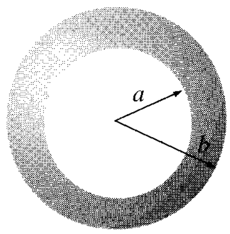
PHYS 507
HANDOUT 3 - *Questions on Potential & Potential Energy*

3.1 Find the potential inside and outside a spherical shell of radius *R,* which carries *a* uniform surface charge. Set the reference point at infinity.

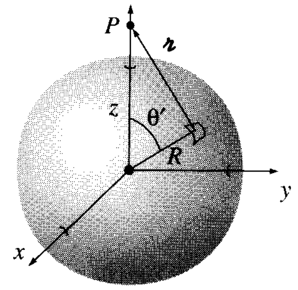
3.2 Find the potential inside and outside a uniformly charged sphere whose radius is *R,* and whose total charge is *q*. Set the reference point at infinity. Compute the gradient of *V* in each region, and check that it yields the correct field. Plot *V*(*r*).

3.3 Find the potential a distance *s* from an infinitely long straight wire that carries a uniform line charge λ. Compute the gradient of your potential, and check that it yields the correct field.

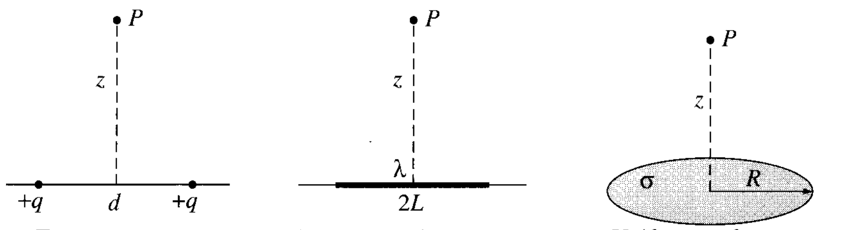
3.4 A hollow spherical shell carries charge density $\rho = k / r^2$ in the region $a \leq r \leq b$. Find the potential in the center using infinity as your reference point.



3.5 Find the potential of a uniformly charged spherical shell of radius *R*.



3.6 Find the potential at a dsitance *z* above the centre of the charge distributions. In each case compute $\mathbf{E} = -\nabla V$ and compare with the results you have from problems...in Handout 2.



(a) Two point charges (b) Uniform line charge (c) Uniform surface charge

3.7 A conical surface (an empty ice-cream cone) carries a uniform surface charge σ. The height of the cone is *h*, as is the radius of the top. Find the potential difference

between points **a** (the vertex) and **b** (the center of the top).

3.8 Find the potential on the axis of a uniformly charged solid cylinder, a distance z from the center. The length of the cylinder is L, its radius is R, and the charge density is ρ. Use your result to calculate the electric field at this point. (Assume that $z > L/2$).

3.9 Use the relation $V(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho(\mathbf{r}')}{r} d\tau'$ to calculate the potential inside a charged solid sphere of radius R and total charge q.

3.10 Check that the results in problems 2.15 from Handout 2 are consistent with the realtion $\mathbf{E}_{\text{above}} - \mathbf{E}_{\text{below}} = \frac{\sigma}{\varepsilon_0}\hat{\mathbf{n}}$.

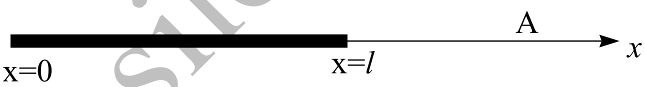
3.11 Show that the electric *potential* at a point on the axis of a ring of charge of radius r, is given by

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{\sqrt{z^2 + r^2}}$$

Derive an expression for the electric field.

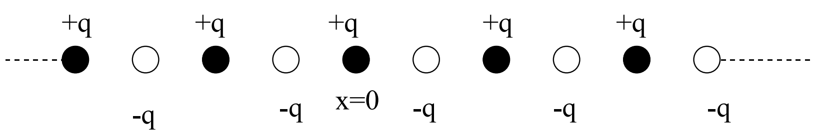
3.12 A spherical drop of water carrying a charge of $3\times10^{-11}C$ has a potential of $500V$ at its surface. (a) What is the radius of the drop? (b) If two such drops of the same charge and radius combine to form a single spherical drop, what is the potential at the surface of the new drop so formed?

3.13 A rod of length l is uniformly charged with a linear charge density λ. Find the electric potential at point A.



3.14 On a straight line we place alternatively an infinite number of charges $+q$ and $-q$ at equal distance as shown in the figure. What is the potential energy of a charge $+q$? You are given that

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots + (-1)^{n+1}\frac{x^n}{n} + \ldots \qquad (-1 < x \leq 1)$$



3.15 Three point charges $q_1 = q$, $q_2 = -q$, $q_3 = 2q$ are placed at the points $A(a,0,0)$, $B(0,-a,0)$, $C(0,0,2a)$ respectively. What is the potential energy of the system?

3.16 Find the electric field inside and outside a non-conducting sphere of radius *R*, with charge spread uniformly throughout its volume. Find the potential $V(r)$ inside and outside the sphere. Plot the potential against the radial distance from the sphere's center.

3.17 Does the superposition principle hold for the energy of the electric field?

3.18 Consider two concentric spherical shells, of radii *a* and *b* respectively. Suppose the inner one carries a charge *q* and the outer one a charge –*q* (both of them uniformly distributed over the surface). Calculate the energy of this configuration, (a) using $W = \frac{\varepsilon_0}{2}\int_{\text{all space}} E^2\, d\tau$, (b) using the superposition principle.

3.19 Find the energy stored in the sphere of problem 3.16. Using a) $W = \frac{\varepsilon_0}{2}\int_{\text{all space}} E^2\, d\tau$, and b) $W = \frac{1}{2}\int_{\substack{\text{charge}\\ \text{distribution}}} \rho V\, d\tau$.

3.20 Find the energy of a uniformly charged spherical shell of a total charge *q* and radius *R*.

3.21 Prove Poisson's and Laplace's equations.

3.22 Prove that the tangential component of the electric field at a boundary is continuous.