

Q1) Find the distance between two planes

$$2x + 6y - 8z + 4 = 0 \text{ and } 3x + 9y - 12z - 4 = 0. \quad [2 \text{ marks}]$$

Answer:  $P_1: 2x + 6y - 8z + 4 = 0$

The normal vector for  $P_1$  is  $\vec{n}_1 = \langle 2, 6, -8 \rangle$

$P_2: 3x + 9y - 12z - 4 = 0$ . The normal vector for  $P_2$  is  $\vec{n}_2 = \langle 3, 9, -12 \rangle$

As  $\frac{2}{3} = \frac{6}{9} = -\frac{8}{-12}$  then  $P_1 \parallel P_2$ .

Let  $A = (-2, 0, 0) \in P_1$

Then  $d = \frac{|3 \times (-2) + 9 \times 0 - 12 \times 0 - 4|}{\sqrt{3^2 + 9^2 + (-12)^2}} = \frac{10}{\sqrt{324}}$

Q2) Sketch the graph of  $y = 6x^2 + z^2$ , and identify the surface. [3 marks]

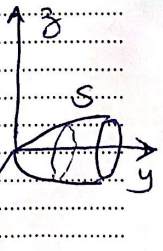
Answer: .....

The equation of the surface  $y = 6x^2 + z^2$  is a paraboloid

When  $y = 0$  the origin  $O(0, 0, 0)$  in the  $xz$ -plane

When  $x = 0$  :  $y = z^2$  Parabola in the  $yz$ -plane

When  $z = 0$  :  $y = 6x^2$  equation of Parabola in the  $xy$ -plane



Q3) Find parametric equations for the tangent line to  $C$  given by

$$\mathbf{r}(t) = (t^2 - t)\mathbf{i} + (2t^2 + 1)\mathbf{j} + (t^3 - 1)\mathbf{k} \text{ at } t = 2. \quad [3 \text{ Marks}]$$

Answer:

$$\mathbf{r}(t) = \langle t^2 - t, 2t^2 + 1, t^3 - 1 \rangle$$

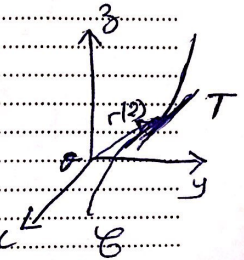
$$\mathbf{r}(2) = \langle 2, 9, 7 \rangle$$

$$\mathbf{v}(t) = \frac{d}{dt} \mathbf{r}(t) = \langle 2t - 1, 4t, 3t^2 \rangle$$

$$\mathbf{v}(2) = \langle 3, 8, 12 \rangle$$

The parametric equations for the tangent line to  $C$  at  $t = 2$

$$\begin{cases} x(t) = 2 + 3t \\ y(t) = 9 + 8t \\ z(t) = 7 + 12t \end{cases}, t \in \mathbb{R}$$



Q4) Show that the  $\lim_{(x,y) \rightarrow (0,0)} \frac{5x^3y^2}{2x^5 + 4y^5}$  does not exist. [2 Marks]

Answer:

Let  $l_m: y = mx$  lines passes through  $(0,0)$

$$\lim_{\substack{(x,y) \in l_m \\ (x,y) \rightarrow (0,0)}} \frac{5x^3y^2}{2x^5 + 4y^5} = \lim_{x \rightarrow 0} \frac{5m^2x^5}{2x^5 + 4m^5x^5} = \frac{5m^2}{2 + 4m^5}$$

It depends of  $m$

So  $\lim_{(x,y) \rightarrow (0,0)} \frac{5x^3y^2}{2x^5 + 4y^5}$  does not exist.