A survey of Probability concepts

Chapter 5

Learning Objectives

- Define probability.
- Explain the terms experiment, event, outcome.
- Define the terms conditional probability and joint probability.
- Calculate probability using the rules of addition and rules of multiplication.

Definition

A probability is a measure of the likelihood that an event in the future will happen. It can only assume a value between 0 and 1.

- A value near zero means the event is unlikely to happen. A value near one means it is likely.
- There are three ways of assigning probability:
  - classical,
  - empirical, and
  - subjective.

Experiment, Outcome, Event

- An experiment is the observation of some activities or the act of taking some measurement.
- An outcome is the particular result of an experiment.
- An event is the collection of one or more outcomes of an experiment.

Experiment, Outcome, Event

		
Experiment	Roll a die	Count the number of members of the board of directors for Fortune 500 companies who are over 60 years of age
All possible outcomes	Observe a 1 Observe a 2 Observe a 3 Observe a 4 Observe a 5 Observe a 6	None are over 60 One is over 60 Two are over 60 ... 29 are over 60 ... ... 48 are over 60 ...
Some possible events	Observe an even number Observe a number greater than 4 Observe a number 3 or less	More than 13 are over 60 Fewer than 20 are over 60

Assigning Probabilities

Three approaches to assigning probabilities

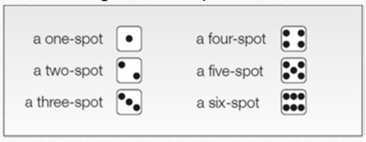
- Classical
- Empirical
- Subjective

Classical Probability

CLASSICAL PROBABILITY $\quad$ $\text{Probability of an event} = \dfrac{\text{Number of favorable outcomes}}{\text{Total number of possible outcomes}}$ **[5-1]**

Consider an experiment of rolling a six-sided die. What is the probability of the event "an even number of spots appear face up"?

The possible outcomes are:



There are three "favorable" outcomes (a two, a four, and a six) in the collection of six equally likely possible outcomes.

Empirical Probability

EMPIRICAL PROBABILITY The probability of an event happening is the fraction of the time similar events happened in the past.

- The empirical approach to probability is based on what is called the law of large numbers. The key to establishing probabilities empirically is that more observations will provide a more accurate estimate of the probability.

LAW OF LARGE NUMBERS Over a large number of trials the empirical probability of an event will approach its true probability.

Empirical Probability

$$Empirical\ probability = \frac{number\ of\ times\ the\ event\ occurs}{Total\ number\ of\ observations}$$

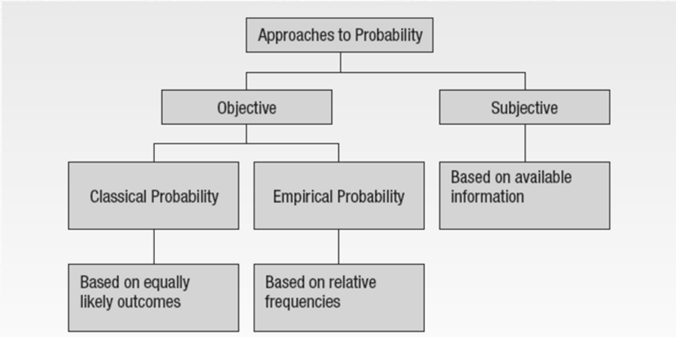
- Example: On February 1, 2003, the Space Shuttle Columbia exploded. This was the second disaster in 113 space missions for NASA. On the basis of this information, what is the probability that a future mission is successfully completed?

Subjective Probability

SUBJECTIVE CONCEPT OF PROBABILITY The likelihood (probability) of a particular event happening that is assigned by an individual based on whatever information is available.

- If there is little or no past experience or information on which to base a probability, it may be arrived at subjectively.

Summary of Types of Probability



Mutually Exclusive and Independent Events

- Events are mutually exclusive if the occurrence of any one event means that none of the others can occur at the same time.
- Events are independent if the occurrence of one event does not affect the occurrence of another.

Collectively Exhaustive Events

- Events are collectively exhaustive if at least one of the events must occur when an experiment is conducted.

