جامعة الملك سعود
King Saud University

CSC 220: Computer Organization

Unit 5
COMBINATIONAL CIRCUITS-1

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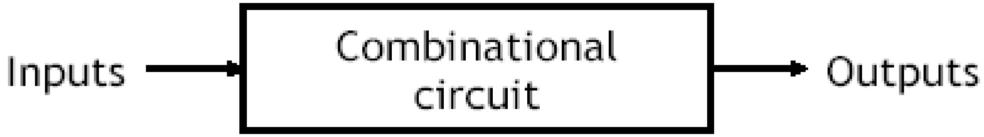
Overview

- Introduction to Combinational Circuits
- Adder
- Ripple Carry Adder
- Subtraction
- Adder/Subtractor

Chapter-3

M. Morris Mano, Charles R. Kime and Tom Martin, **Logic and Computer Design Fundamentals**, Global (5th) Edition, Pearson Education Limited, 2016. ISBN: 9781292096124

Combinational circuits



- So far we've only worked with combinational circuits, where applying the same inputs always produces the same outputs.
  - This corresponds to a mathematical function, where every input has a single, unique output.
  - In programming terminology, combinational circuits are similar to "functional programs" that do not contain variables and assignments.
- Such circuits are comparatively easy to design and analyze.

Binary addition by hand

- You can add two binary numbers one column at a time starting from the right, just like you add two decimal numbers.
- But remember it's binary. For example, 1 + 1 = 10 and you have to carry!

The initial carry in is implicitly 0

	1	1	1	0		Carry in
		1	0	1	1	Augend
+		1	1	1	0	Addend
	1	1	0	0	1	Sum

most significant bit (MSB)

least significant bit (LSB)

Adder

- Design an Adder for 1-bit numbers?
- **1. Specification:**

  2 inputs (X,Y)

  2 outputs (C,S)

Adder …

- Design an Adder for 1-bit numbers?
- **1. Specification:**
  2 inputs (X,Y)
  2 outputs (C,S)
- **2. Formulation:**

X	Y	C	S
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

Adder ...

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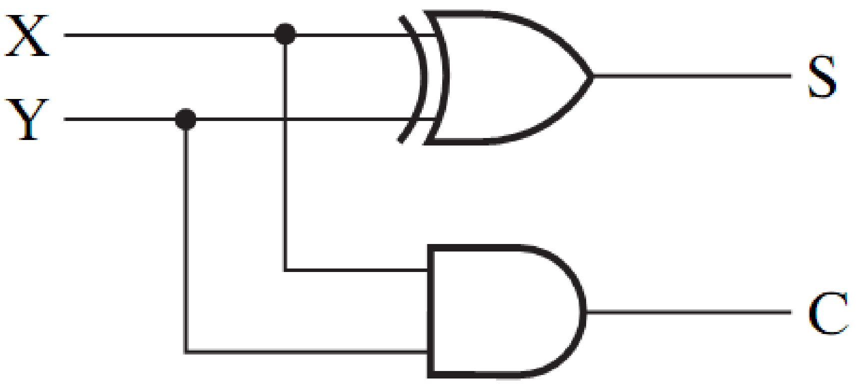
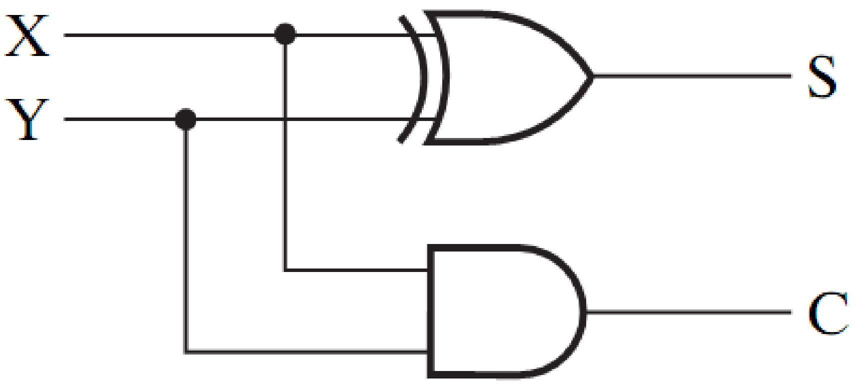
3. Optimization/Circuit



Half Adder …

- This adder is called a Half Adder
- **Q: Why?**

X	Y	C	S
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0



Full Adder

- A combinational circuit that adds 3 input bits to generate a Sum bit and a Carry bit

- A truth table and sum of minterm equations for C and S are shown below.

