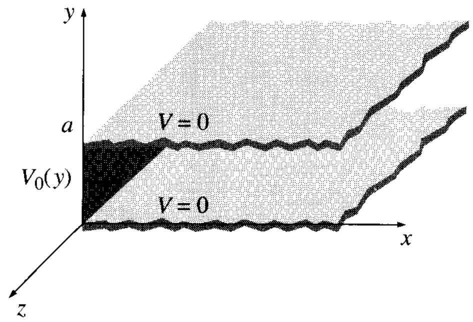
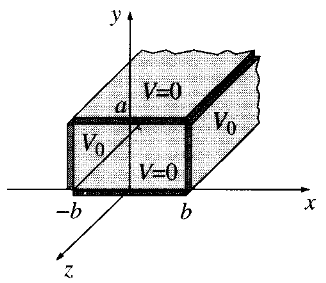
PHYS 507
HANDOUT 6 - *Questions on Laplace Equation and Mulitipole Expansion*

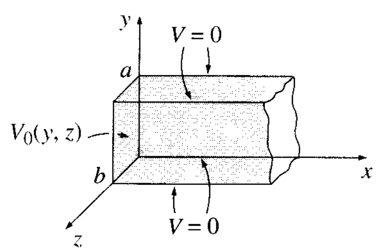
6.1 Two infinite grounded metal plates lie parallel to the xz plane, one at y=0, the other at y=a. The left end, at x=0, is closed off with an infinite strip insulated from the two plates and maintained at a specific potential $V_0(y)$. Find the potential inside this "slot".



6.2 Two infinitely long grounded metal plates, again at y=0, and y=a, are connected at $x = \pm b$ by metal strips maintained at a constant potential V_0 (a thin layer of insulation at each corner prevents them from shorting out). Find the potential inside the resulting rectangular pipe.



6.3 An infinitely long rectangular metal pipe (sides a and b) is grounded, but one end, at x =0, is maintained at a specified potential $V_0(y,z)$. Find the potential inside the pipe.

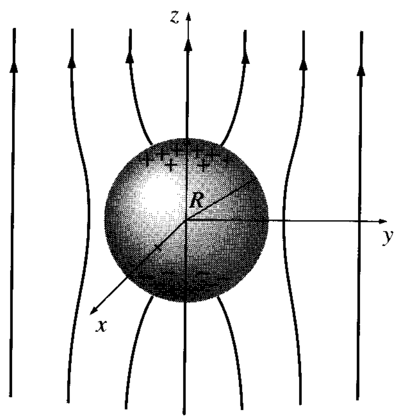


6.4 Find the general solution of the Laplace equation in spherical coordinates when the potential has an **azimuthal symmetry** (does not depend on the angle φ).

6.5 Find the general solution of the Laplace equation in spherical coordinates when the potential has the form $V_0(\theta)$ on the sphere. Study the special case where $V_0(\theta) = k\sin^2\theta/2$. We are interested in the potential **inside** the sphere.

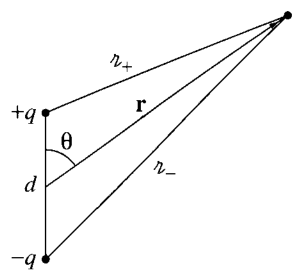
6.6 Solve the previous problem for the potential **outside** the sphere.

6.7 An uncharged metal sphere of radius R is placed in an otherwise uniform field $\mathbf{E} = E_0\hat{\mathbf{z}}$. The field will push positive charge to the "northern" surface of the sphere, leaving a negative charge on the "southern" surface. The induced charge, in turn, distorts the field in the neighborhood of the sphere. Find the potential in the region outside the sphere.

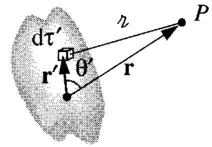


6.8 A spherical shell of radius R has a charge distributed on its surface with a surface charge density $\sigma_0(\theta)$. Find the resulting potential inside and outside the shell.

6.9 Find the potential at a point away from a dipole as shown in the figure.



6.10 Prove the multipole expansion formula for the distribution shown in the figure:



6.11 Study the dipole distribution in a mutipole expansion.

6.12 Prove the when the total charge is zero the dipole moment is independent of the choice of the origin.

6.13 Calculate the electric field of an electric dipole starting from the formula of its potential in spherical coordinates:

$$V_{dip}\left(r,\theta\right)=\frac{\hat{\mathbf{r}}\cdot\mathbf{p}}{4\pi\varepsilon_0 r^2}$$