جامعة الملك سعود King Saud University	Second quiz Math218 (Differential Calculus_ - First Semester 2021-2022- Duration 45 min Name:.........Answer......... ID:........................

Q1: a) Find rectangular coordinates for the point has polar coordinates $(4, \frac{2\pi}{3})$. (2 marks)

$r = 4 \ , \ \theta = \frac{2\pi}{3}$

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- $x = r\cos\theta = 4\cos(\frac{2\pi}{3}) = 4\times(-1/2) = -2$
- $y = r\sin\theta = 4\sin(2\pi/3) = 4\times(\sqrt{3}/2) = 2\sqrt{3}$

b) Find polar coordinates for the point that has rectangular coordinates (2,-2).

$x = 2 \ ; \ y = -2$

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- $r = \sqrt{x^2+y^2} = \sqrt{4+4} = 2\sqrt{2}$
- $\theta = \tan^{-1}(y/x) = \tan^{-1}(-2/2) = \tan^{-1}(-1) = -\frac{\pi}{4}$

Q2: Write the complex number $z = 1+i$ in polar form and find $(1+i)^{10}$. (2 marks)

$x = 1 \ ; \ y = 1$

1 $\quad z = 1+i = \sqrt{2}\, e^{i\pi/4} = \sqrt{2}\left[\cos(\frac{\pi}{4}) + i\sin(\frac{\pi}{4})\right]$

$z^{10} = (1+i)^{10} = (\sqrt{2}\, e^{i\pi/4})^{10} = 2^5\, e^{i(\frac{10\pi}{4})}$

1 $\quad z^{10} = 2^5\, e^{i(2\pi + \pi/2)} = 2^5\, e^{i\pi/2} = 2^5 i = 32i$

Q3: Find the limit, if there exists: (3 marks)

1 $\quad$ (a) $\lim_{x\to 1} \frac{x-1}{x^2-1} = \lim_{x\to 1} \frac{(x-1)}{(x-1)(x+1)} = \lim_{x\to 1} \frac{1}{x+1} = \frac{1}{2}$

(b) $\lim\limits_{x\to 0}\dfrac{\sqrt{x^2+9}-3}{x^2} = \frac{0}{0}$ I F

$= \lim\limits_{x\to 0} \dfrac{(\sqrt{x^2+9}-3)(\sqrt{x^2+9}+3)}{x^2(\sqrt{x^2+9}+3)}$

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$= \lim\limits_{x\to 0} \dfrac{x^2+\cancel{9}-\cancel{9}}{x^2(\sqrt{x^2+9}+3)} = \lim\limits_{x\to 0}\dfrac{1}{\sqrt{x^2+9}+3} = \dfrac{1}{6}$

(c) $\lim\limits_{x\to 0^-} e^{1/x} = e^{\lim\limits_{x\to 0^-} 1/x} = e^{-\infty} = 0$

because the exponential function is continuous on $\mathbb{R}$.

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Q4: Show that $\lim\limits_{x\to +\infty}\dfrac{\sin x}{x} = 0$. (1 mark)

By Squeeze thm: for $x \in \mathbb{R}$ $\quad -1 \le \sin x \le 1$

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for $x>0$ $\quad ; -1/x \le \dfrac{\sin x}{x} \le \dfrac{1}{x}$

As $\lim\limits_{x\to +\infty}\dfrac{1}{x} = 0$ then $\lim\limits_{x\to +\infty}\dfrac{\sin x}{x} = 0$

Q5: Find an equation of tangent line for the curve $y=\ln x$ at the point P$(e,1)$. (2 marks)

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$T_P : \quad y - y_P = f'(x_P)(x - x_P)$

$x_P = e \quad ; \quad y_P = 1 \quad ; \quad f(x) = \ln x$

$f'(x) = 1/x \quad$ so $\quad f'(e) = \dfrac{1}{e}$

The equation of tangent line to the curve $y = \ln x$ at P

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is $\quad y - 1 = \dfrac{1}{e}(x - e)$

$y - 1 = \dfrac{1}{e}x - 1$

so $\quad y = 1/e\, x$.