X	Y	Z	C	S
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

0 + 1 + 1 = 10 → (row 0 1 1)

1 + 1 + 1 = 11 → (row 1 1 1)

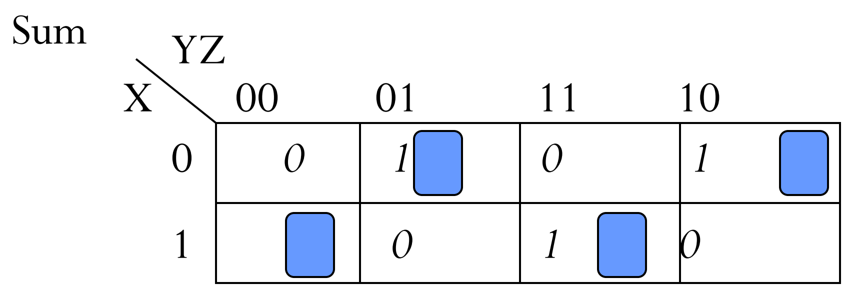
$C(X,Y,Z) = \Sigma m(3,5,6,7)$

$S(X,Y,Z) = \Sigma m(1,2,4,7)$

Full Adder

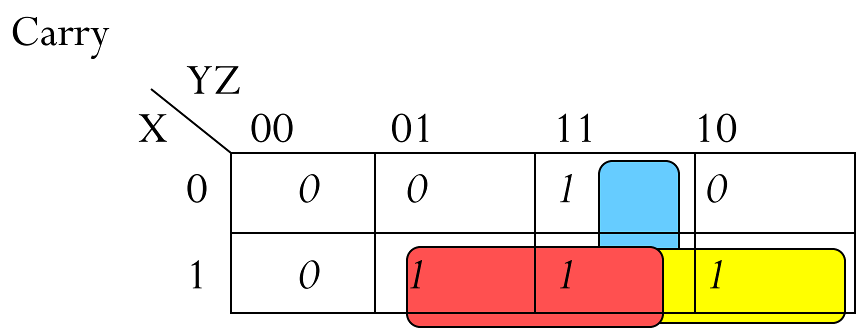
- A combinational circuit that adds 3 input bits to generate a Sum bit and a Carry bit

X	Y	Z	C	S
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1



$S = X'Y'Z + X'YZ' + XY'Z' + XYZ$

$= X \oplus Y \oplus Z$



$C = XY + YZ + XZ$

Full Adder

Full Adder = 2 Half Adders

Manipulating the Equations:

