***Additional Topics***

***Additional Mathematics Topics in General insurance and Life Insurance***

1. ***Risk Measurement***

At the beginning I would like to ask a question, may be this question in your mind, *How can you measure the risk?* *The answer of this question* based on, How to manage the risk which may occur in future? Estimates of future typically are based primarily on historical and/or theoretical data. These data are used to develop probabilities of future occurrence of each event. Representations of all possible outcomes (probable risks) along with their associated probabilities are called ***probabilities distributions*** ***of risks***.

Probabilities distributions of risks are those associated with frequency and severity of losses, ***frequency*** is a measure of how often accidents occur (i.e. the number of collisions of cars that occur in a factory during a specified period). And,***Severity*** is a measure of the amount of damage of cars caused by each accident.

In order to measure the risk, we have to measure ***expectations*** of the probable risks (i.e. average value) and ***variability*** of those expectations (variance). Table (1.1) shows the loss value for 10 cars.

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Cars (observations)** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** | **Total** |
| **Loss value (L.E)** | **200** | **300** | **200** | **200** | **300** | **500** | **1000** | **500** | **1000** | **1000** | **5200** |

**Table (1.1) Loss value of ten cars**

From table (1.1), we can determine loss probabilities as indicated in table (1.2)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Value loss category** | **200** | **300** | **500** | **1000** | **Total** |
| **Number of cars (Observations)** | **3** | **2** | **2** | **3** | **10** |
| **Probabilities** | **3/10 = 0.30** | **2/10 = 0.20** | **2/10 = 0.20** | **3/10 = 0.30** | **1** |

From tables (1.1) and (1.2) both expectations of risk (average value or mean) and the variability of risk (variance) can be calculated as follows:

1. ***The average value (mean):***

Average value or mean = 

Where:

xi  = Value of observation i

n = number of observations

*That is*, the mean = 5200 / 10 = 520

Or 

Where

xj  = value of category j (possible events)

m = number of categories

p(xj) = probability (relative frequency) of category j

*That is*, the mean = (200 x 0.30) + (3000 x 0.20) + (500 x 0.20) + (10000 x 0.30) = 520 L.E

1. ***Variance ()***

The variance of a probability distribution of risk can be measured by the following equations:

Variance = 

Where xi = value of observation i

 =  = mean of the probability distribution

n = number of observations

Or

Variance = 

Where xj = value of category j of possible events.

P (xj) = probability of category j

So, the variance ***()*** = [(200 – 520)2 + (200 – 520)2 + (200 – 520)2 + (300 – 520)2 + (300 – 520)2 + (500 – 520)2 + (500 – 520)2 + (1000 – 520)2 + (1000 – 520)2 + (1000 – 520)2 = 109600

And *standard* *deviation* (σ) = = 331.058

By looking at the value of variance we find it 109600. This value is *a large value*. So statisticians often use the square root of variance (that is, σ standard deviation), to provide some meaningful measure of risk. The standard deviation, in our example equals 331.058 L. E. . On average, each observation is approximately 331 L. E away from the mean of 520 L. E.

If we are comparing two distributions with different means and standard deviations in order to know which distribution is riskier than others, it is better to consider *the coefficient of variation*, which equals the standard deviation of a distribution divided by its mean, that is:

*Coefficient of variation (****CV****)* = 

*The coefficient of variation(****CV****)* give us ***a relative value of risk*** in our example, the coefficient of variation equals 63.6.

The smaller the value, the lower the relative riskiness of distribution.

***A notice****:* A risk manager uses the law of large numbers to estimate future outcomes for planning purposes. The law of large numbers holds that" as a sample of observations is increased in size, the relative variation about the mean declines" *In other words*, "The larger the sample size, the lower the relative risk". So, it is better for risk manager to gather many observations of the risk to be able to make his decision about it.

1. ***Net or Pure Premium***

If the loss distribution follows *the normal distribution*, the pure premium P will be calculated as follow:

P =  + **z** 99.9%

If the number of insured = 1 then:

P1 = x1 + **z** 99.9% where **z%** 99.9%is the standard value for the confidence degree 99.9% & z = 3.09 (we will approximate it to 3 to facilitate the calculation).

P2 = 2 + **z** 99.9%

P3 = 3 + **z** 99.9%

P100 = 100 + **z** 99.9%

.

.

P1000 = 1000 + **z** %9.99

P10000 = 10000 + **z** %9.99

P1000000 = 1000000 + **z** %9.99

Based on the previous value of  =1000 & =1500 then

If n = 1 (i.e. if there is only one insured), then the premium for one insured would be:

P1 = 1 + **z** %9.99

P1 = 1000 + **3** = 1000 + 4500 =5500

If n = 2 (i.e. if there is only two insureds), then the premium for two insureds would be:

P2 = 2 + **z** %9.99

P2 = 1000×2 + × ) × 3)

= 2000 + ((1500× 1.4142) ×3)

=2000 + 6363.96 =8363.96

Then the premium for each insured = = = 4141.98

If n = 3 (i.e. if there is only three insureds) then the premium for three insureds would be:

P3 = 3 + **z** %9.99

P3 = 1000× 3 + × ) × 3)

=3000 + ((1500× 1.732051) ×3)

= 3000 + 7794.23 = 10794.23

Then the premium for each insured = = = = 3598.08

If n = 100 (i.e. if there is 100 insureds) then the premium for 100 insureds would be:

P100 = 100 + **z** %9.99

P100 = 1000× 100 + × ) × 3)

= 100000 + ((1500× 10) ×3)

= 1000000 +45000 = 145000

Then the premium for each insured = = = 1450

If n = 10000 (i.e. if there is 10000 insureds), then the premium for 10000 insureds would be:

P10000 = 10000 + **z** %9.99

P10000 = 1000× 10000+ × ) × 3)

= 10000000+ ((1500× 100) ×3)

= 10000000 +450000 = 145000 = 10450000

Then the premium for each insured = = = 1045

If n = 1000000 (i.e. if there is 1000000 insureds), then the premium for 1000000 insureds would be:

P1000000 = 1000000 + **z** %9.99

P1000000  = (1000× 1000000 ) + × ) × 3 )

= 1000000000+ ((1500× 1000) ×3)

= 1000000000 +4500000 = 145000 = 1004500000

Then the premium for each insured = = = 1004.5

And,

The gross premium =

Given = 30 %

So, the gross premium = =1435 (1004.2 pure premium and 430.5 loadings)