Complement Probability

- P(A) means the probability that event A occurs.
- P(~A) means the probability that event A does **NOT** occur.
- To calculate P(~A)

$$P(\sim A) = 1 - P(A)$$

Recall NASA's example, what is the probability that a future mission is unsuccessful?

Complement Probability

- Example: A coin is tossed four times. what is the probability that all four tosses will result in a head face up? What is the probability that not all four tosses will result in a head face up?

A: All four tosses result in a head face up.

$$P(A) = \frac{1}{16} = 0.0625$$
$$P(\sim A) = 1 - 0.0625 = 0.9375$$

Joint Probability

- Joint probability is a probability that measures the likelihood two or more events will happen concurrently.
- For two events A and B we write the joint probability as $P(A\ and\ B)$, $P(A \cap B)$, or $P(AB)$.
- What is P(A and B) if A and B are mutually exclusive.
- What is P(A and B) if A and B are independent.
- How do we calculate the joint probability?

Union of the Events

- If we have two events A and B the union of the events is either event A or event B or both occur on a single performance of an experiment.
- For two events A and B we write the union probability as $P(A\ or\ B)$ or $P(A \cup B)$.
- What is P(A or B) if A and B are mutually exclusive.
- What is P(A or B) if A and B are independent.
- How do we calculate the union probability?

Conditional Probability

- Conditional probability is the probability of a particular event occurring, given that another event has occurred.
- For two events A and B we write the conditional probability of event A occurring, given that event B has occurred as $P(\mathrm{A}|\mathrm{B})$.
- What is $P(\mathrm{A}|\mathrm{B})$ if A and B are mutually exclusive.
- What is $P(\mathrm{A}|\mathrm{B})$ if A and B are independent.
- How do we calculate $P(\mathrm{A}|\mathrm{B})$.

Rules of Addition

- Special rule of Addition:

If two events A and B are mutually exclusive, the probability of one or the other event's occurring equals the sum of their probabilities.

$$P(A\ or\ B) = P(A) + P(B)$$

Example: In a class we have 40 students from three countries. 10 Saudis, 20 Indians, and 10 Egyptians. What is the probability that a selected student is either from India or from Egypt?

Rules of Addition

- General rule of Addition:

If we have two events A and B the probability that either event A or event B or both occur is:

$$P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)$$

Example: In a class of 35 students, 15 students speak English, 25 students speak Arabic, 10 students speak both Arabic and English, and 5 students speak French. What is the probability that a selected student speaks Arabic or English?

Rules of Multiplication

- Special rule of Multiplication:

If two events A and B are independent, the probability that the two events will happen concurrently is the probability of one event multiply the probability of the other event.

$$P(A \text{ and } B) = P(A) * P(B)$$

Example: In an experiment of tossing a fair coin, what is the probability that a head will face up in the first trial and in the second trial?

Rules of Multiplication

- General rule of Multiplication:

If we have two events A and B the probability that the two events will happen concurrently is:

$$P(A \text{ and } B) = P(\mathrm{A}|\mathrm{B}) * P(B)$$
$$P(A \text{ and } B) = P(\mathrm{B}|\mathrm{A}) * P(A)$$

Rules of Multiplication

Example: A golfer has 15 golf shirts in his closet. Suppose 10 of these shirts are white and the others blue. He gets dressed in the dark, so he just grabs a shirt and puts it on. He plays golf two days in a row and does not do laundry. What is the likelihood both shirts selected are white?

Let A_1 first shirt selected is white.

And A_2 second shirt selected is white.

We want $P(A_1 \text{ and } A_2)$. We have is $P(A_1) = \frac{10}{15}$, and $P(A_2|A_1) = \frac{9}{14}$

Example 1

- Example: A manufacturer of preassembled windows produced 50 windows yesterday. This morning the quality assurance inspector reviewed each window for all quality aspects. Each was classified as acceptable or unacceptable and by the shift on which it was produced. Thus we reported two variables on a single item. The two variables are shift and quality. The results are reported in the following table.

	Shift			
	Day	Afternoon	Night	Total
Defective	3	2	1	6
Acceptable	17	13	14	44
Total	20	15	15	50

Example 1

- What is the probability that a randomly selected window is defective and produced in a day shift?
- What is the probability that a randomly selected window is acceptable and produced in a night shift?
- What is the probability that a randomly selected window is defective given that it was produced in a day shift?
- What is the probability that a randomly selected window is produced in a night shift given it is acceptable?