$$S = (X \oplus Y) \oplus Z$$

$$C = XY + XZ + YZ$$

$$= XY + XZ(Y + Y') + YZ(X + X')$$

$$= XY + XYZ + XY'Z + X'YZ + X\cancel{Y}Z$$

$$= XY(1 + Z) + Z(XY' + X'Y)$$

$$= XY + Z(X \oplus Y)$$

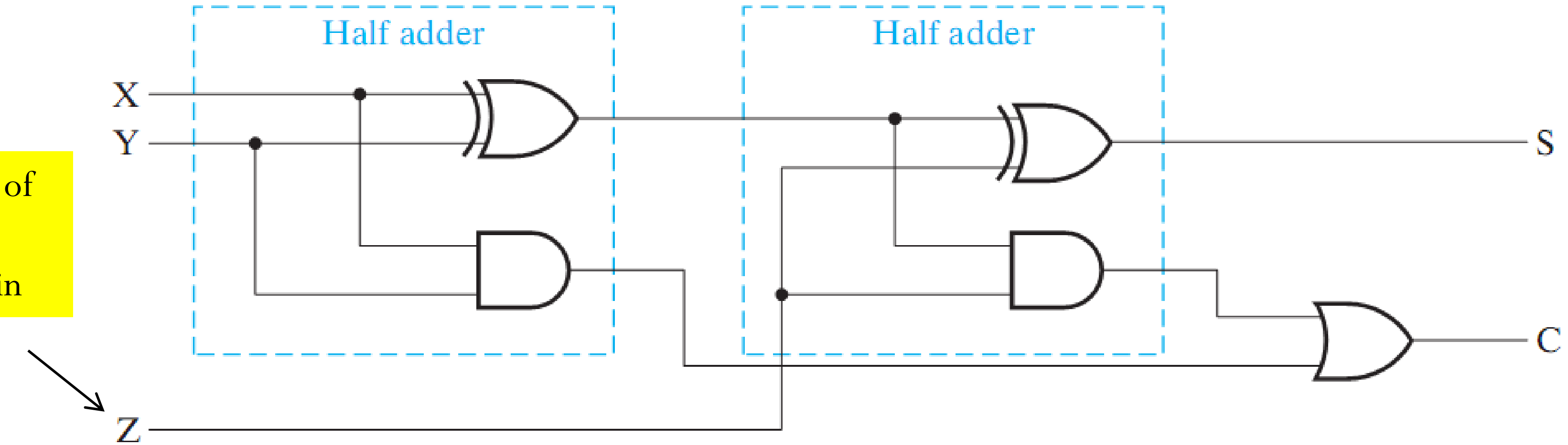
Full Adder = 2 Half Adders

Manipulating the Equations:

$$S = (X \oplus Y) \oplus Z$$

$$C = XY + XZ + YZ = XY + Z(X \oplus Y)$$

Think of Z as a carry in



Src: Mano's Book

n-bit Adder

- How to build an adder for n-bit numbers?
  - Example: 4-Bit Adder
    - Inputs ?
    - Outputs ?
    - What is the size of the truth table?
    - How many functions to optimize?

n-bit Adder …

- How to build an adder for n-bit numbers?
  - Example: 4-Bit Adder
    - Inputs ? 9 inputs
    - Outputs ? 5 outputs
    - What is the size of the truth table? 512 rows!
    - How many functions to optimize? 5 functions

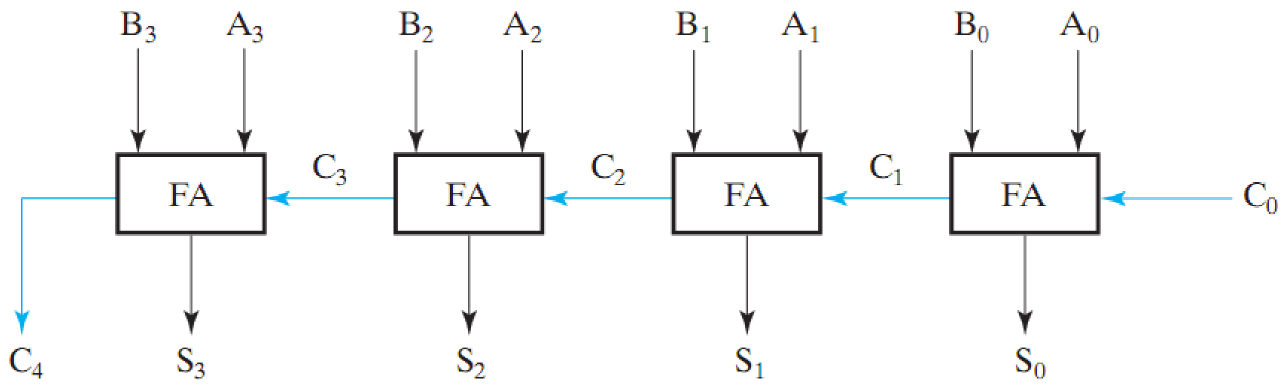
Binary Parallel Adder

- To add n-bit numbers:
- Use n Full-Adders in parallel
- The carries propagates as in addition by hand
- Use Z in the circuit as a C_{in}

- 1 0 0 0
- 0 1 0 1
- 0 1 1 0
- 1 0 1 1

Binary Parallel Adder ..

- To add n-bit numbers:
- Use n Full-Adders in parallel
- The carries propagates as in addition by hand



