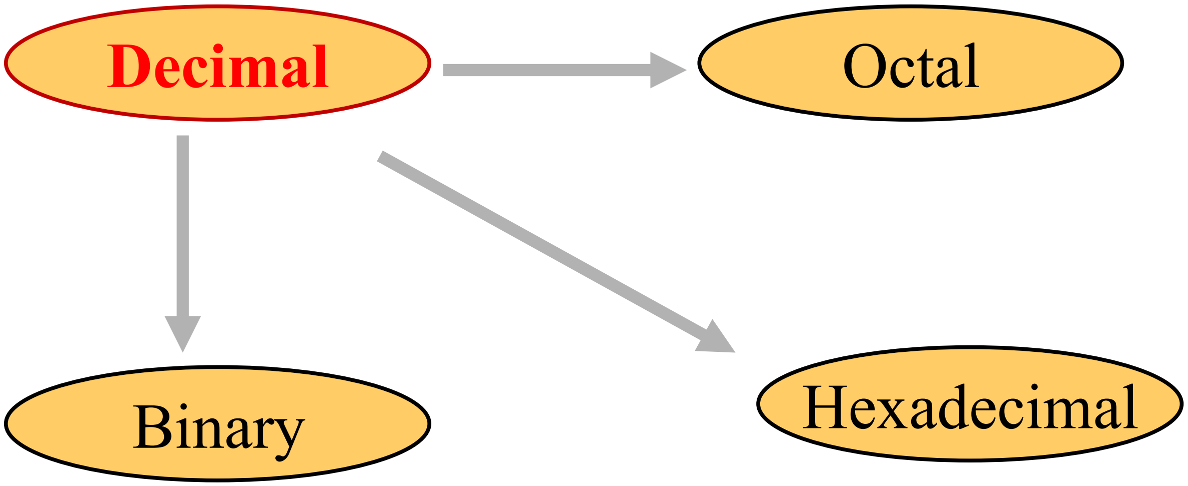
Decimal	Binary	Hex
0	0000	0
1	0001	1
2	0010	2
3	0011	3
4	0100	4
5	0101	5
6	0110	6
7	0111	7
8	1000	8
9	1001	9
10	1010	A
11	1011	B
12	1100	C
13	1101	D
14	1110	E
15	1111	F

Example

$$
\begin{aligned}
ABC_{16} \Rightarrow \quad & C \times 16^0 = 12 \times 1 = 12 \\
& B \times 16^1 = 11 \times 16 = 176 \\
& A \times 16^2 = 10 \times 256 = 2560 \\
\hline
& 2748_{10}
\end{aligned}
$$

Group 2: From Decimal

- Technique
  - Divide by the **base**, keep track of the remainder
  - First remainder is bit 0 (LSB, least-significant bit)
  - Second remainder is bit 1
  - Etc.



Why does this work?

- This same idea works for converting from decimal to any other base.
- Think about "converting" 162 from decimal to decimal:

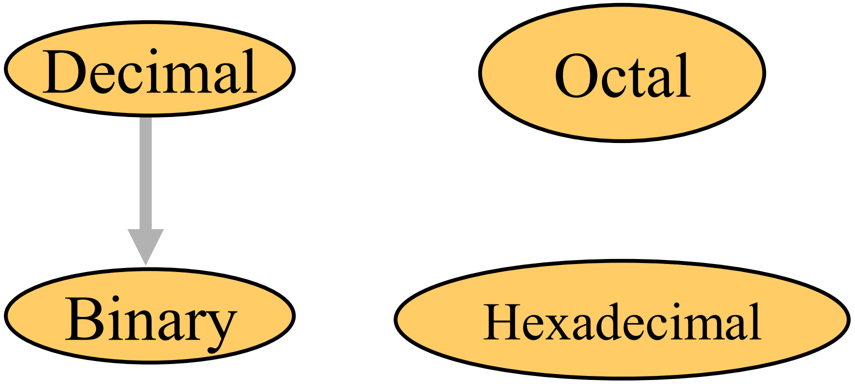
$$
\begin{aligned}
162 / 10 &= 16 \quad \text{rem } 2 \\
16 / 10 &= 1 \quad \text{rem } 6 \\
1 / 10 &= 0 \quad \text{rem } 1
\end{aligned}
$$

