## MASS TRANSFER BETWEEN PHASES

We consider the mass transfer of solute A from one fluid phase by convection and then through a second fluid phase by convection. For example, the solute may diffuse through a gas phase and then diffuse through and be absorbed in an adjacent and immiscible liquid phase. This occurs in the case of absorption of ammonia from air by water. The two phases are in direct contact with each other, such as in a packed, tray, or spray-type tower, and the interfacial area between the phases is usually not well defined. In two-phase mass transfer, a concentration gradient will exist in each phase, causing mass transfer to occur. At the interface between the two fluid phases, equilibrium exists in most cases. In such cases, equilibrium relations, e.g. Henry's law and equilibrium distribution coefficients, are important to determine concentration profiles for predicting rates of mass transfer.

The concentration in the bulk gas phase $y_{A G}$ decreases to $y_{A i}$ at the interface. The liquid concentration starts at $x_{A i}$ at the interface and falls to $x_{A L}$ in the bulk liquid phase. At the interface, since there would be no resistance to transfer across this interface, $y_{A i}$ and $x_{A i}$ are in equilibrium and are related by the equilibrium distribution relation (e.g. Henry's law),

$$
\begin{equation*}
y_{A i}=f\left(x_{A i}\right) \tag{10.4-1}
\end{equation*}
$$

We consider here any two-phase system where y stands for the one phase and $x$ for the other phase. For example, for the extraction of the solute acetic acid (A) from water (y phase) by isopropyl ether (x phase), the same relations will hold.

## Mass Transfer Using Film Mass-Transfer Coefficients and Interface Concentrations

## Case 1. Equi-molar counter-diffusion

For A diffusing from the gas to liquid and Bin equi-molar counter-diffusion from liquid to gas,

$$
\begin{equation*}
N_{A}=k_{y}^{\prime}\left(y_{A G}-y_{A i}\right)=k_{x}^{\prime}\left(x_{A i}-x_{A L}\right) \tag{10.4-2}
\end{equation*}
$$

The driving force in the gas phase is $\left(y_{A G}-y_{A i}\right)$ and in the liquid phase it is ( $x_{A i}-x_{A L}$ ). Here, $k_{y}^{\prime}$ is gas-phase mass-transfer coefficient in $\mathrm{kg} \mathrm{mol} / \mathrm{s} \cdot \mathrm{m}^{2} \cdot \mathrm{~mol}$ frac and $k_{x}^{\prime}$ is liquid-phase mass-transfer coefficient.

Rearranging,

$$
\begin{equation*}
-\frac{k_{x}^{\prime}}{k_{y}^{\prime}}=\frac{y_{A G}-y_{A i}}{x_{A L}-x_{A i}} \tag{10.4-3}
\end{equation*}
$$

In Fig. 10.4-2, point P represents the bulk phase compositions $y_{A G}$ and $x_{A L}$ of the two phases and point M the interface concentrations $y_{A i}$ and $x_{A i}$. The slope of the line PM is $-k_{x}^{\prime} / k_{y}^{\prime}$. Therefore, if the two film coefficients are known, the interface compositions, i.e. $x_{A i}, y_{A i}$ can be determined by drawing line PM with a slope $-k_{x}^{\prime} / k_{y}^{\prime}$ intersecting the equilibrium line.

Fig 10.4-2. For the case of equimolar counterdiffusion, conc. driving forces and interface conc. in interphase mass transfer


## Case 2. Diffusion of A through stagnant or non-diffusing B.

For the common case of A diffusing through a stagnant gas phase and then through a stagnant liquid phase, the equations for A diffusing through a stagnant gas and then through a stagnant liquid are

$$
\begin{equation*}
N_{A}=k_{y}\left(y_{A G}-y_{A i}\right)=k_{x}\left(x_{A i}-x_{A L}\right) \tag{10.4-4}
\end{equation*}
$$

The driving force in the gas phase is $\left(y_{A G}-y_{A i}\right)$ and $\left(x_{A i}-x_{A L}\right)$ in the liquid phase. Here, the gas-phase mass-transfer coefficient ( $\mathrm{kg} \mathrm{mol} / \mathrm{s} \cdot \mathrm{m}^{2} \cdot \mathrm{~mol} \mathrm{frac}$ ) is

$$
\begin{equation*}
k_{y}=\frac{k_{y}^{\prime}}{\left(1-y_{A}\right)_{i M}}=\frac{k_{y}^{\prime}}{y_{B_{i M}}} \tag{10.4-5a}
\end{equation*}
$$

and the liquid-phase mass-transfer coefficient is

$$
\begin{equation*}
k_{x}=\frac{k_{x}^{\prime}}{\left(1-x_{A}\right)_{i M}}=\frac{k_{x}^{\prime}}{x_{B_{i M}}} \tag{10.4-5a}
\end{equation*}
$$

