Answer sheet Midterm Exam Math 218 (Dr Wo...)

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Name:..................................
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Num of attendance:.....

Time: 2 hours Midterm 1 - Math 218 - Semester I - 1444 H Marks: 30

Question	1	2	3	4	5	6	7	8	9	10
Answer	D	D	C	A	C	D	D	C	D	D

I) Choose the correct answer (write it in the table above): [1 × 10 Marks]

1) The distance between the points $A(-2,0)$ and $B(2,3)$ is equal to

(a) $\sqrt{13}$	(b) $\sqrt{8}$	(c) 3	(d) 5

$d(A,B) = AB = \sqrt{(x_B-x_A)^2+(y_B-y_A)^2} = \sqrt{4^2+3^2} = 5$

2) The remainder when $P(x) = 8x^4 + 6x^2 - 3x + 1$ is divided by $2x^2 - x + 2$ is

(a) $x-5$	(b) $7x-1$	(c) $-x+5$	(d) $-7x+1$

$8x^4+6x^2-3x+1 \quad | \; 2x^2-x+2$

$-8x^4+4x^3-8x^2 \quad | \; 4x^2+2x$

$4x^3-2x^2-3x+1$

$-4x^3+2x^2-4x$

$-7x+1$

3) $\ln(27)$ is equal to

(a) $9\ln 3$	(b) $27\ln 1$	(c) $\ln 9 + \ln 3$	(d) $4\ln 3$

$\ln(27) = \ln(9\times 3) = \ln 9 + \ln 3$

4) The domain of the function $y = \ln(4-2x)$ is

(a) $(-\infty, 2)$	(b) $(-\infty, 2]$	(c) $(2, \infty)$	(d) $[2, \infty)$

$f(x) = \ln(4-2x)$

$D_f = \{x \in \mathbb{R} \mid 4-2x>0\} = \{x\in\mathbb{R} \mid 2>x\}$

5) The solution of the equation $\ln(\ln x) = 0$ is

(a) 0	(b) 1	(c) e	(d) e^2

$D_E = (1, \infty)$

$\ln(\ln x) = 0$

$e^{\ln(\ln x)} = e^0 = 1$

$\ln x = 1 \iff e^{\ln x} = e^1 \iff x = e$

6) The solution of equation $e^{2x} - e^{x} - 6 = 0$ is

(a) 2	(b) $\ln 2$	(c) 3	(d) $\ln 3$

$e^{2x} - e^{x} - 6 = 0 \iff (e^{x})^2 - (e^{x}) - 6 = 0$

put $u = e^{x} > 0$ $\quad u^2 - u - 6 = (u+2)(u-3) = 0$

$u = -2 = e^{x}$ ~~y~~ or $u = 3$

$\iff e^{x} = 3$

$x = \ln 3$

7) $\cos(\frac{7\pi}{6}) =$

(a) $\frac{1}{2}$	(b) $-\frac{1}{2}$	(c) $\frac{\sqrt{3}}{2}$	(d) $-\frac{\sqrt{3}}{2}$

$\cos\left(\frac{7\pi}{6}\right) = -\cos(\pi/6) = -\frac{\sqrt{3}}{2}$

8) If $\cos t = -\frac{3}{5}$ and t in the third quadrant then $\sin t$ is equal to

(a) $-\frac{3}{5}$	(b) $\frac{3}{5}$	(c) $-\frac{4}{5}$	(d) $\frac{4}{5}$

II | I
III | IV

As $\cos^2 t + \sin^2 t = 1$ then $\sin t = \pm\sqrt{1-\cos^2 t}$

As $t \in$ III then $\sin t = -\sqrt{1-(-\frac{3}{5})^2} = -4/5$

9) $\sin^{-1}(\frac{1}{2})$ is equal to

(a) $-\frac{\pi}{3}$	(b) $-\frac{\pi}{6}$	(c) $\frac{\pi}{3}$	(d) $\frac{\pi}{6}$

$\sin^{-1}: [-1,1] \to [-\frac{\pi}{2}, \frac{\pi}{2}]$

$\sin^{-1}(\frac{1}{2}) = \sin^{-1}(\sin\frac{\pi}{6}) = \frac{\pi}{6}$

10) $\sin^{-1}(\sin(\frac{13\pi}{6}))$ equals

(a) $-\frac{\pi}{3}$	(b) $-\frac{\pi}{6}$	(c) $\frac{\pi}{3}$	(d) $\frac{\pi}{6}$

$\sin^{-1}\left(\sin\left(\frac{13\pi}{6}\right)\right) = \sin^{-1}\left(\sin\left(\frac{\pi}{6} + 2\pi\right)\right)$

$= \sin^{-1}\left(\sin\frac{\pi}{6}\right) = \frac{\pi}{6}$

II) A) i) Determine whether $f(x) = 2x^2 - 12x + 3$ has a maximum or a minimum and find this value.

$f(x) = 2(x-3)^2 - 15$, f has a minimum

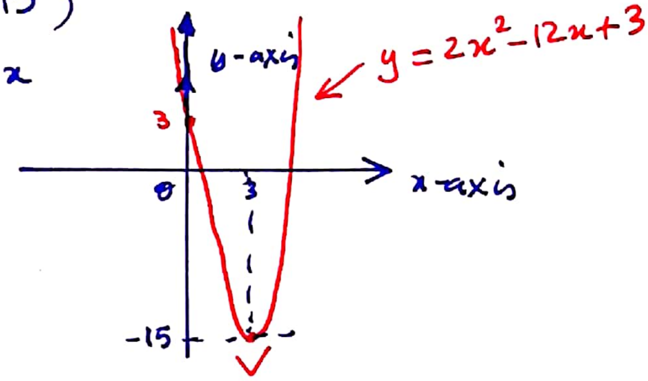
2 at $x = 3$

The vertex is: $V(3, -15)$

$f(x) \geq -15$ for every x

ii) Sketch the graph of f.

