PHYS 502
HANDOUT 1-*Revision questions*

1. In gases theory the differential relation between pressure and volume is $dV/dP=-(V/P)$. Deduce Boyle's law.

2. In a direct current circuit which includes a resistor R and a capacitor C the application of 1st Kirchhoff's rule gives the following DE for the current:

$$R\frac{dI}{dt}+\frac{1}{C}I=0.$$

   Find $I(t)$.

3. The Laplace transform of Bessel's equation leads to

$$\left(s^2+1\right)f'(s)+sf(s)=0.$$

   Solve for $f(s)$.

4. The decay of population by catastrophic two body collisions is described by

$$dN/dt=-kN^2.$$

   Derive the solution

$$N(t)=N_0\left(1+\frac{t}{\tau_0}\right)^{-1},$$

   where $\tau_0=\left(kN_0\right)^{-1}$.

5. The rate of a particular chemical reaction $A+B\to C$ is proportional to the concentration of the reactions A and B:

$$dC(t)/dt=a\left[A(0)-C(t)\right]\left[B(0)-C(t)\right].$$

   a) Find $C(t)$ for $A(0)\neq B(0)$.
   b) Find $C(t)$ for $A(0)=B(0)$.
   The initial condition is that $C(0)=0$.

6. A boat, coasting through the water, experiences a resisting force given by

$$F=-kv^n$$

   Find the expression for the velocity as a function of time and as a function of distance. The initial conditions are $v(0)=0,\ x(0)=0$.

7. In the first-order differential equation $dy/dx=f(x,y)$ the function $f(x,y)$ is a function of the ratio y/x:

$$dy/dx=g(y/x).$$

Show that the substitution $u=y/x$ leads to a separable equation in u and x.

8. The motion of a body falling in a resisting medium may be described by

$$m\frac{dv}{dt} = mg - bv$$

when the retarding force is proportional to the velocity v. Find the velocity as a function of time. The initial condition is that $v(0) = 0$.

9. Radioactive nuclei decay according to the law The state of a particle is given by

$$\frac{dN}{dt} = -\lambda N .$$

N being the concentration of a given nuclide and λ the particular decay constant. In a radioactive series of n different nuclides, starting with N_1,

$$\frac{dN_1}{dt} = -\lambda_1 N_1,$$

$$\frac{dN_2}{dt} = \lambda_1 N_1 - \lambda_2 N_2, \text{ and so on.}$$

Find $N_2(t)$ for the conditions $N_1(0) = N_0$ and $N_2(0) = 0$.

10. The evaporation from a particular spherical drop of liquid (constant density) is proportional to its surface area. Assuming this to be the sole mechanism of mass loss, find the radius of the drop as a function of time.

11. Bernoulli's equation

$$\frac{dy}{dx} + f(x)y = g(x)y^n$$

is nonlinear for $n \neq 0$ or 1. Show that the substitution $u = y^{1-n}$ reduces Bernoulli's equation to a linear equation.

12. The differential equation

$$P(x,y)dx + Q(x,y)dy = 0$$

is *exact*. Construct a solution

$$\phi(x,y)=\int_{x_0}^{x} P(x,y)dx+\int_{y_0}^{y} Q(x_0,y)dy=\text{constant}$$

13. Classify the following PDE as a parabolic, as hyperbolic or as elliptic.

$$u_t=u_{xx},\quad u_{tt}=u_{xx},\quad u_{tx}=0,\quad u_{xx}+u_{yy}=0,\quad yu_{xx}+u_{yy}=0\,.$$

14. How many solutions to the PDE $u_t=u_{xx}$ can you find? Try solutions of the form $u(x,t)=e^{ax+bt}$.

15. Probably the easiest of all PDE to solve is the equation

$$\frac{\partial u(x,y)}{\partial x}=0$$

Can you solve this equation?