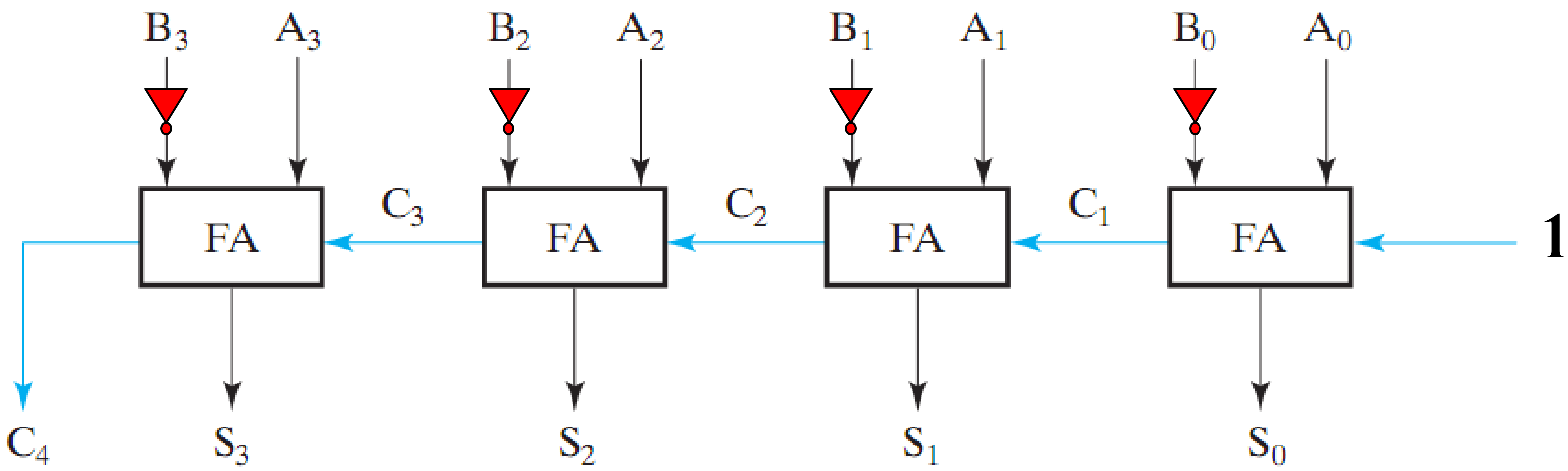
This adder is called *ripple carry adder*

Src: Mano's Book

Subtraction (2's Complement)

- How to build a subtractor using 2's complement?

