PHYS 507
HANDOUT 1 - *Questions on Vectors*

1.1 *Direction of a vector* : Given a vector **A**= −**i**+ 2**j** – 2**k** in Cartesian coordinates, find the expression for the unit vector in the direction of **A**.

1.2 *Relation between two vectors*: Show that, if **A**·**B** = **A**·**C** and **A**×**B**=**A**×**C**, where **A** is not a null vector, then **B**=**C**.

1.3 *Multiple applications of · and ×* : Consider arbitrary vectors **A**, **B**, **C** and **D**.

(a) Is $\mathbf{A}\times(\mathbf{B}\times\mathbf{C}) = (\mathbf{C}\times\mathbf{B})\times\mathbf{A}$? Explain.

(b) Is $\mathbf{A}\cdot(\mathbf{B}\times\mathbf{C}) = (\mathbf{C}\times\mathbf{B})\cdot\mathbf{A}$? Explain.

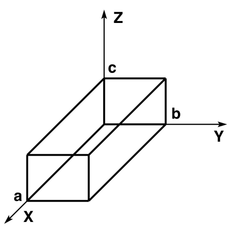
(c) Does $\mathbf{A}\times\mathbf{B} = \mathbf{A}\times\mathbf{C}$, implies $\mathbf{B}=\mathbf{C}$? Explain.

(d) Does $\nabla\times\mathbf{B} = \nabla\times\mathbf{C}$, implies $\mathbf{B}=\mathbf{C}$? Explain.

1.4 *Vector components and directions*: Find the relative position vector **R** of the point $P(2,-2,3)$ with respect to $Q(-3,1,4)$. What are the direction angles of **R**? (The direction angles are the angles between the vector **R** and the axes x, y and z respectively).

1.5 *Vector components and directions*: Given that **A**= **i**+ 2**j** +3**k** and **B**= 4**i**-5**j** +6**k**. Find the angle between the two vectors. Find the component of **A** in the direction of **B**.

1.6 *Flux of a vector field*: Given the vector field **A**= (xy)**i**+ (yz)**j** +(zx)**k**, evaluate directly from the definition, the flux of **A** through the surface of the following rectangular parallelepiped.



1.7 *Flux of a vector field*: For the figure of previous problem evaluate $\int \nabla\cdot\mathbf{A}\,dV$ and show that it is equal to the calculated flux as predicted by Gauss's Theorem.

1.8 *Component in the direction of a gradient:* (a) A family of hyperbolas in the xy plane is given by $u = xy$. Find ∇u. (b) Given the vector **A**=3**i**+ 2**j** +4**k**, find the component in the direction of ∇u at the point on the curve for which $u = 3$ and $x= 2$.

1.9 *Unit normal to ellipsoids:* The equation giving a family of ellipsoids:

$$u = \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2}.$$

Find the unit vector normal to each point of the surface of these ellipsoids.

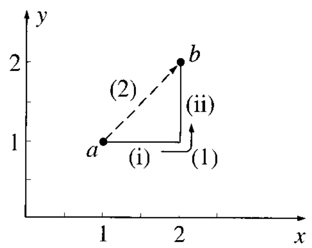
1.10 *Divergence of a radial field*: For fields of the form $r^n \hat{r}$ ($r \neq 0$), find for which values of n the divergence is zero.

1.11 *Field of cylindrical form*: For fields of the form $\rho^n \hat{\rho}$ ($\rho \neq 0$), find for which values of n the curl is zero.

1.12 *Line integral of a vector*: You are given the vector field,

$$\mathbf{A} = y^2\mathbf{i} + 2x(y+1)\mathbf{j}.$$

Evaluate directly the line integral of **A** around the closed path shown in figure below.



Evaluate the surface integral $\nabla \times \mathbf{A}$ over the area enclosed by the path. Show that the two are related as expected by Stokes' Theorem.

1.13 *Dirac Delta function*: Evaluate the integral $\int_0^3 x^3 \delta(x-2)\,dx$.

1.14 *Dirac Delta function*: Show that $x\frac{d}{dx}\left(\delta(x)\right) = -\delta(x)$.

1.15 *Dirac Delta function in three dimensions:* Evaluate the integral $J = \int_V \left(r^2+2\right)\nabla\cdot\left(\frac{\hat{\mathbf{r}}}{r^2}\right)dV$, where V is a sphere of radius R centered at the origin.