- After each division, the remainder contains the rightmost digit of the dividend, while the quotient holds the remaining digits.
- Similarly when converting fractions, each multiplication strips off the leftmost digit as the integer result, leaving the remaining digits in the fractional part.

$$
\begin{aligned}
0.375 \times 10 &= 3.750 \\
0.750 \times 10 &= 7.500 \\
0.500 \times 10 &= 5.000
\end{aligned}
$$

Decimal to Binary

- Technique
  - Divide by two, keep track of the remainder
  - First remainder is bit 0 (LSB, least-significant bit)
  - Second remainder is bit 1
  - Etc.



Example

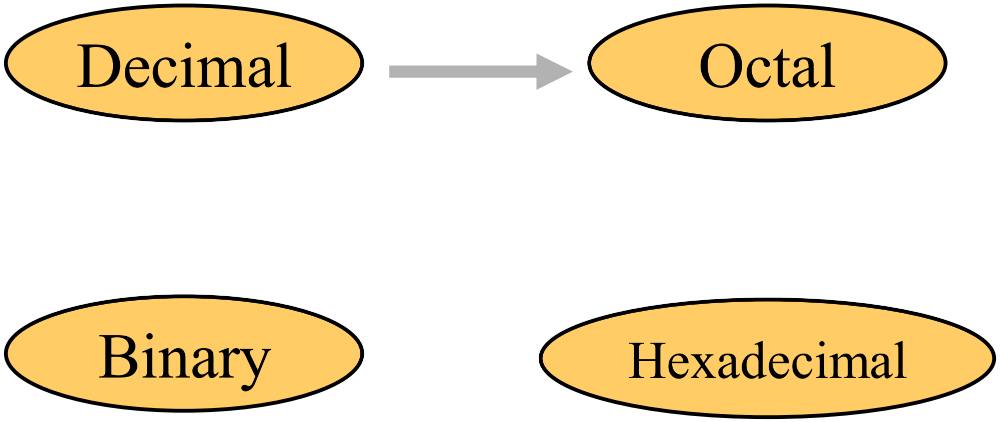
$125_{10} = ?_2$

2	125	
2	62	1
2	31	0
2	15	1
2	7	1
2	3	1
2	1	1
	0	1

$125_{10} = 1111101_2$

Decimal to Octal

- Technique
  - Divide by 8
  - Keep track of the remainder



Example

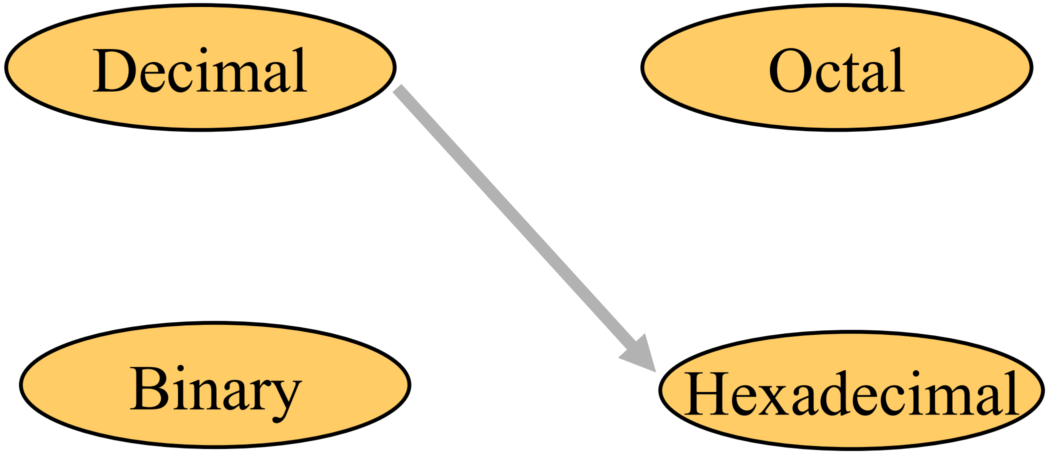
$1234_{10} = ?_8$

8	1234	
8	154	2
8	19	2
8	2	3
	0	2

$1234_{10} = 2322_8$

