جامعة الملك سعود
King Saud University

Name: Answer Sheet
ID:............................
Num of attendance:.....

Time: 35 min **Quiz 1 - Math 205 - Trimester 3 - 1444 H** Marks: 10

Q1) Determine whether the sequence $\left\{\dfrac{\cos n}{n^2+2n}\right\}$ converges or diverges. [3 marks]

*Answer:*

$-1 \le \cos n \le 1$ for every $n$

1,5 $$-\frac{1}{n^2+2n} \le \frac{\cos n}{n^2+2n} \le \frac{1}{n^2+2n}$$

As $\displaystyle\lim_{n\to+\infty} \frac{1}{n^2+2n} = 0$

1,5 We deduce that $\displaystyle\lim_{n\to+\infty} \frac{\cos n}{n^2+2n} = 0$

Q2) Determine whether the infinite series $\displaystyle\sum_{n=1}^{\infty} \frac{3}{n(n+3)}$ converges or diverges. If it converges find the sum $S$. [3 marks]

*Answer:*

let $N \ge 1$, $\displaystyle S_N = \sum_{n=1}^{N} \frac{3}{n(n+3)}$

$$f(x) = \frac{3}{x(x+3)} = \frac{A}{x} + \frac{B}{x+3} = \frac{1}{x} - \frac{1}{x+3}$$

1
- $\displaystyle A = \lim_{x\to 0} x f(x) = \lim_{x\to 0} \frac{3}{x+3} = 1$
- $\displaystyle B = \lim_{x\to -3} (x+3) f(x) = \lim_{x\to -3} 3/x = -1$

So, $$S_N = \sum_{n=1}^{N} \left[\frac{1}{n} - \frac{1}{n+3}\right] = \left(\sum_{n=1}^{N} \frac{1}{n}\right) - \left(\sum_{n=1}^{N} \frac{1}{n+3}\right)$$

2 $$S_N = \left(\sum_{n=1}^{N} \frac{1}{n}\right) - \left(\sum_{n=4}^{N+3} \frac{1}{n}\right) = \left(1 + \frac{1}{2} + \frac{1}{3}\right) - \left(\frac{1}{N+1} + \frac{1}{N+2} + \frac{1}{N+3}\right)$$

As $\displaystyle\lim_{N\to+\infty} S_N = 1 + \frac{1}{2} + \frac{1}{3} = \frac{11}{6}$, then $\left(\displaystyle\sum_{n=1}^{\infty} \frac{3}{n(n+3)}\right)$ is convergent to the sum $S = 11/6$.

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Q3) Determine whether the infinite series $\sum_{n=1}^{\infty} n^3 e^{-n^4}$ converges or diverges. [4 Marks]

*Answer:* $\sum_{n=1}^{\infty} n^3 e^{-n^4}$ is a positive Serie

Using ~~In~~ tegral test:

2

- $f(x) = x^3 e^{-x^4}$ is continuous on $[1, +\infty)$

$$f'(x) = 3x^2 e^{-x^4} + x^3(-4x^3) e^{-x^4}$$

$$= x^2 e^{-x^4}[3 - 4x^4] < 0$$

so $f$ is a decreasing function on $[1, +\infty)$

- $$\int_1^{+\infty} x^3 e^{-x^4} dx = \lim_{t \to +\infty} \left( \int_1^t x^3 e^{-x^4} dx \right)$$

$u = -x^4$

$du = -4x^3 dx$

so $x^3 dx = -\frac{1}{4} du$

2

$$= \lim_{t \to +\infty} \left( \int_{-1}^{-t^4} -\frac{1}{4} e^u du \right)$$

$$= -\frac{1}{4} \lim_{t \to +\infty} \left[ e^u \right]_{-1}^{-t^4} = -\frac{1}{4} \lim_{t \to +\infty} \left[ e^{-t^4} - e^{-1} \right]$$

$$= \frac{1}{4e} \quad \text{because} \quad \lim_{t \to +\infty} e^{-t^4} = e^{-\infty} = 0$$

We deduce that $\left( \sum_{n=1}^{\infty} n^3 e^{-n^4} \right)$ is Convergent.



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