1



B) i) Use long division to find the quotient and the remainder when $P(x) = 6x^2 - 26x + 12$ is divided by $(x - 4)$.

3

$$\begin{array}{r|l} 6x^2 - 26x + 12 & x-4 \\ \cline{2-2} -6x^2 + 24x & 6x-2 \\ \hline -2x + 12 & \\ 2x - 8 & \\ \hline 4 & \end{array}$$

So $6x^2 - 26x + 12 = (6x-2)(x-4) + \boxed{4}$

- The quotient is $q(x) = 6x - 2$.
- The remainder is $r(x) = 4 = P(4)$

ii) Find all zeros of $P(x) = x^3 - 3x + 2$. As the coefficients are integers then $x = p/q$ with $p|2$ & $q|1$ So $x = \{\pm 1 ; \pm 2\}$

2

$P(1) = 0$; $P(-1) = 4$; $P(-2) = 0$; $P(2) = 4$

$$\begin{array}{r|l} x^3 - 3x + 2 & x-1 \\ \cline{2-2} -x^3 + x^2 & x^2+x-2 \\ \hline x^2 - 3x + 2 & \\ -x^2 + x & \\ \hline -2x + 2 & \\ 2x - 2 & \\ \hline 0 & \end{array}$$

$P(x) = x^3 - 3x + 2 = (x-1)(x^2 + x - 2)$

The zeros of P are 1 & (-2) $P(x) = (x-1)(x+2)(x-1)$

iii) Find a polynomial of degree 4 that has zeros (-2), 0, 1 and 2 and the coefficient of x^3 is (-8).

$P(x) = a(x+2)x(x-1)(x-2)$.

2

$P(x) = a(x^2 - x)(x^2 - 4)$.

$P(x) = a(x^4 - 4x^2 - x^3 + 4x)$.

$P(x) = ax^4 - ax^3 - 4ax^2 + 4ax$.

As the coefficient of x^3 is (-8) then $\boxed{a = 8}$

So $P(x) = 8x(x-1)(x+2)(x-2)$.

C) Solve the following equations:

i) $\log(x+2)+\log(x-1)=1$. The domain of equation is $(1,\infty)$.

for $x>1$, $\log(x+2)+\log(x-1)=\log((x+2)(x-1))=1$

1,5

So $(x+2)(x-1)=10$

$x^2+x-12=0$; $a=1$; $b=1$; $c=-12$

$\Delta=b^2-4ac$
$=1+48=(7)^2$

So The solution is $\{3\}$ $x_1=\frac{-1-7}{2}=-4$ ↯ $x_2=\frac{-1+7}{2}=3$

ii) $5^{3-2x}=4$.

- Domain of the equation is $\mathbb{R}$

1,5

$5^{3-2x}=4$

$\ln(5^{3-2x})=\ln 4 \Leftrightarrow 3-2x=\ln 4 \Leftrightarrow 3-\ln 4=2x$

So $x=\frac{1}{2}(3-\ln 4)=\frac{3}{2}-\ln 2$.

iii) $2\cos^2\theta-7\cos\theta+3=0$. Put $u=\cos\theta$. Then $2u^2-7u+3=0$

$a=2$; $b=-7$; $c=3$

Discriminant: $D=49-24$
$=25=5^2$

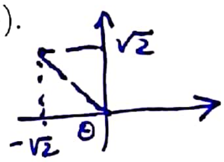
2

$u_1=\frac{7-5}{4}=\frac{1}{2}=\cos\theta$

$u_2=\frac{7+5}{4}=3=\cos\theta$ ↯

$\cos\theta=\frac{1}{2}$ then $\theta=\frac{\pi}{3}+2n\pi$ or $\theta=-\frac{\pi}{3}+2n\pi$ with $n\in\mathbb{Z}$

D) i) Find the rectangular coordinates for the point has polar coordinates $(2,\frac{3\pi}{4})$.

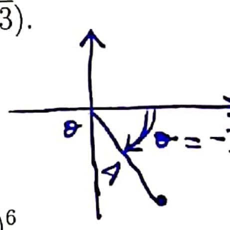


$r=2$; $\theta=\frac{3\pi}{4}$

1,5

$$\begin{cases} x=r\cos\theta = 2(-\sqrt{2}/2) = -\sqrt{2} \\ y=r\sin\theta = 2(\sqrt{2}/2) = \sqrt{2} \end{cases}$$

ii) Find the polar coordinates for the point has rectangular coordinates $(2,-2\sqrt{3})$.



$x=2$; $y=-2\sqrt{3}$

1,5

then $\cdot\ r=\sqrt{x^2+y^2}=\sqrt{4+12}=4$

$$\begin{cases} \cos\theta = x/r = 1/2 \\ \sin\theta = y/r = -\sqrt{3}/2 \end{cases}$$ So $\theta=-\pi/3$

iii) Write the complex number $z=-1+i\sqrt{3}$ in polar form and find $(-1+i\sqrt{3})^6$.

$z=-1+i\sqrt{3}$: $|z|=\sqrt{1^2+(\sqrt{3})^2}=2$

$\theta=Arg(z)$; $\cos\theta=-1/2$ $\sin\theta=\sqrt{3}/2$ $\boxed{\theta=\frac{2\pi}{3}}$

1

$z=2e^{i\frac{2\pi}{3}}$

1

So $z^6=\left(2e^{i\frac{2\pi}{3}}\right)^6=2^6(e^{i4\pi})=2^6$.

End of exam