Rearranging,

$$
\begin{equation*}
-\frac{k_{x}}{k_{y}}=-\frac{\left[\frac{k_{x}^{\prime}}{\left(1-x_{A}\right)_{i M}}\right]}{\left[\frac{k_{y}^{\prime}}{\left(1-y_{A}\right)_{i M}}\right]}=\frac{y_{A G}-y_{A i}}{x_{A L}-x_{A i}} \tag{10.4-3}
\end{equation*}
$$

where,

$$
\begin{equation*}
\left(1-y_{A}\right)_{i M}=\frac{\left(1-y_{A i}\right)-\left(1-y_{A G}\right)}{\ln \left[\left(1-y_{A i}\right) /\left(1-y_{A G}\right)\right]}=y_{B_{i M}}=\frac{y_{B_{i}}-y_{B_{G}}}{\ln \left[y_{B_{i}} / y_{B_{G}}\right]} \tag{10.4-6,7}
\end{equation*}
$$

In Figure 10.4-3, the slope of the line PM is $-k_{x} / k_{y}$. Therefore, if the two film coefficients are known, the interface compositions, i.e. $x_{A i}, y_{A i}$ can be determined by drawing line PM with a slope $-k_{x} / k_{y}$ intersecting the equilibrium line.

However, since $\left(1-x_{A}\right)_{i M}$ and $\left(1-y_{A}\right)_{i M}$ are unknowns, a trial and error method is needed. For the first trial $\left(1-x_{A}\right)_{i M}$ and $\left(1-y_{A}\right)_{i M}$ are assumed to be 1.0 and Eq. (10.4-9) is used to get the slope and $x_{A i}, y_{A i}$. Then for the second trial, these values of $x_{A i}, y_{A i}$ are used to calculate a new slope to get new values of $x_{A i}, y_{A i}$. This is repeated until the interface compositions do not change.

Figure 10.4-3: For the case of A diffusing through stagnant B, conc. driving forces and interface concentrations in interphase mass transfer


## Example 10.4-1 (CJG):

## Determination of Interface Composition in Interphase Mass Transfer

The solute $A$ is being absorbed from a gas mixture of $A$ and $B$ in a wettedwall tower with the liquid flowing as a film downward along the wall. At a certain point in the tower, bulk gas concentration is $\boldsymbol{y}_{\boldsymbol{A G}}=\mathbf{0 . 3 8} \mathrm{mol}$ fraction and the bulk liquid concentration is $\boldsymbol{x}_{\boldsymbol{A L}}=\mathbf{0 . 1 0}$.
The tower is operating at 298 K and $101.325 \times 10^{3} \mathrm{~Pa}$ and the equilibrium data are given in the table.

| $x_{A}$ | 0 | 0.050 | 0.10 | 0.15 | 0.20 | 0.25 | 0.30 | 0.35 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y_{A}$ | 0 | 0.022 | 0.052 | 0.087 | 0.131 | 0.187 | 0.265 | 0.385 |

The solute A diffuses through a stagnant B in the gas phase and then through a non-diffusing liquid.

Using correlations for dilute solutions in wetted-wall towers, the film masstransfer coefficients for A in the gas and liquid phases are predicted as:

$$
\begin{aligned}
& k_{y}=1.465 \times 10^{-3} \mathrm{~kg} \mathrm{~mol} \mathrm{~A} / \mathrm{s} \cdot \mathrm{~m}^{2} \cdot \text { molfrac } \\
& k_{x}=1.967 \times 10^{-3} \mathrm{~kg} \mathrm{~mol} \mathrm{~A} / \mathrm{s} \cdot \mathrm{~m}^{2} \cdot \text { molfrac }
\end{aligned}
$$

Note that for dilute solution, $k_{x}^{\prime}=k_{x} ; k_{y}^{\prime}=k_{y}$.
Calculate the interface concentrations, $\left(x_{A i}, y_{A i}\right)$ and flux of component A.

## SOLUTION:

Step 1: Plot the equilibrium data for the given temperature and pressure. Next, locate the bulk compositions, $\mathrm{P}\left(x_{A L}, y_{A G}\right)$.

