PHYS 454
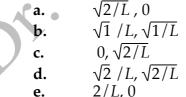## HANDOUT 4-*Schrödinger equation and Infinite Square Well*

1. A system at $t=0$ has a wave function $|\psi\rangle = 3N|\psi_1\rangle + 4N|\psi_2\rangle$, where $|\psi_1\rangle$, $|\psi_2\rangle$ the eigenstates of the energy with corresponding eigenvalues E_1, E_2. What is the time evolution of the system?

2. A system at $t=0$ has a wave function $|\psi\rangle = N|\psi_1\rangle + 2iN|\psi_2\rangle + iN|\psi_3\rangle$, where $|\psi_1\rangle$, $|\psi_2\rangle$ and $|\psi_3\rangle$ the eigenstates of the energy with corresponding eigenvalues E_1, E_2 and E_3. What is the time evolution of the system?

3. The state of a system is described by the normalized wave function

$$|\psi(t=0)\rangle = \sum_{n=0}^{\infty} c_n |\psi_n\rangle,$$

where $|\psi_n\rangle$ are the eigenstates of the energy with corresponding eigenvalues E_n. Find the average value of the operator $\hat{A}$ at time t. Also find $d\langle A\rangle / dt$. You are given that $\langle \psi_n|\hat{A}|\psi_m\rangle = A_{nm}\delta_{nm}$.

4. Find the energy eigenvalues for an infinite square well without solving the Schroedinger equation.

5. Calculate the ground state energy (in eV) for an electron in a box having a width of 0.05 nm.

6. The wave function for a particle confined to a one-dimensional box of length L is given by $\Psi(x) = A \sin(n\pi x/L) + B \cos(n\pi x/L)$. The constants A and B are determined to be

   a. $\sqrt{2/L}$, 0
   b. $\sqrt{1}/L$, $\sqrt{1/L}$
   c. 0, $\sqrt{2/L}$
   d. $\sqrt{2}/L$, $\sqrt{2/L}$
   e. $2/L$, 0

7. A particle of mass m is in the ground state of the infinite square well. Suddenly the well expands to twice its original size – the right wall moving from a to $2a$ – leaving the wavefunction (momentarily) undisturbed. The energy of the particle is now measured. What is the most probable result?

8. A particle is at the eigenstate $\psi_n(x) = \sqrt{\dfrac{2}{L}}\sin\left(\dfrac{n\pi x}{L}\right)$. Calculate the following quantities: a) $\langle x \rangle$, b) $\langle x^2 \rangle$, c) $\langle p \rangle$, d) $\langle p^2 \rangle$ and e) $\Delta x \cdot \Delta p$.

9. Show that, in one dimensional problems, the energy spectrum of the bound states is always non-degenerate.

10. Show that the wavefunction of a particle in the infinite square well returns to its original form after a quantum revival time $T = 4ma^2 / \pi\hbar$. That is $\Psi(x,T) = \Psi(x,0)$ for any state (not just a stationary state).

11. A particle in the infinite square well has the initial wave function

$$\Psi(x,0) = Ax(a - x), \qquad \left(0 \le x \le a\right)$$

    for some constant A. Find $\Psi(x,t)$.

12. A particle in the infinite square well has its initial wave function an even mixture of the first two stationary states:

$$\Psi(x,0) = A\left[\psi_1(x) + \psi_2(x)\right].$$

    (a) What is A?
    (b) Find $\Psi(x,t)$ and $|\Psi(x,t)|^2$
    (c) Compute $\langle x \rangle$.
    (d) Compute $\langle p \rangle$.
    (e) If you measured the energy of this particle, what values might you get, and what is the probability of getting each of them? Find the expectation (average) value of H. How does it compare with E_1, E_2?

13. Show that the overall phase constant of the wave function is of no physical significance (it cancels out whenever you calculate a measurable quantity). However if we consider the wave function $\Psi(x,0) = A\left[\psi_1(x) + e^{i\varphi}\psi_2(x)\right]$, where there is a relative phase, this phase does matter. Find $\Psi(x,t)$ and $|\Psi(x,t)|^2$.

14. At time $t=0$ the state of a particle is described by the wavefunction

$$\psi(x) = N\sin^3\left(\frac{\pi x}{a}\right)$$

a) What are the possible results if we try to measure its energy and the relative probabilities of them?
b) Calculate the average energy and the energy uncertainty.
c) What is the average position of the particle at $t=0$ and at a later time t?

15. The ground state for a particle in an infinite well has energy equal to 3 eV. If the state of the particle is $\psi = \dfrac{1}{\sqrt{3}}\psi_1 + i\sqrt{\dfrac{2}{3}}\psi_2$, what is the average energy in this state and the uncertainty in energy? (Ans: *9 eV, $3\sqrt{2}$ eV*)

16. A proton and an electron are found in the same infinite well (separate wells, of course) at the ground state. Which has the smaller energy?

17. A particle is found at the second excited state of an infinite well of width a. What is the probability of finding the particle at the region $(0, a/3)$? (Ans: 1/3)

18. A particle is at the state with $n=3$ of an infinite well and "falls" on the state with $n=2$ by emitting a photon with wavelength 1200 $\overset{\circ}{A}$. What is the minimum energy of the particle in this well. (Ans: 2 eV)

19. Using the theory of the infinite well could you explain why the energy of a nucleus is far larger than that of an atom?