**Chapter 3: Collective risk model.**

We consider a sequence of insurance policies with claims and define the total claim , where is the random number of claims in a given period of time. We suppose that the ’s are independent and identically distributed (iid) and independent with . The aim of chapter 3 is to compute or approximate the distribution of the portfolio value . is called in this case a compound random variable.

1. Compound distribution.

Let be a compound random variable with claim and claim number . Then we have the following:

For a real function we get: .

Since the ’s and are independent, then

.

We conclude in particular that

* ,
* ,
* .

**Example 1.1**. Suppose and . Compute the mean, the variance, the mgf and the cdf of S.

**Example 2.1**. Suppose and . Compute the mean, the variance, the mgf and the cdf of S.

We recall that and with and .

1. Convolution formula.

Let be a compound random variable with claim and claim number . Then the cdf of S is given by

,

And

.

Since

.

**Note:**, , for and for , for .

**Example 2.1**. Suppose and with respective probabilities . Compute the mean, the mgf and the cdf of S.

**Example 2.2**. Suppose and for . Compute the mean, the mgf and the cdf of S.

1. Compound Poisson distribution.

* is called a compound Poisson random variable when the claim number has a Poisson distribution with parameter . In this case we get

1. ,
2. ,
4. .

* Let be independent compound Poisson random variables with respective parameters and respective claim distributions ( is the cdf of the claims associated to ). Then is a compound Poisson random variable with parameter and claim distribution

.

* Special case: Suppose real numbers and independent Poisson random variables with respective parameters . Then is a compound Poisson random variable with parameter and claim mass function for .

**Example 3.1**. Suppose two independent compound Poisson random variables and with and . Compute the second mean of .

* Sparse vector algorithm: Assume that is compound Poisson distributed with parameter and with discrete claim distribution for . Then where are independent Poisson random variables with respective parameters .

**Example 3.2**. Suppose and . Compute the mass function of S at .

1. Panjer recursion.

Consider a compound distribution with integer-valued non-negative claims with mass function and claim number distribution satisfying (#)

for and for some real numbers *a* and *b*.

Then the mass function of the total claim is given by:

and

for

**Example 4.1**. Compute the Panjer recursive formula for the total claim S mass function in the following cases: , and .

**Example 4.2**. Compute the Panjer recursive formula for the mass function for the total claim S when and with respective probabilities and .

1. Application to stop-loss premium.

For an integer-valued random variable , we can write the stop-loss premium in an integer retention as follows:

.

**Example 5.1**. Consider with . Compute .

1. Approximations for Compound Poisson distributions.

Central limit Theorem: For a compound Poisson random variable with parameter , mean and finite variance , we get for large values of that

.

**Example 6.1**. Assume that is compound Poisson distributed with parameter and uniform(0,1) distributed claims. Approximate with the CLT approximation, the translated gamma approximation and the NP approximation.