Step 2: One needs to plot a straight line from point $\mathrm{P}\left(x_{A L}, y_{A G}\right)$ with the slope

$$
-\frac{k_{x}}{k_{y}}=-\frac{\left[\frac{k_{x}^{\prime}}{\left(1-x_{A}\right)_{i M}}\right]}{\left[\frac{k_{y}^{\prime}}{\left(1-y_{A}\right)_{i M}}\right]}
$$

Its intersection with equilibrium curve gives interface composition $\left(x_{A i}, y_{A i}\right)$. Since $\left(x_{A i}, y_{A i}\right)$ is not known, one need to its value for computing the slope. Therefore, iterative approach is required.

## Trial 1

Assume $\left(x_{A i}=0, y_{A i}=0\right)$, and compute the slope

$$
\begin{gathered}
\left(1-x_{A}\right)_{i M}=1 ;\left(1-y_{A}\right)_{i M}=1 \\
-\frac{k_{x}}{k_{y}}=-\frac{\left[\frac{1.967 \times 10^{-3}}{1}\right]}{\left[\frac{1.465 \times 10^{-3}}{1}\right]}=-1.342
\end{gathered}
$$

Use the following straight line equation,

$$
\begin{gathered}
y-y_{0}=m\left(x-x_{0}\right) \\
y-y_{A G}=-\frac{k_{x}}{k_{y}}\left(x-x_{A L}\right) \\
y-0.38=-1.342(x-0.1)
\end{gathered}
$$

Choose any arbitrary value for $x$ to get $y$. Choosing, $x=0.2$ gives $y=$ $0.38-0.1342=0.246$. Now draw a straight line with points $(0.10,0.38)$ and $(0.2,0.246)$. Its intersection with the equilibrium line from the graph gives

$$
\left(x_{A i}=0.247, y_{A i}=0.183\right)
$$

## Trial 2

From first trial, use $\left(x_{A i}=0.247, y_{A i}=0.183\right)$, and compute the slope

$$
\begin{gathered}
\left(1-x_{A}\right)_{i M}=\frac{\left(1-x_{A L}\right)-\left(1-x_{A i}\right)}{\ln \left[\left(1-x_{A L}\right) /\left(1-x_{A i}\right)\right]}=\frac{(1-0.10)-(1-0.247)}{\ln [(1-0.10) /(1-0.247)]}=0.825 \\
\left(1-y_{A}\right)_{i M}=\frac{\left(1-y_{A i}\right)-\left(1-y_{A G}\right)}{\ln \left[\left(1-y_{A i}\right) /\left(1-y_{A G}\right)\right]}=\frac{(1-0.183)-(1-0.38)}{\ln [(1-0.183) /(1-0.38)]}=0.715 \\
\quad-\frac{k_{x}}{k_{y}}=-\frac{\left[\frac{1.967 \times 10^{-3}}{0.825}\right]}{\left[\frac{1.465 \times 10^{-3}}{0.715}\right]}=-1.163
\end{gathered}
$$

Trial 1 slope $=-1.342$
Trial 2 slope $=-1.163$
Use the following straight line equation,

$$
\begin{gathered}
y-y_{0}=m\left(x-x_{0}\right) \\
y-y_{A G}=-\frac{k_{x}}{k_{y}}\left(x-x_{A L}\right) \\
y-0.38=-1.163(x-0.1)
\end{gathered}
$$

Choose any arbitrary value for $x$ to get $y$. Choosing, $x=0.2$ gives $y=0.38-$ $0.1163=0.264$. Now draw a straight line with points $(0.10,0.38)$ and ( $0.2,0.264$ ). Its intersection with the equilibrium line from the graph gives

$$
\left(x_{A i}=0.257, y_{A i}=0.197\right)
$$

## Trial 3

From second trial, use ( $x_{A i}=0.257, y_{A i}=0.197$ ), and compute the slope

$$
\begin{gathered}
\left(1-x_{A}\right)_{i M}=\frac{\left(1-x_{A L}\right)-\left(1-x_{A i}\right)}{\ln \left[\left(1-x_{A L}\right) /\left(1-x_{A i}\right)\right]}=\frac{(1-0.10)-(1-0.257)}{\ln [(1-0.10) /(1-0.257)]}=0.820 \\
\left(1-y_{A}\right)_{i M}=\frac{\left(1-y_{A i}\right)-\left(1-y_{A G}\right)}{\ln \left[\left(1-y_{A i}\right) /\left(1-y_{A G}\right)\right]}=\frac{(1-0.197)-(1-0.38)}{\ln [(1-0.197) /(1-0.38)]}=0.709 \\
-\frac{k_{x}}{k_{y}}=-\frac{\left[\frac{1.967 \times 10^{-3}}{0.825}\right]}{\left[\frac{1.465 \times 10^{-3}}{0.715}\right]}=-1.16
\end{gathered}
$$

