$\boldsymbol{Name}$ **:** $\boldsymbol{Student\ ID}$ **:**

$\underline{\boldsymbol{Question\ 1}}$

Aakash has a liability of 6000 due in four years. This liability will be met with payments of A in two years and B in six years. Aakash is employing a full immunization strategy using an annual effective interest rate of 5%.

Calculate.

$|A-B|$

- $P_{Assets} = P_{\text{Liability}}$

$$\frac{6000}{1.05^4} = P_{\text{Liability}}$$

$$\frac{6000}{1.05^4} = \frac{A}{1.05^2} + \frac{B}{1.05^6}$$

$$6000(1.05^2) = A(1.05^4) + B$$

$$B = 6000(1.05^2) - A(1.05^4) \qquad (1)$$

- $\bar{d}_{Assets} = \bar{d}_{\text{Liability}}$

$$4 = 2 \cdot \frac{\frac{A}{1.05^2}}{\frac{6000}{1.05^4}} + 6 \cdot \frac{\frac{B}{1.05^6}}{\frac{6000}{1.05^4}}$$

$$4\left[\frac{6000}{1.05^4}\right] = 2 \cdot \frac{A}{1.05^2} + 6 \cdot \frac{B}{1.05^6}$$

$$4[6000(1.05^2)] = 2 \cdot A(1.05^4) + 6 \cdot B$$

$$from\ \ (1)$$

$$4[6000(1.05^2)] = 2 \cdot A(1.05^4) + 6 \cdot 6000(1.05^2) - 6 \cdot A(1.05^4)$$

$$4 \cdot A(1.05^4) = 2 \cdot 6000(1.05^2)$$

$$A = \frac{2 \cdot 6000(1.05^2)}{4 \cdot (1.05^4)}$$

$$A = 2721.088435$$

$$Going\ Back\ to\ (1)$$

$$B = 6000(1.05^2) - A(1.05^4)$$

$$B = 6000(1.05^2) - (2721.088435)(1.05^4)$$

$$B = 3307.5$$

$$|A-B| = |2721.088435 - 3307.5| = 586.411565\ \blacksquare$$

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Question 2

Trevor has assets at time 2 of A and at time 9 of B. He has a liability of 95,000 at time 5. Trevor has achieved Redington immunization in his portfolio using an annual effective interest rate of 4%.
Calculate .

$\frac{A}{B}$

- $P_{Assets} = P_{\text{Liability}}$

$$\frac{95{,}000}{1.04^5} = P_{\text{Liability}}$$

$$\frac{95{,}000}{1.04^5} = \frac{A}{1.04^2} + \frac{B}{1.04^9}$$

$$95{,}000(1.04^4) = A(1.04^7) + B$$

$$B = 95{,}000(1.04^4) - A(1.04^7) \qquad (1)$$

- $\bar{d}_{Assets} = \bar{d}_{\text{Liability}}$

$$2 \cdot \frac{\frac{A}{1.04^2}}{\frac{95{,}000}{1.04^5}} + 9 \cdot \frac{\frac{B}{1.04^9}}{\frac{95{,}000}{1.04^5}} = 5$$

$$2 \cdot \frac{A}{1.04^2} + 9 \cdot \frac{B}{1.04^9} = 5 \cdot \frac{95{,}000}{1.04^5}$$

$$2 \cdot A(1.04^2) + 9 \cdot B = 5 \cdot 95{,}000(1.04^4)$$

$$from \quad (1)$$

$$2 \cdot A(1.04^2) + 9 \cdot 95{,}000(1.04^4) - 9 \cdot A(1.04^7) = 5 \cdot 95{,}000(1.04^4)$$

$$4 \cdot 95{,}000(1.04^4) = 7 \cdot A(1.04^7)$$

$$A = \frac{4 \cdot 95{,}000(1.04^4)}{7 \cdot (1.04^7)}$$

$$A = 48{,}259.80233$$

$$Going\ Back\ to \quad (1)$$

$$B = 95{,}000(1.04^4) - A(1.04^7)$$

$$B = 95{,}000(1.04^4) - (48{,}259.80233)(1.04^7)$$

$$B = 47{,}629.95565$$

$$\frac{A}{B} = 1.013223751\ \blacksquare$$

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Question 3

A company must pay liabilities of 4000 and 6000 at the end of years one and two, respectively. The only investments available to the company are one-year zero-coupon bonds with an annual effective yield of 8% and two-year zero-coupon bonds with an annual effective yield of 11%.
Determine how much the company must invest today to exactly match its liabilities.

$c_1 = 4{,}000$

$c_2 = 6{,}000$

$$\frac{4{,}000}{1.08} + \frac{6{,}000}{1.11^2} = 8573.438303$$

Question 4

Porter makes three-year loans that include inflation protection. The annual interest rate compounded continuously that must be paid is 3.2% plus the rate of inflation.
The U.S. government borrows 100,000 for three years from Porter. The actual annual inflation rate during the first year was 2.4% compounded continuously. The actual annual inflation rates for the second and third years respectively was 2.8% and 4.2% compounded continuously.
The U.S. government is considered a risk free borrower, which means there is no chance of default.
Calculate the amount that the U.S. government will owe Porter at the end of three years.

The first year, the government pays $0.032 + 0.024 = 0.056$ *compounded continuously.*

In the next two years the rates will be $0.032 + 0.028 = 0.060$ *and* $0.032 + 0.042 = 0.074$ *respectively.*

The amount owed after three years is $100{,}000e^{0.056}e^{0.060}e^{0.074} = 120{,}925$