	Shift			
	Day	Afternoon	Night	Total
Defective	3	2	1	6
Acceptable	17	13	14	44
Total	20	15	15	50

Example 2

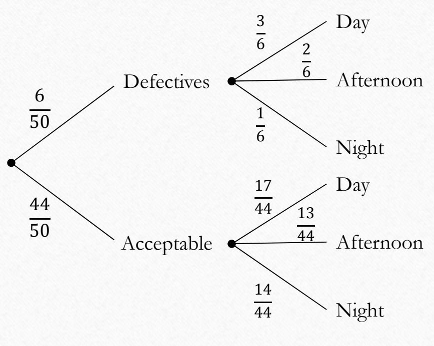
- In a small village of 416 people. Each one has a black cow or yellow cow or both. If the number of people who owned black cow is 316 and the number of people who owned a yellow cow 280 people. What is the probability that a randomly selected person:

1. Do have black cow and yellow cow?
2. Does not have a yellow cow?
3. Do have a black cow given that he/she has a yellow cow?

Tree Diagram

The **tree diagram** is a graph that is helpful in organizing calculations that involve several stages. Each segment in the tree is one stage of the problem. The branches of a tree diagram are weighted by probabilities.

Tree Diagram



$$\frac{6}{50} * \frac{3}{6} = \frac{3}{50}$$

$$\frac{6}{50} * \frac{2}{6} = \frac{2}{50}$$

$$\frac{6}{50} * \frac{1}{6} = \frac{1}{50}$$

$$\frac{44}{50} * \frac{17}{44} = \frac{17}{50}$$

$$\frac{44}{50} * \frac{13}{44} = \frac{13}{50}$$

$$\frac{44}{50} * \frac{14}{44} = \frac{14}{50}$$

Bayes' Theorem

$$P(A_1|B) = \frac{P(B|A_1) * P(A_1)}{P(B|A_1) * P(A_1) + P(B|A_2) * P(A_2)}$$

- Assume in the pervious formula the events A_1 and A_2 are mutually exclusive and collectively exhaustive, and A_1 refers to either event A_1 or A_2. Hence A_1 and A_2 are in this case complements. The meaning of the symbols used is illustrated by the following example

Bayes' Theorem

- Suppose 5 percent of the population of Umen, a fictional Third World country, have a disease that is peculiar to that country. We will let A_1 refer to the event "has the disease" and A_2 refer to the event "does not have the disease." Thus, we know that if we select a person from Umen at random, the probability the individual chosen has the disease is .05, or $P(A_1) = 0.05$ This probability, $P(A_1) = \text{P(has the disease)} = 0.05$ is called the **prior probability.** It is given this name because the probability is assigned before any empirical data are obtained.

Bayes' Theorem

- The prior probability a person is not afflicted with the disease is therefore .95, or $P(A_2) = 0.95$ found by $1 - 0.05$
- There is a diagnostic technique to detect the disease, but it is not very accurate. Let *B* denote the event "test shows the disease is present." Assume that historical evidence shows that if a person actually has the disease, the probability that the test will indicate the presence of the disease is .90. Using the conditional probability definitions developed earlier in this chapter, this statement is written as:

$$P(B|A_1) = 0.90$$

- Assume the probability is .15 that for a person who actually does not have the disease the test will indicate the disease is present. $P(B|A_2) = 0.15$

Bayes' Theorem

- Let's randomly select a person from Umen and perform the test. The test results indicate the disease is present. What is the probability the person actually has the disease? In symbolic form, we want to know $P(A_1|B)$ which is interpreted as: $P(has\ the\ disease \mid the\ test\ results\ are\ positive)$. The probability is called a **posterior probability.**

Bayes' Theorem

If there are n mutually exclusive and collectively exhaustive events $A_1, A_2, A_3 \ldots A_n$ then the Bayes' theorem becomes:

$$P(A_i|B) = \frac{P(A_i)P(B|A_i)}{P(A_1)P(B|A_1) + P(A_2)P(B|A_2) + \cdots + P(A_n)P(B|A_n)}$$

PRIOR PROBABILITY The initial probability based on the present level of information.

POSTERIOR PROBABILITY A revised probability based on additional information.