Trial 1 slope $=-1.342$
Trial 2 slope $=-1.163$
Trial 3 slope $=\mathbf{- 1 . 1 6 0}$
Therefore,

$$
\begin{aligned}
x_{A i} & =0.257, y_{A i}=0.197 \\
x_{A}^{*} & =0.349, y_{A}^{*}=0.052
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
& \left.\begin{array}{l}
N_{A}=k_{y}\left(y_{A G}-y_{A i}\right)=\left[\frac{1.465 \times 10^{-3}}{0.715}\right](0.38-0.197) \\
=3.78
\end{array}\right) \\
& \left.\begin{array}{rl}
N_{A}=k_{x}\left(x_{A i}-x^{-4} \mathrm{~kg} \mathrm{~mol} \mathrm{~A} / \mathrm{s} \cdot \mathrm{~m}^{2}\right.
\end{array}\right)=\left[\frac{1.967 \times 10^{-3}}{0.825}\right](0.257-0.10) \\
& =3.78 \times 10^{-4} \mathrm{~kg} \mathrm{~mol} \mathrm{~A} / \mathrm{s} \cdot \mathrm{~m}^{2}
\end{aligned}
$$

## Summary

| Trial | $x_{A i}$ | $y_{A i}$ | $\left(1-x_{A}\right)_{i M}$ | $\left(1-y_{A}\right)_{i M}$ | $-\boldsymbol{k}_{x} / \boldsymbol{k}_{\boldsymbol{y}}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | 0 | 0 | 1.00 | 1.00 | -1.342 |
| 2 | 0.247 | 0.183 | 0.825 | 0.715 | -1.163 |
| 3 | 0.257 | 0.197 | 0.820 | 0.709 | -1.160 |
| 4 | 0.257 | 0.197 |  |  |  |




Figure 10.4-4. Location of interface concentrations for Example 10.4-1.

## Question 4 (Fall 20182019 Test 2):

The solute $A$ is being absorbed from a gas mixture of $A$ and $B$ in a wetted-wall tower with the liquid flowing as a film downward along the wall. At a certain point in the tower, bulk gas concentration is $y_{A G}=0.38 \mathrm{~mol}$ fraction and the bulk liquid concentration is $x_{A L}=0.10$. The solute A diffuses through a stagnant B in the gas phase and then through a non-diffusing liquid. Using correlations for dilute solutions in wetted-wall towers, the film mass-transfer coefficients for $A$ in the gas and liquid phases are predicted as:
$k_{y}=1.00 \times 10^{-3} \mathrm{~kg} \mathrm{~mol} \mathrm{~A} / \mathrm{s} \cdot \mathrm{m}^{2} \cdot$ molfrac $; k_{x}=2.00 \times 10^{-3} \mathrm{~kg} \mathrm{~mol} \mathrm{~A} / \mathrm{s} \cdot \mathrm{m}^{2} \cdot \mathrm{molfrac}$
Note that for dilute solution, $k_{x}^{\prime}=k_{x} ; k_{y}^{\prime}=k_{y}$. The tower is operating at 298 K and $101.325 \times$ $10^{3} \mathrm{~Pa}$. The equilibrium data are given in the figure, which also show the interface concentration. Determine

- Mass transfer coefficient, $k_{y}$, in the present case.
- Molar flux of component $\mathrm{A}, \mathrm{N}_{\mathrm{A}}, \mathrm{kg} \mathrm{mol} \mathrm{A} / \mathrm{s} \cdot \mathrm{m}^{2}$



## Question 4 (Fall 20182019 Test 2): Solution

From the figure,

$$
\begin{gathered}
x_{A i}=0.228 \\
y_{A i}=0.158 \\
\left(1-y_{A}\right)_{i M}=\frac{\left(1-y_{A i}\right)-\left(1-y_{A G}\right)}{\ln \left[\left(1-y_{A i}\right) /\left(1-y_{A G}\right)\right]}=\frac{(1-0.158)-(1-0.38)}{\ln [(1-0.158) /(1-0.38)]}=0.725 \\
N_{A}=k_{y}\left(y_{A G}-y_{A i}\right)=\left[\frac{1.0 \times 10^{-3}}{0.725}\right](0.38-0.158)=3.06 \times 10^{-4} \mathrm{~kg} \mathrm{~mol} \mathrm{~A} / \mathrm{s} \cdot \mathrm{~m}^{2}
\end{gathered}
$$