Decimal to Hexadecimal

- Technique
  - Divide by 16
  - Keep track of the remainder



Example

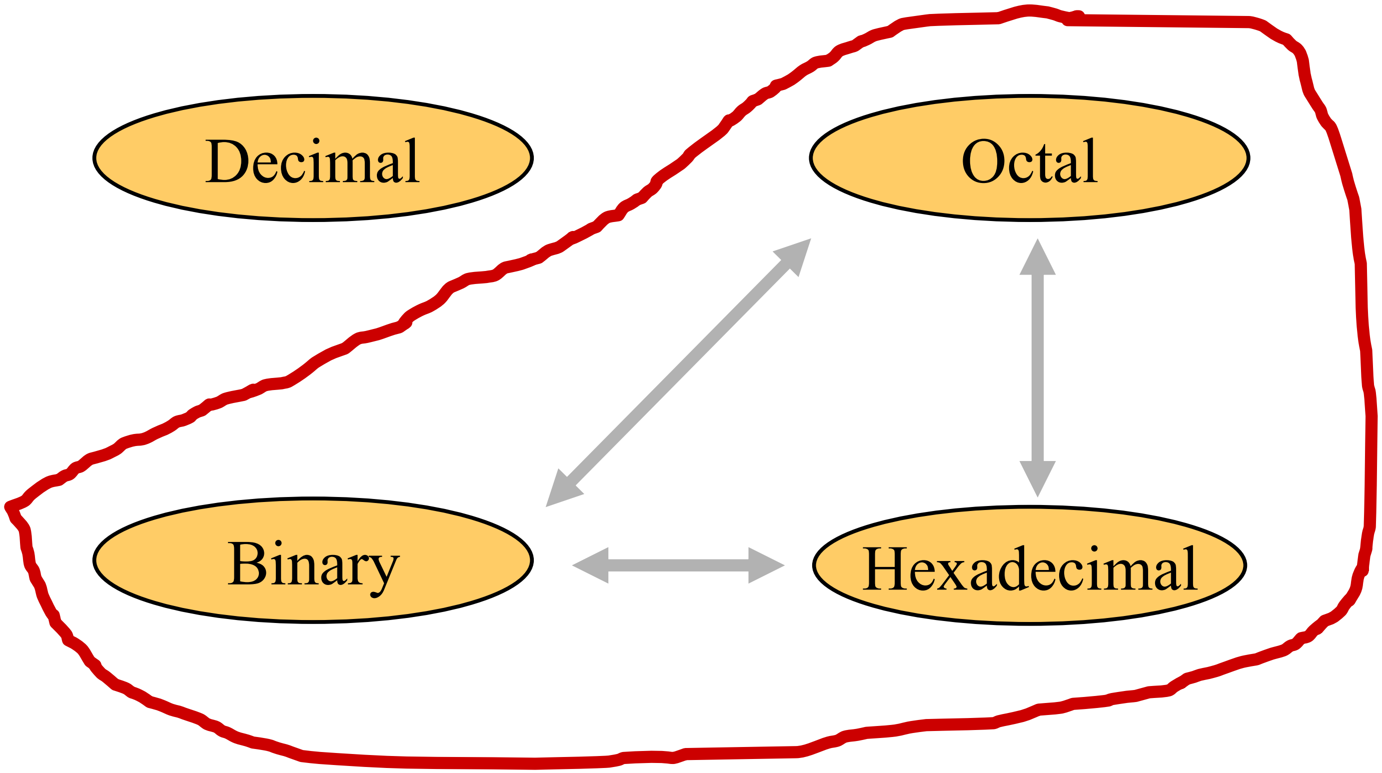
$1234_{10} = ?_{16}$

16	1234	
16	77	2
16	4	13 = D
	0	4

$1234_{10} = 4D2_{16}$

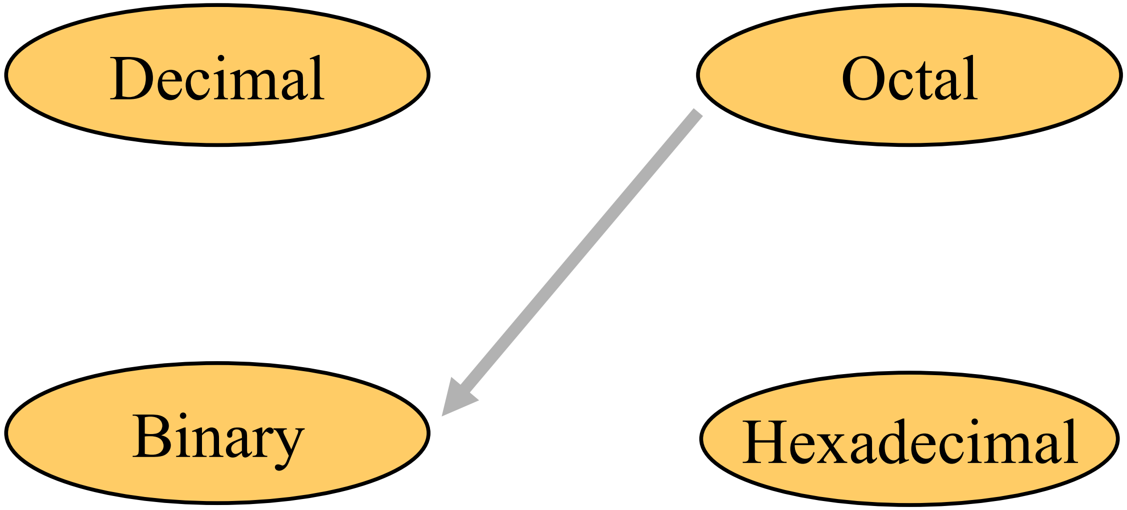
Group-3: Except Decimal

- Technique
  - Convert each digit to a equivalent binary representation



Octal to Binary

- Technique
  - Convert each octal digit to a 3-bit equivalent binary representation



