

Time: 45 min

Quiz 2 - Math 218 - Semester I - 1444 H

Marks: 10

Question	1	2	3	4	5	6	7	8	9	10
Answer	C	A	B	D	C	D	B	B	B	D

I) Choose the correct answer (write it in the table above):

$$1) \lim_{x \rightarrow 1} \frac{x^2 - 1}{x - 1} = \lim_{x \rightarrow 1} \frac{(x-1)(x+1)}{(x-1)} = 2$$

(a) 0	(b) 1	(c) 2	(d) 3
-------	-------	-------	-------

$$2) \lim_{x \rightarrow 0} x^2 \sin\left(\frac{1}{x}\right) = 0 \text{ because } -1 \leq \sin\left(\frac{1}{x}\right) \leq 1$$

$$-x^2 \leq x^2 \sin\left(\frac{1}{x}\right) \leq x^2$$

$$\lim_{x \rightarrow 0} x^2 = 0$$

(a) 0	(b) 1	(c) 2	(d) 3
-------	-------	-------	-------

$$3) \lim_{x \rightarrow \infty} \frac{x + 4x^2}{1 - 2x^2} = \lim_{x \rightarrow \infty} \frac{x^2\left(\frac{1}{x} + 4\right)}{x^2(1 - 2x^2)} = \frac{4}{-2} = -2$$

(a) -4	(b) -2	(c) 2	(d) 4
--------	--------	-------	-------

$$4) \lim_{x \rightarrow 4} \frac{x^2 - 2x - 8}{x - 4} = \frac{0}{0} = \lim_{x \rightarrow 4} \frac{(x-4)(x+2)}{(x-4)} = 6$$

(a) $\infty$	(b) 2	(c) 4	(d) 6
--------------	-------	-------	-------

$$5) \lim_{x \rightarrow 0} \frac{(3+x)^2 - 9}{x} = \frac{0}{0} = \lim_{x \rightarrow 0} \frac{9 + x^2 + 6x - 9}{x} = \lim_{x \rightarrow 0} (x+6) = 6$$

(a) 0	(b) 3	(c) 6	(d) 9
-------	-------	-------	-------

6)  $\lim_{x \rightarrow 0} \frac{|x|}{x} =$

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

$$\frac{|x|}{x} = \begin{cases} 1 & \text{if } x > 0 \\ -1 & \text{if } x < 0 \end{cases}$$

(a) -1

(b) 0

(c) 1

(d) doesn't exist

7) If  $f(x) = \frac{1}{\sqrt{1-x^2}}$  then  $f$  is continuous on the interval  $D_f = \{x \in \mathbb{R} / 1-x^2 > 0\} = (-1, 1)$ (a)  $(-\infty, -1)$ (b)  $(-1, 1)$ (c)  $[-1, 1]$ (d)  $(1, +\infty)$ 8) If  $f(x) = \sqrt{2+x^2}$  then  $f'(\sqrt{2}) = \frac{\sqrt{2}}{\sqrt{2+2}} = \frac{\sqrt{2}}{2}$ 

$$f'(x) = \frac{\frac{2x}{2}}{2\sqrt{2+x^2}}$$

(a)  $\frac{1}{2}$ (b)  $\frac{\sqrt{2}}{2}$ (c)  $\frac{\sqrt{2}}{4}$ 

(d) 2

9) If  $f$  is a differentiable function and  $f(1) = 1$ ,  $f'(1) = 1$ , and  $g(x) = xf(x)$  then  $g'(1) =$ 

$$g'(x) = f(x) + x f'(x) \text{ so } g'(1) = f(1) + f'(1)$$

(a) 1

(b) 2

(c) 6

(d) 7

10) The slope of the tangent line to the graph  $f(x) = 3x^2 + 2$  at the point  $P(1, 5)$  is

(a) 1

(b) 3

(c) 5

(d) 6

$$\text{Eq of tangent } T_P : y - 5 = \underbrace{f'(1)}_{\text{slope}} (x - 1)$$

$$f'(x) = 6x$$

$$f'(1) = 6$$

II) A) Differentiate the following functions:

i)  $f(x) = 2x^5 - 5x^2 + \sqrt{x}$ .

①  $f'(x) = 10x^4 - 10x + \frac{1}{2\sqrt{x}}$

ii)  $g(x) = (x + x^2)e^{3x}$ . (product of 2 functions)

①  $g'(x) = (1+2x)e^{3x} + (x+x^2)(3)e^{3x}$

$g'(x) = [3x^2 + 3x + 1]e^{3x}$

iii)  $h(x) = \frac{\sin x}{2 + \cos x}$ . (quotient of 2 functions)

①  $h'(x) = \frac{(\cos x)(2 + \cos x) - (\sin x)(-\sin x)}{(2 + \cos x)^2} = \frac{2\cos x + \cos^2 x + \sin^2 x}{(2 + \cos x)^2}$

$h'(x) = \frac{2\cos x + 1}{(2 + \cos x)^2}$

iv)  $k(x) = [\ln(1 + x^3)]^4$ .

(Chain Rule)

①  $k'(x) = 4(\ln(1 + x^3))^3 \frac{d}{dx} \ln(1 + x^3)$

$k'(x) = 4(\ln(1 + x^3))^3 \left( \frac{3x^2}{1 + x^3} \right)$

B) Find an equation of tangent line for the curve  $y = 3^x$  at the point  $P(0, 1)$ .

The equation of tangent line is

$T_p: y - 1 = \frac{d}{dx}(3^x) \Big|_{x=0} (x - 0)$

①

$T_p: y = (\ln 3)x + 1$