From the figure,

$$
\begin{gathered}
x_{A i}=0.228 \\
y_{A i}=0.158 \\
\left(1-x_{A}\right)_{i M}=\frac{\left(1-x_{A L}\right)-\left(1-x_{A i}\right)}{\ln \left[\left(1-x_{A L}\right) /\left(1-x_{A i}\right)\right]}=\frac{(1-0.10)-(1-0.228)}{\ln [(1-0.10) /(1-0.228)]}=0.835 \\
N_{A}=k_{x}\left(x_{A i}-x_{A L}\right)=\left[\frac{2.0 \times 10^{-3}}{0.835}\right](0.228-0.10)=3.06 \times 10^{-4} \mathrm{~kg} \mathrm{~mol} \mathrm{~A} / \mathrm{s} \cdot \mathrm{~m}^{2}
\end{gathered}
$$

## Overall Mass-Transfer Coefficients and Driving Forces

Film or local single-phase mass-transfer coefficients are often difficult to measure experimentally. Therefore, overall mass-transfer coefficients $K_{y}^{\prime}$ and $K_{x}^{\prime}$ are measured based on the overall gas phase/liquid phase driving forces,

$$
\begin{equation*}
N_{A}=K_{y}^{\prime}\left(y_{A G}-y_{A}^{*}\right)=K_{x}^{\prime}\left(x_{A}^{*}-x_{A L}\right) \tag{10.4-10}
\end{equation*}
$$

$K_{y}^{\prime}$ : overall gas-phase mass-transfer coefficient in $\mathrm{kg} \mathrm{mol} / \mathrm{s} \cdot \mathrm{m}^{2} \cdot \mathrm{~mol}$ frac $K_{x}^{\prime}$ : overall liquid-phase mass-transfer coefficient in $\mathrm{kg} \mathrm{mol} / \mathrm{s} \cdot \mathrm{m}^{2} \cdot \mathrm{~mol}$ frac $y_{A}^{*}$ : gas-phase value that would be in equilibrium with $x_{A L}$
$x_{A}^{*}$ : liquid-phase value that would be in equilibrium with $y_{A G}$

Case 1: Equi-molar Counter-diffusion

$$
\begin{equation*}
N_{A}=k_{y}^{\prime}\left(y_{A G}-y_{A i}\right)=k_{x}^{\prime}\left(x_{A i}-x_{A L}\right) \tag{10.4-2}
\end{equation*}
$$

| Overall gas-phase driving forces: | $y_{A G}-y_{A}^{*}=\left(y_{A G}-y_{A i}\right)+\left(y_{A i}-y_{A}^{*}\right)$ |
| :--- | :--- |


| $m^{\prime}=\frac{y_{A i}-y_{A}^{*}}{x_{A i}-x_{A L}}$ | $10.4-$ <br> 13 |
| :--- | :--- |
| $y_{A G}-y_{A}^{*}=\left(y_{A G}-y_{A i}\right)$ <br> $+m^{\prime}\left(x_{A i}-x_{A L}\right)$ | $10.4-$ <br> 14 |
| $\frac{N_{A}}{K_{y}^{\prime}}=\frac{N_{A}}{k_{y}^{\prime}}+\frac{m^{\prime} N_{A}}{k_{x}^{\prime}}$ | $10.4-$ <br> $\frac{1}{K_{y}^{\prime}}=\frac{1}{k_{y}^{\prime}}+\frac{m^{\prime}}{k_{x}^{\prime}}$ |



If slope $\mathrm{m}^{\prime}$ is quite small, so that the equilibrium curve in Fig. 10.4-2 is almost horizontal, a small value of $y_{A}$ in the gas will give a large value of $x_{A}$ in equilibrium in
the liquid. The gas solute A is then very soluble in the liquid phase, and hence the term $\frac{m^{\prime}}{k_{x}^{\prime}}$ in Eq. (10.4-15) is very small. Then,
$\frac{1}{K_{y}^{\prime}} \cong \frac{1}{k_{y}^{\prime}}$
and the major resistance is in the gas phase, or the "gas phase is controlling." The point $M$ has moved down very close to $E$, so that
$y_{A G}-y_{A}^{*} \cong\left(y_{A G}-y_{A i}\right)$

Overall liquid phase driving force:
$x_{A}^{*}-x_{A L}=\left(x_{A}^{*}-x_{A i}\right)+\left(x_{A i}-x_{A L}\right)$

| $m^{\prime \prime}=\frac{y_{A G}-y_{A i}}{x_{A}^{*}-x_{A i}}$ | $10.4-$ <br> 17 |
| :---: | :---: |
| $x_{A}^{*}-x_{A L}=\left(y_{A G}-y_{A i}\right) / m^{\prime \prime}$ <br> $+\left(x_{A i}-x_{A L}\right)$ |  |
| $\frac{1}{K_{x}^{\prime}}=\frac{1}{m^{\prime \prime} k_{y}^{\prime}}+\frac{1}{k_{x}^{\prime}}$ | $10.4-$ <br> 18 |



