PHYS 453
HANDOUT 2-*Schroedinger equation*

1. Show that the average value of the momentum $\langle p \rangle$ is zero in the following two cases: a) when the wavefunction $\psi(x)$ is real and b) when $\psi(x)$ is even or odd.

2. The wavefunction of a particle has the form $\psi(x) = Ne^{-\lambda|x|}$. a) Find *N*. b) What is the probability of finding the particle in the region $-1 \le x \le 1$ if we consider $\lambda = 1$?

3. Show that the average value of the square of the momentum in an one dimensional problem could always be written in the form

$$\langle p^2 \rangle = \hbar^2 \int_{-\infty}^{+\infty} \left|\psi'(x)\right|^2 dx .$$

4. Calculate the product of uncertainties $(\Delta x)(\Delta p)$ for the Gaussian function

$$\psi(x) = \sqrt[4]{\frac{\lambda}{\pi}} e^{-\lambda x^2/2} .$$

5. Calculate the uncertainty in momentum and position for a particle with a wavefunction equal to

$$\psi(x) = Nxe^{-\lambda x^2/2} .$$

6. Find the average value of the momentum for any wavefunction of the form

$$\Psi(x) = \psi(x) e^{ikx}$$

where $\psi(x)$ is a real and square integrable wavefunction.

7. Let $P_{ab}(t)$ be the probability of finding the particle in the range $(a < x < b)$, at time *t*. Show that

$$\frac{dP_{ab}}{dt} = J(a,t) - J(b,t)$$

where

$$J(x,t) \equiv \frac{i\hbar}{2m}\left(\Psi\frac{\partial\Psi^*}{\partial x} - \Psi^*\frac{\partial\Psi}{\partial x}\right).$$

What are the units of *J*? [$J(x,t)$ is called the **probability current**, because it tells you the rate at which probability is "flowing" past the point *x*. If $P_{ab}(t)$ is increasing, then more probability is flowing into the region at one end than flows at the other.]

8. A particle is represented (at time *t*=0) by the wave function

$$\Psi(x,0) = \begin{cases} A\left(a^2 - x^2\right), & -a \le x \le a \\ 0 & \text{otherwise.} \end{cases}$$

(a) Determine the normalization constant *A*.
(b) What is the expectation value of *x* (at time *t*=0)?
(c) What is the expectation value of *p* (at time *t*=0)?
(d) Find the uncertainty in *x*.
(e) Find the expectation value of p^2.
(f) Find the uncertainty in *p*.
(g) Check that your results are consistent with uncertainty principle.

9. A particle of mass *m* is in the state

$$\Psi(x) = Ae^{-a\left[\left(mx^2/\hbar\right)+it\right]}$$

(a) Find A.
(b) For what potential energy function $V(x)$ does Ψ satisfy the Schroedinger equation?
(c) Calculate the expectation values for *x*, *p*, p^2 and x^2.
(d) Find the uncertainties in *x* and *p*. Are they consistent with the uncertainty principle?

10. Show that the momentum operator is given by $\hat{p} = -i\hbar\partial/\partial x$.