Example

$705_8 = ?_2$

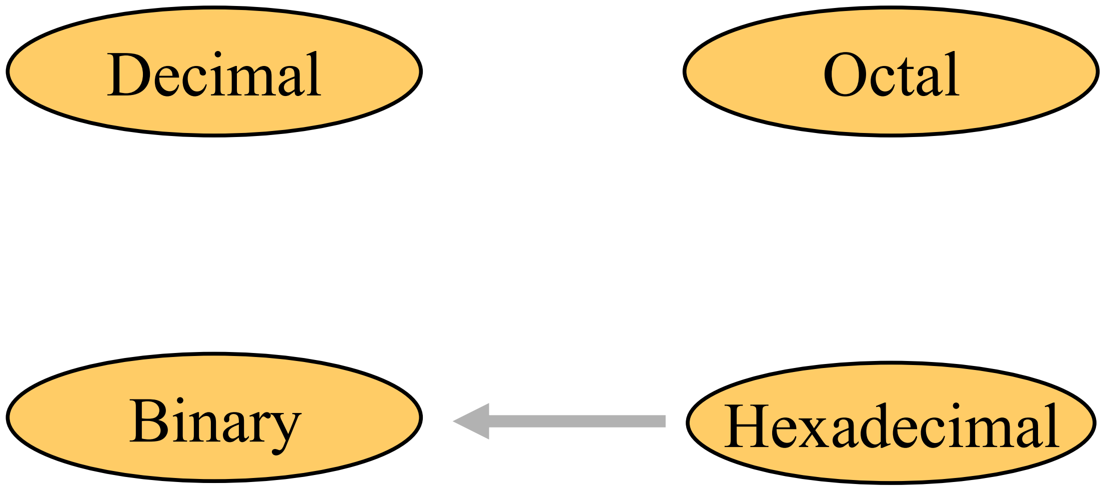
7 0 5

111 000 101

$705_8 = 111000101_2$

Hexadecimal to Binary

- Technique
  - Convert each hexadecimal digit to a 4-bit equivalent binary representation



Binary and hexadecimal conversions

- Converting from hexadecimal to binary is easy: replace each hex digit with its equivalent four-bit binary value.

$$261.A5_{16} = 2 \quad 6 \quad 1 \quad . \quad A \quad 5_{16}$$
$$= 0010 \quad 0110 \quad 0001 \quad . \quad 1010 \quad 0101_{2}$$

- To convert from binary to hexadecimal, partition the binary number into groups of four bits, starting from the point. (Add 0s to the ends if needed.) Then replace each four-bit group by the corresponding hex digit.

$$10110100.001011_{2} = 1011 \quad 0100 \quad . \quad 0010 \quad 1100_{2}$$
$$B \quad 4 \quad . \quad 2 \quad C_{16}$$

Binary	Hex
0000	0
0001	1
0010	2
0011	3
0100	4
0101	5
0110	6
0111	7
1000	8
1001	9
1010	A
1011	B
1100	C
1101	D
1110	E
1111	F

Example

$10AF_{16} = ?_2$

1 0 A F

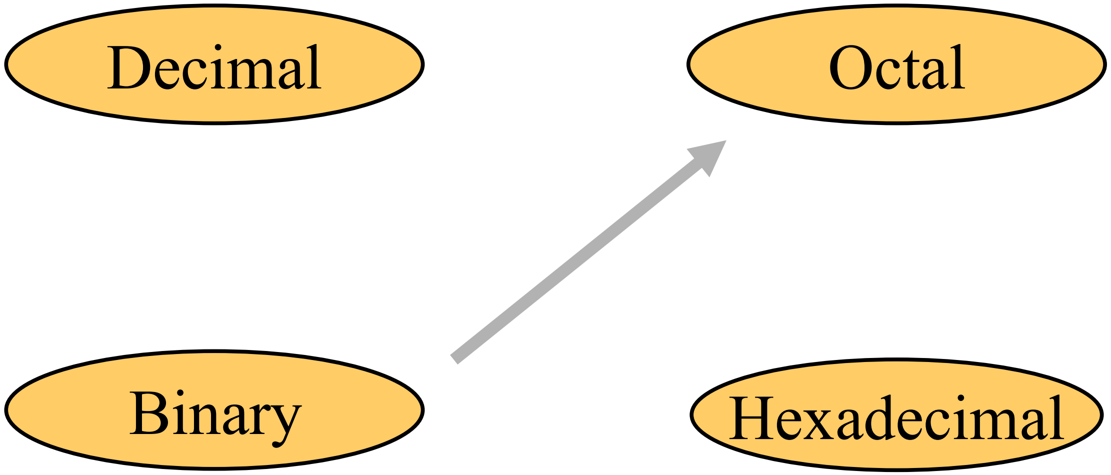
↓ ↓ ↓ ↓

0001 0000 1010 1111

$10AF_{16} = 0001000010101111_2$

Binary to Octal

- Technique
  - Group bits in threes, starting on right
  - Convert to octal digits



