

**PHYS 454****HANDOUT 8 – The quantum mechanical simple harmonic oscillator**

1. The energy of the ground state of a quantum SHO is 1 eV. What is its energy in its fifth excited state?
2. A particle is in the second excited state of a quantum SHO. What is the probability of finding the particle in the region  $(0, \infty)$ ?
3. A quantum SHO is the state with  $n=3$ . How many nodes has its wavefunction?
4. A quantum SHO, at a certain time instant  $t$ , is at a state

$$\psi = N(\psi_0 + \sqrt{2}\psi_1).$$

- a) Find  $N$ .
  - b) Compute the function  $\psi(x, t)$  for all values of  $t$ .
  - c) What is the average energy of the oscillator at times  $t=0$ ,  $t=\pi/\omega$ , and  $t=2\pi/\omega$ ?
  - d) What is the average position of the particle at a time  $t$ ?
5. Find the eigenstates and eigenvalues of the Hamiltonian of a particle with a potential energy:

$$V(x) = \begin{cases} \infty, & -\infty < x < 0 \\ \frac{1}{2}m\omega^2 x^2, & x \geq 0 \end{cases}$$

6. The *Stark effect* is the shift of the energies of a quantum system due to the presence of a uniform electric field. When a SHO is charged and we place it inside a uniform electric field parallel to the oscillation axis then its potential energy is given by:

$$V(x) = \frac{1}{2}kx^2 - qEx$$

Where  $q$  is the charge and  $E$  is the electric field strength. Find the energies of this quantum SHO.

7. Use the parabolic approximation to calculate the first energy levels of the potential  $V(x) = V_0 e^{\lambda x^2}$ .
8. A particle is at the ground state of a SHO. Calculate the probability of finding the particle in the classical forbidden region.

9. Show that the eigenfunctions of the harmonic oscillator in the ground state and in the first excited state have inflection points where the condition  $V(x) = E$  is satisfied, i.e.,

$$\frac{m\omega^2}{2}x^2 = \hbar\omega\left(n + \frac{1}{2}\right)$$

10. A particle of mass  $m$  in the harmonic oscillator potential starts out in the state

$$\Psi(x,0) = A\left(\frac{m\omega}{\pi\hbar}\right)^{1/4}\left(1 - 2\sqrt{\frac{m\omega}{\hbar}}x\right)^2 \exp(-m\omega x^2 / 2\hbar)$$

for some constant  $A$ . What is the expectation value of the energy?

11. Show that the probability of finding the harmonic oscillator beyond the classical turning points  $x = \pm x_0$  decreases with increasing  $n$ . This example shows that the classical and quantum pictures becomes less and less marked with increasing  $n$ , in agreement with the correspondence principle.
12. A particle moves in the potential of problem 5 above and is at the ground state of the potential. What is the probability to find it in the forbidden region?