Similarly, when $\mathrm{m}^{\prime \prime}$ is very large, the solute A is very insoluble in the liquid, $\frac{1}{m^{\prime \prime} k_{y}^{\prime}}$ becomes small,

$$
\frac{1}{K_{x}^{\prime}} \cong \frac{1}{k_{x}^{\prime}}
$$

The "liquid phase is controlling" and

$$
x_{A}^{*}=x_{A i}
$$

Systems for absorption of oxygen or carbon dioxide from air by water are similar to (10.4-21).

## Case 2: Diffusion of A through stagnant or nondiffusing B

$$
N_{A}=\frac{k_{y}^{\prime}}{\left(1-y_{A}\right)_{i M}}\left(y_{A G}-y_{A i}\right)=\frac{k_{x}^{\prime}}{\left(1-x_{A}\right)_{i M}}\left(x_{A i}-x_{A L}\right)
$$

Since the overall gas-phase driving forces can be written as,

$$
y_{A G}-y_{A}^{*}=\left(y_{A G}-y_{A i}\right)+\left(y_{A i}-y_{A}^{*}\right)
$$

But, as before

$$
m^{\prime}=\frac{y_{A i}-y_{A}^{*}}{x_{A i}-x_{A L}}
$$

Therefore,

$$
y_{A G}-y_{A}^{*}=\left(y_{A G}-y_{A i}\right)+m^{\prime}\left(x_{A i}-x_{A L}\right)
$$

Define,

$$
N_{A}=\frac{K_{y}^{\prime}}{\left(1-y_{A}\right)_{* M}}\left(y_{A G}-y_{A}^{*}\right)=\frac{K_{x}^{\prime}}{\left(1-x_{A}\right)_{* M}}\left(x_{A}^{*}-x_{A L}\right)
$$

or,

$$
N_{A}=K_{y}\left(y_{A G}-y_{A}^{*}\right)=K_{x}\left(x_{A}^{*}-x_{A L}\right)
$$

Here,
$K_{y}$ : overall gas-phase mass-transfer coefficient for A diffusing through stagnant B $K_{x}$ : overall liquid-phase mass-transfer coefficient for A diffusing through stagnant B $y_{A}^{*}$ : gas-phase value that would be in equilibrium with $x_{A L}$
$x_{A}^{*}$ : liquid-phase value that would be in equilibrium with $y_{A G}$

$$
\begin{align*}
& \left(1-y_{A}\right)_{* M}=\frac{\left(1-y_{A}^{*}\right)-\left(1-y_{A G}\right)}{\ln \left[\left(1-y_{A}^{*}\right) /\left(1-y_{A G}\right)\right]}=y_{B_{* M}}=\frac{y_{B}^{*}-y_{B G}}{\ln \left[y_{B}^{*} / y_{B G}\right]}  \tag{10.4-25}\\
& \left(1-x_{A}\right)_{* M}=\frac{\left(1-x_{A L}\right)-\left(1-x_{A}^{*}\right)}{\ln \left[\left(1-x_{A L}\right) /\left(1-x_{A}^{*}\right)\right]}=x_{B * M}=\frac{x_{B L}-x_{B}^{*}}{\ln \left[x_{B L} / x_{B}^{*}\right]} \tag{10.4-27}
\end{align*}
$$

Note: Overall mass transfer coefficients are concentration dependent

$$
\begin{equation*}
\frac{1}{K_{y}}=\frac{1}{k_{y}}+\frac{m^{\prime}}{k_{x}} \tag{24}
\end{equation*}
$$

$$
\begin{aligned}
\frac{1}{K_{x}}=\frac{1}{k_{y} m^{\prime \prime}} & \frac{1}{K_{x}^{\prime} /\left(1-x_{A}\right)_{* M}}=\left(m^{\prime \prime} \frac{k_{y}^{\prime}}{\left(1-y_{A}\right)_{i M}}\right)^{-1}+\left(\frac{k_{x}^{\prime}}{\left(1-x_{A}\right)_{i M}}\right)^{-1} 26 \\
& 2 \frac{1}{k_{x}}
\end{aligned}
$$

$\left.\begin{array}{l|c|}\hline & \begin{array}{l}\text { Equi-molar Counter- } \\ \text { diffusion }\end{array}\end{array} \begin{array}{l}\text { Diffusion of A through } \\ \text { stagnant or nondiffusing B }\end{array}\right]$