Example

$1011010111_2 = ?_8$

1 011 010 111

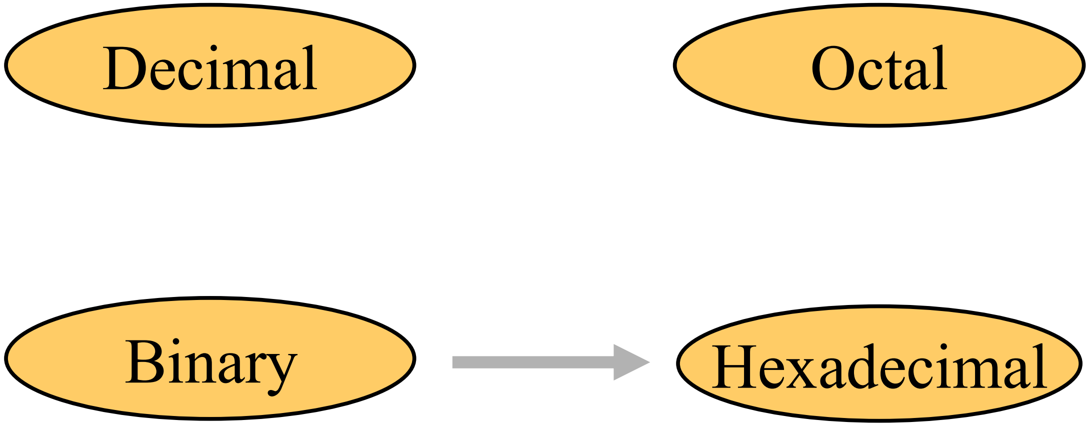
↓ ↓ ↓ ↓

1 3 2 7

$1011010111_2 = 1327_8$

Binary to Hexadecimal

- Technique
  - Group bits in fours, starting on right
  - Convert to hexadecimal digits