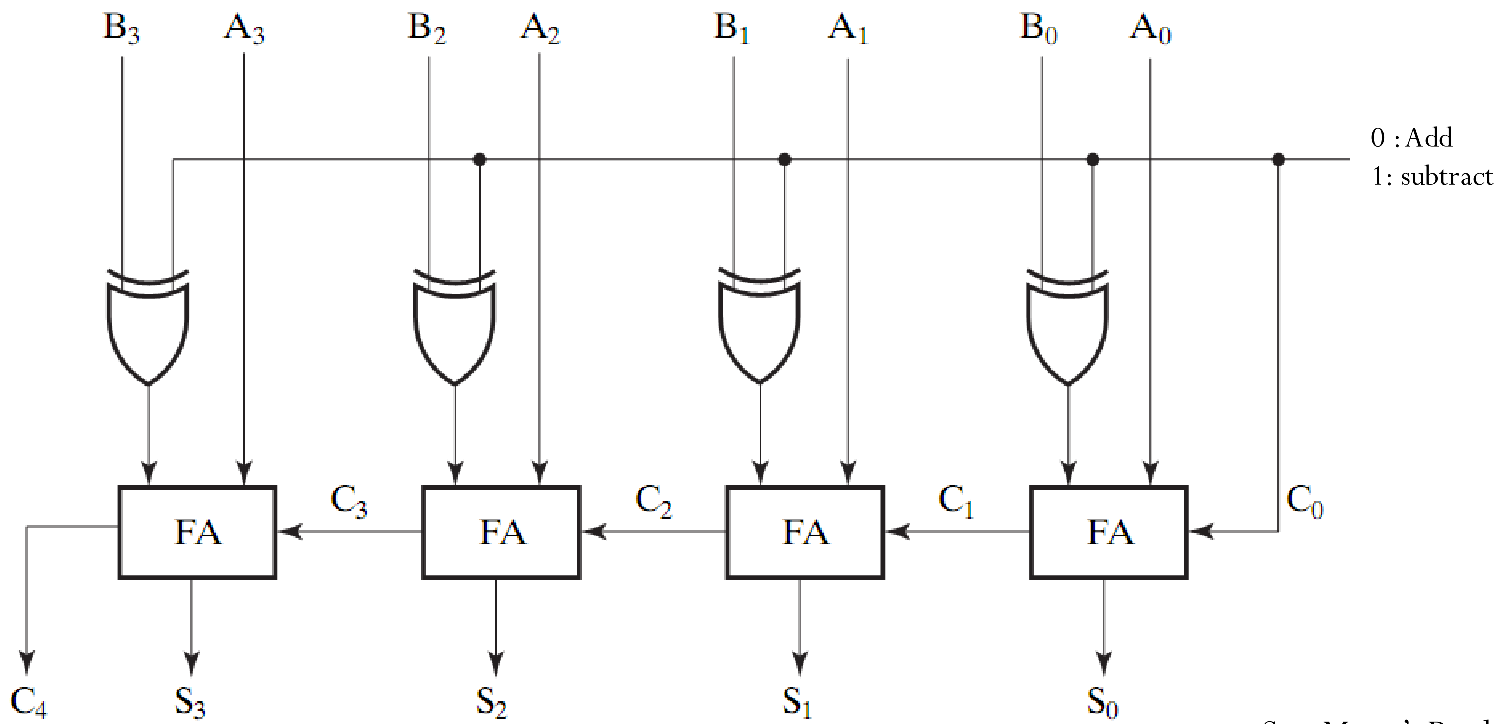
$$S = A - B$$
$$= A + (-B)$$



Src: Mano's Book

Adder-Subtractor

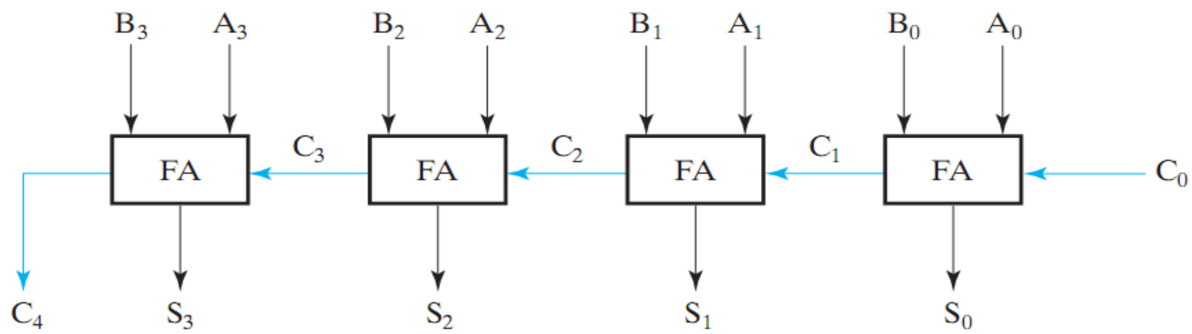
- How to build a circuit that performs both addition and subtraction?



Src: Mano's Book

Using full adders and XOR we can build an Adder/Subtractor!

Carry Look Ahead Adder



- How to reduce propagation delay of ripple carry adders?
- **Carry look ahead adder:** All carries are computed as a function of C_0 (independent of n !)
- It works on the following standard principles:
  - A carry bit is generated when both input bits Ai and Bi are 1, or
  - When one of input bits is 1, and a carry in bit exists

Carry bits → Cn Cn-1…….Ci……….C2C1C0

An-1…….Ai……….A2A1A0

Bn-1…….Bi……….B2B1B0

Carry Out →

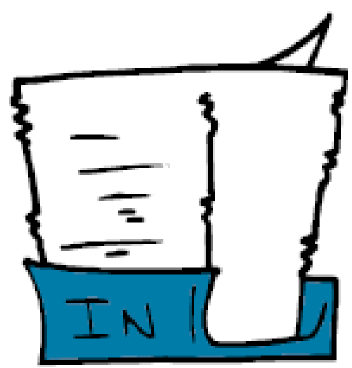
Sn Sn-1…….Si……….S2S1S0

Detecting signed overflow

- The easiest way to detect signed overflow is to look at all the sign bits.

```
       0 1 0 0      (+4)              1 1 0 0      (-4)
  +    0 1 0 1    + (+5)         +    1 0 1 1    + (-5)
  -----------    -------         -----------    -------
     0 1 0 0 1      (-7)            1 0 1 1 1      (+7)
```

- Overflow occurs only in the two situations above.
  1. If you add two *positive* numbers and get a *negative* result.
  2. If you add two *negative* numbers and get a *positive* result.
- Overflow can never occur when you add a positive number to a negative number. (Do you see why?)



Detecting Sign Overflow ...