Overall liquid-phase mass-transfer coefficient

$$
\frac{1}{K_{x}^{\prime}}=\frac{1}{m^{\prime \prime} k_{y}^{\prime}}+\frac{1}{k_{x}^{\prime}} \quad \frac{1}{K_{x}}=\frac{1}{k_{y} m^{\prime \prime}}+\frac{1}{k_{x}}
$$

$$
\boldsymbol{K}_{y}=\frac{\boldsymbol{K}_{y}^{\prime}}{\left(1-\boldsymbol{y}_{A}\right)_{* M}} ; \boldsymbol{K}_{x}=\frac{\boldsymbol{K}_{x}^{\prime}}{\left(1-x_{A}\right)_{* M}} ; \boldsymbol{k}_{y}=\frac{\boldsymbol{k}_{y}^{\prime}}{\left(1-\boldsymbol{y}_{A}\right)_{i M}} ; \boldsymbol{k}_{x}=\frac{\boldsymbol{k}_{x}^{\prime}}{\left(1-x_{A}\right)_{i M}}
$$

## Ex. 10.4-2 (CJG): Overall Mass-Transfer Coefficients from Film Coefficients

The solute A is being absorbed from a gas mixture of A and B in a wetted-wall tower with the liquid flowing as a film downward along the wall. At a certain point in the tower, bulk gas concentration is $y_{A G}=0.38 \mathrm{~mol}$ fraction and the bulk liquid concentration is $\boldsymbol{x}_{\boldsymbol{A L}}=\mathbf{0} \mathbf{1 0}$. The tower is operating at 298 K and $101.325 \times 10^{3} \mathrm{~Pa}$ and the equilibrium data are given in the table.

| $x_{A}$ | 0 | 0.050 | 0.10 | 0.15 | 0.20 | 0.25 | 0.30 | 0.35 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y_{A}$ | 0 | 0.022 | 0.052 | 0.087 | 0.131 | 0.187 | 0.265 | 0.385 |

The solute $A$ diffuses through a stagnant $B$ in the gas phase and then through a nondiffusing liquid. Using correlations for dilute solutions in wetted-wall towers, the film mass-transfer coefficients for A in the gas and liquid phases are predicted as:

$$
\begin{aligned}
& k_{y}=1.465 \times 10^{-3} \mathrm{~kg} \mathrm{~mol} A / \mathrm{s} \cdot \mathrm{~m}^{2} \cdot \text { molfrac } \\
& k_{x}=1.967 \times 10^{-3} \mathrm{~kg} \mathrm{~mol} A / \mathrm{s} \cdot \mathrm{~m}^{2} \cdot \text { molfrac }
\end{aligned}
$$

Note that for dilute solution, $k_{x}^{\prime}=k_{x} ; k_{y}^{\prime}=k_{y}$.
Calculate the overall mass transfer coefficient $K_{y}^{\prime}$, the flux, and the percent resistance in the gas and liquid films. Do this for the case of A diffusing through stagnant B.

## Solution:

$$
\begin{aligned}
& y_{A G}^{*}=0.052 \text { (See figure in next page) } \\
& m^{\prime}=\frac{y_{A i}-y_{A}^{*}}{x_{A i}-x_{A L}}=\frac{0.197-0.052}{0.257-0.100}=0.923 \\
& k_{y}=\frac{k_{y}^{\prime}}{\left(1-y_{A}\right)_{i M}}=\frac{1.465 \times 10^{-3}}{0.709} \\
& k_{x}=\frac{k_{x}^{\prime}}{\left(1-x_{A}\right)_{i M}}=\frac{1.967 \times 10^{-3}}{0.820} \\
& \left(1-y_{A}\right)_{* M}=\frac{\left(1-y_{A}^{*}\right)-\left(1-y_{A G}\right)}{\ln \left[\left(1-y_{A}^{*}\right) /\left(1-y_{A G}\right)\right]}=\frac{(1-0.052)-(1-0.380)}{\ln [(1-0.052) /(1-0.380)]}=0.773
\end{aligned}
$$