Example

$1010111011_2 = ?_{16}$

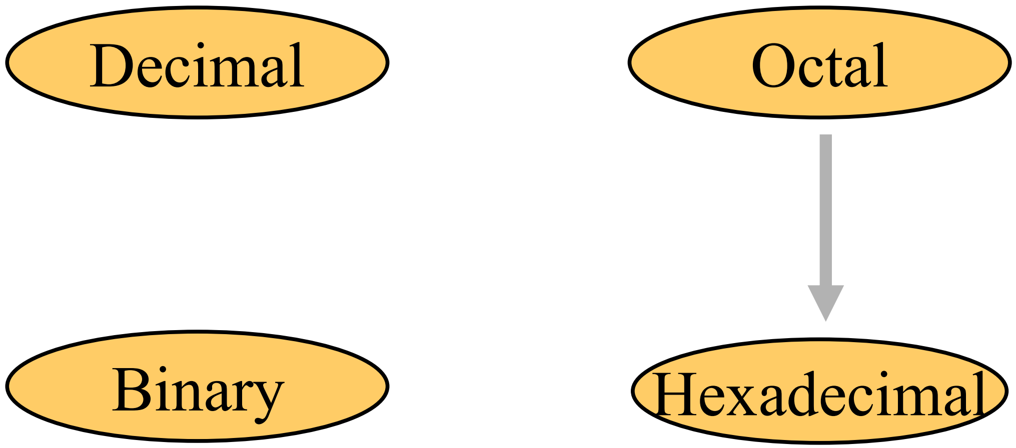
10 1011 1011

2 B B

$1010111011_2 = 2BB_{16}$

Octal to Hexadecimal

- Technique
  - Use binary as an intermediary



Example

$1076_8 = ?_{16}$

1 0 7 6

↓ ↓ ↓ ↓

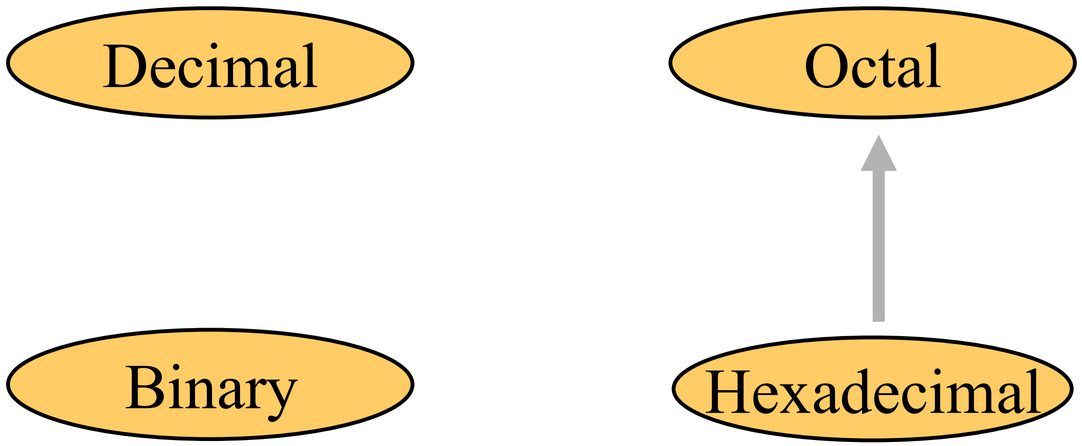
001 0|00 11|1 110

2 | 3 | E

$1076_8 = 23E_{16}$

Hexadecimal to Octal

- Technique
  - Use binary as an intermediary



Example

$1F0C_{16} = ?_8$

1 F 0 C

0|001 | 111|1 00|00 1|100

| 1 | 7 | 4 1 | 4

$1F0C_{16} = 17414_8$

Exercise – Convert ...

Decimal	Binary	Octal	Hexa-decimal
33			
	1110101		
		703	
			1AF

Don't use a calculator!

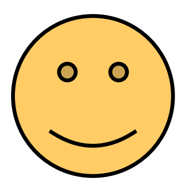
Skip answer

Answer

Exercise – Convert …

Answer

Decimal	Binary	Octal	Hexa-decimal
33	100001	41	21
117	1110101	165	75
451	111000011	703	1C3
431	110101111	657	1AF



Fractions

- Decimal to decimal (just for fun)

```
3.14  =>   4 x 10^-2 = 0.04
           1 x 10^-1 = 0.1
           3 x 10^0   = 3
                        3.14
```

$$3.14 \Rightarrow \begin{array}{l} 4 \times 10^{-2} = 0.04 \\ 1 \times 10^{-1} = 0.1 \\ 3 \times 10^{0} = 3 \\ \qquad\qquad 3.14 \end{array}$$

