PHYS 507
HANDOUT 5 - *Questions on Laplace Equation*

5.1 Prove the basic property of a function that satisfies the Laplace equation: the average value on a spherical region is equal to the value at the center of this region.

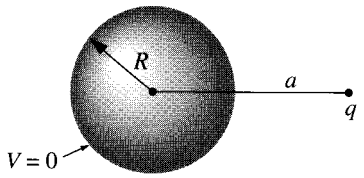
5.2 Prove the first uniqueness theorem.

5.3 Prove the second uniqueness theorem.

5.4 Find the general solution of the Laplace equation in spherical and cylindrical coordinates in the case where the potential spends only on the radial coordinate.

5.5 Show that the potential is *constant* inside an enclosure completely surrounded by conducting material, provided that there is no charge within the enclosure.

5.6 A point charge q is situated a distance a from the center of a grounded conducting sphere of radius R. Find the potential outside the sphere.

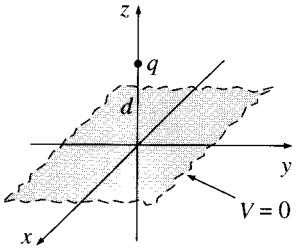


5.7 Find the average potential over a sphere of radius R due to a point charge q located *inside* (same as above, in other words, only with z < R). (In this case, of course, Laplace's equation does not hold within the sphere.) Show that, in general,

$$V_{\text{ave}} = V_{\text{center}} + \frac{Q_{\text{enc}}}{4\pi\varepsilon_0 R}$$

where V_{center} is the potential at the center due to all the external charges, and Q_{enc} is the total enclosed charge.

5.8 Find a) the charge induced on the conducting plane and b) the force exerted on the charge q by the charged induced on the conducting plane c) the energy of the system.



5.9 Solve the problem 5.7 considering that the sphere is kept at a constant potential V_0.

5.10 For the configuration in problem 5.6 find a) the induced charge on the sphere b) the total energy.

5.11 Find the force on the charge $+q$ by the charged induced on the conducting plane which is at the plane xy.