Total gas-phase resistance is sum of gas film and liquid film, i.e.
$\frac{1}{K_{y}}($ Total resistance $)=\frac{1}{k_{y}}($ Gas film resistance $)+\frac{m^{\prime}}{k_{x}}($ Liquid film resistance $)$
$\frac{1}{K_{y}^{\prime} /\left(1-y_{A}\right)_{* M}}=\frac{1}{k_{y}^{\prime} /\left(1-y_{A}\right)_{i M}}+\frac{m^{\prime}}{k_{x}^{\prime} /\left(1-x_{A}\right)_{i M}}$
$\frac{1}{\frac{K_{y}^{\prime}}{0.773}}=\frac{1}{\frac{1.465 \times 10^{-3}}{0.709}}+\frac{0.923}{\frac{1.967 \times 10^{-3}}{0.820}}=484(56 \%)+384.8(44 \%)=868.8$
$K_{y}^{\prime}=8.90 \times 10^{-4} \mathrm{~kg} \mathrm{~mol} A / \mathrm{s} \cdot \mathrm{m}^{2} \cdot$ molfrac

$$
\begin{gathered}
N_{A}=\frac{K_{y}^{\prime}}{\left(1-y_{A}\right)_{* M}}\left(y_{A G}-y_{A}^{*}\right)=\frac{8.90 \times 10^{-4}}{0.773}(0.38-0.052) \\
=3.78 \times 10^{-4} \frac{\mathrm{~kg} \mathrm{~mol} \mathrm{~A}}{\mathrm{~s} \cdot \mathrm{~m}^{2}}
\end{gathered}
$$

## Final Exam Winter 20172018:

The solute A is being absorbed from a gas mixture of A and B in a wetted-wall tower with the liquid flowing as a film downward along the wall. At a certain point in the tower the bulk gas concentration $\mathrm{y}_{\mathrm{AG}}=0.30 \mathrm{~mol}$ fraction and the bulk liquid concentration is $\mathrm{x}_{\mathrm{AL}}=0.10$. The tower is operating at 298 K and 1013 kPa and the equilibrium data given in the figure. The solute A diffuses through stagnant B in the gas phase and then through a non-diffusing liquid. Using correlations for dilute solutions in wetted-wall towers, the film mass-transfer coefficient for A in the gas phase is predicted as:
$k_{x}^{\prime} a=1.0 \times 10^{-2} \mathrm{~kg} \mathrm{~mol} / \mathrm{s} \cdot \mathrm{m}^{3} \cdot \mathrm{~mol}$ frac ; $k_{y}^{\prime} a=5.0 \times 10^{-2} \mathrm{~kg} \mathrm{~mol} / \mathrm{s} \cdot \mathrm{m}^{3} \cdot \mathrm{~mol}$ frac
Compute the slope of the tie lie using $-\left[\frac{k_{x}^{\prime} a}{\left(1-x_{A L}\right)}\right] /\left[\frac{k_{y}^{\prime} a}{\left(1-y_{A G}\right)}\right]$
If required, assume $a=10 \mathrm{~m}^{2} / \mathrm{m}^{3}$, and answer the following by filling up the table

| $x_{i}$ | $y_{i}$ | $x^{*}$ | $y^{*}$ |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |
| $\left(1-x_{i}\right)_{M}$ | $\left(1-y_{i}\right)_{M}$ | $k_{x} a$ | $k_{y} a$ |  |
|  |  |  |  |  |
| $K_{x} a$ | Gas film resist. (\%) | Liquid film resist. (\%) | Molar flux |  |
|  |  |  |  |  |



## Final Exam Fall 2017-2018:

The solute $A$ is being absorbed from a gas mixture of $A$ and $B$ in a wetted-wall tower with the liquid flowing as a film downward along the wall. At a certain point in the tower the bulk gas concentration $y_{A G}=0.35 \mathrm{~mol}$ fraction and the bulk liquid concentration is $x_{A L}=0.20$. The tower is operating at 298 K and 1013 kPa and the equilibrium data given in the figure. The solute A diffuses through stagnant Bin the gas phase and then through a non-diffusing liquid.

Using correlations for dilute solutions in wetted-wall towers, the film mass-transfer coefficient for A in the gas phase is predicted as:

$$
\begin{aligned}
k_{y}^{\prime} a & =6.16 \times 10^{-2} \mathrm{~kg} \mathrm{~mol} / \mathrm{s} \cdot \mathrm{~m}^{3} \cdot \mathrm{~mol} \text { frac } \\
k_{x}^{\prime} a & =6.16 \times 10^{-2} \mathrm{~kg} \mathrm{~mol} / \mathrm{s} \cdot \mathrm{~m}^{3} \cdot \mathrm{~mol} \text { frac }
\end{aligned}
$$

Calculate the overall mass transfer coefficient $K_{y}^{\prime} a$ and the percent resistance in the gas and the liquid films and the flux $N_{A}$. If required, assume $a=10 \mathrm{~m}^{2} / \mathrm{m}^{3}$. Use the given figure showing the equilibrium line and make only one trial to obtain interface concentration assuming $\left(1-y_{A}\right)_{i M}=\left(1-x_{A}\right)_{i M}=1$.