pp. 46-50

Fractions

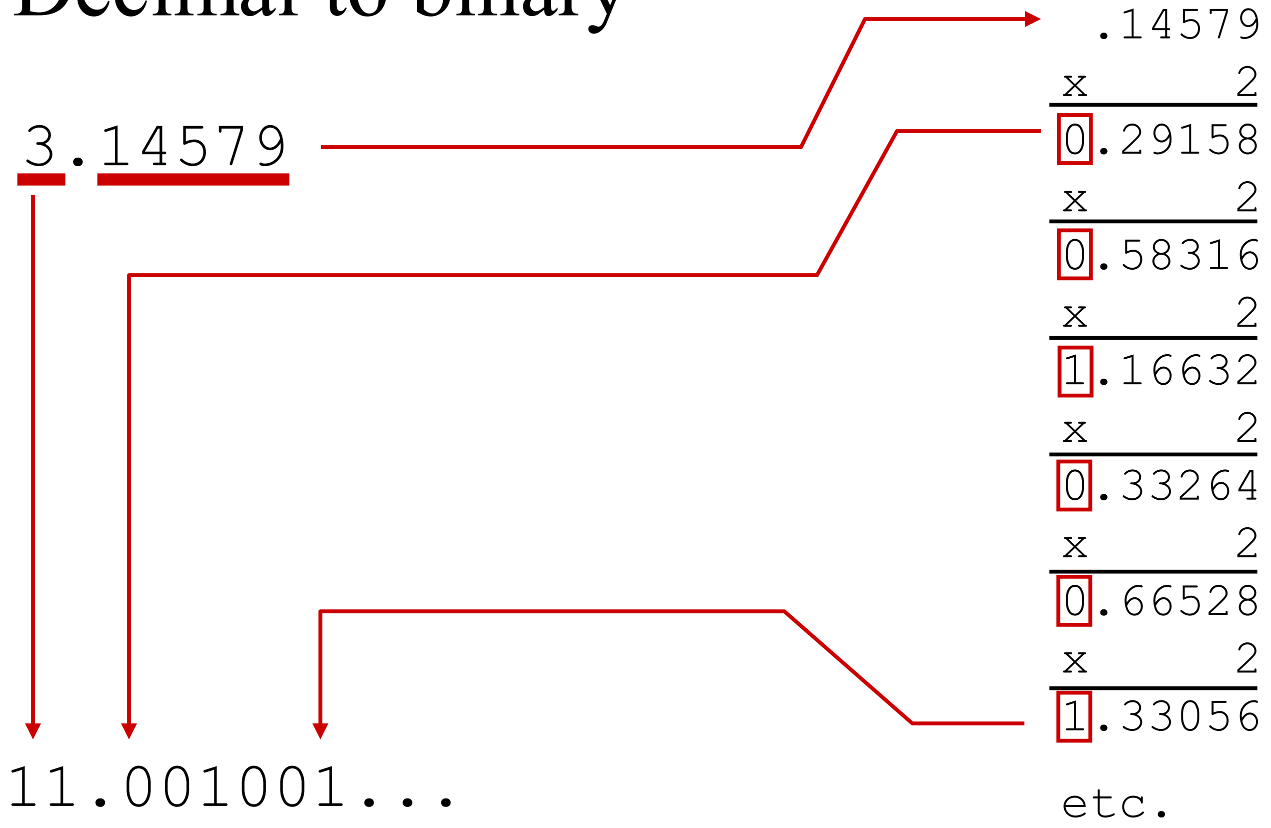
- Binary to decimal

$$
\begin{array}{llcl}
10.1011 \Rightarrow & 1 \times 2^{-4} & = & 0.0625 \\
& 1 \times 2^{-3} & = & 0.125 \\
& 0 \times 2^{-2} & = & 0.0 \\
& 1 \times 2^{-1} & = & 0.5 \\
& 0 \times 2^{0} & = & 0.0 \\
& 1 \times 2^{1} & = & 2.0 \\
\hline
& & & 2.6875
\end{array}
$$

pp. 46-50

Fractions

- Decimal to binary



p. 50

Exercise – Convert ...

Decimal	Binary	Octal	Hexa-decimal
29.8			
	101.1101		
		3.07	
			C.82

Don't use a calculator!

Skip answer

Answer

Exercise – Convert …

Answer

Decimal	Binary	Octal	Hexa-decimal
29.8	11101.110011…	35.63…	1D.CC…
5.8125	101.1101	5.64	5.D
3.109375	11.000111	3.07	3.1C
12.5078125	1100.10000010	14.404	C.82

4-Bit Binary Coded Decimal (BCD) Systems

- The 4-bit BCD system is usually employed by the computer systems to represent and process numerical data only.
- In the 4-bit BCD system, each digit of the decimal number is encoded to its corresponding 4-bit binary sequence.

Decimal digits	Weighted 4-bit BCD code
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001

4-Bit BCD Code

- Represent the decimal number 5327 in BCD code.

4-bit BCD representation of decimal digit 5 is 0101
4-bit BCD representation of decimal digit 3 is 0011
4-bit BCD representation of decimal digit 2 is 0010
4-bit BCD representation of decimal digit 7 is 0111

Therefore, the BCD representation of decimal number 5327 is 0101001100100111.