جامعة الملك سعود
King Saud University

Name: Answer (Dr Borhen)
ID: ....
Num of attendance: .....

Time: 1h 30 min **Midterm 2 - Math 218- Semester II- 1443 H** Marks: 25

Question	1	2	3	4	5	6	7	8	9	10
Answer	D	C	D	D	C	D	C	B	D	C

I) Choose the correct answer (write it in the table above):

1) $\cos(\frac{7\pi}{6}) = \cos\left(\pi + \frac{\pi}{6}\right) = -\cos\left(\frac{\pi}{6}\right)$

(a) $\frac{1}{2}$	(b) $-\frac{1}{2}$	(c) $\frac{\sqrt{3}}{2}$	(d) $-\frac{\sqrt{3}}{2}$

2) If $\cos t = -\frac{3}{5}$ and t in the third quadrant then $\sin t$ is equal to

$t \in \text{III}$ so $\sin t < 0$
$\sin t = -\sqrt{1-\cos^2 t}$

(a) $-\frac{3}{5}$	(b) $\frac{3}{5}$	(c) $-\frac{4}{5}$	(d) $\frac{4}{5}$

3) $\sin^{-1}(\frac{1}{2})$ is equal to $\sin^{-1}\left(\frac{1}{2}\right) = \sin^{-1}\left(\sin\frac{\pi}{6}\right) = \frac{\pi}{6} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

(a) $-\frac{\pi}{3}$	(b) $-\frac{\pi}{6}$	(c) $\frac{\pi}{3}$	(d) $\frac{\pi}{6}$

4) $\sin^{-1}(\sin(\frac{13\pi}{6}))$ equals $\sin^{-1}\left(\sin\left(2\pi + \pi/6\right)\right) = \sin^{-1}\left(\sin\left(\frac{\pi}{6}\right)\right) = \frac{\pi}{6}$

(a) $-\frac{\pi}{3}$	(b) $-\frac{\pi}{6}$	(c) $\frac{\pi}{3}$	(d) $\frac{\pi}{6}$

5) $\lim_{x\to 0} \frac{(3+x)^2 - 9}{x} = \lim_{x\to 0} \frac{9+6x+x^2-9}{x} = \lim_{x\to 0} (x+6) = 6$

(a) 0	(b) 3	(c) 6	(d) 9

6) $\lim\limits_{x\to 0}\frac{|x|}{x} =$ $\begin{cases} 1 & \text{if } x>0 \\ -1 & \text{if } x<0 \end{cases}$ So it doesn't exit.

(a) -1	(b) 0	(c) 1	(d) doesn't exist

7) If $f(x) = \sqrt{1-x^2}$ then f is continuous on the interval

The domain of f
$D_f = \{x \in \mathbb{R} / 1-x^2 \geq 0\} = [-1, 1]$

(a) $(-\infty, -1]$	(b) $(-1, 1)$	(c) $[-1, 1]$	(d) $[1, +\infty)$

8) If $f(x) = \sqrt{1+x^2}$ then $f'(\sqrt{3}) =$ $\frac{\sqrt{3}}{\sqrt{1+3}} = \frac{\sqrt{3}}{2}$

$f(x) = (1+x^2)^{1/2}$ so $f'(x) = \frac{1}{2}(1+x^2)^{-1/2} \cdot 2x = \frac{x}{\sqrt{1+x^2}}$

(a) $\frac{3}{2}$	(b) $\frac{\sqrt{3}}{2}$	(c) $\frac{\sqrt{3}}{4}$	(d) 2

9) If f is a differentiable function and $f(1) = 1$, $f'(1) = 6$, and $g(x) = xf(x)$ then $g'(1) =$ $f(1) + f'(1)$

$g'(x) = f(x) + xf'(x)$

(a) 1	(b) 2	(c) 6	(d) 7

10) The slope of the tangent line to the graph $f(x) = 3x^2 - 4$ at the point $P(1, -1)$ is

$m = f'(1)$; $f'(x) = 6x$ so $m = 6$

(a) -1	(b) 1	(c) 6	(d) 7

II) A) i) Solve the equation $2\cos^2\theta - 7\cos\theta + 3 = 0$.

put $u = \cos\theta$

It becomes $2u^2 - 7u + 3 = 0$; $a = 2$; $b = -7$; $c = 3$

• Discriminant $D = b^2 - 4ac = 49 - 24 = 25 = 5^2$

2 $u_1 = \frac{7-5}{4} = \frac{1}{2}$

$u_2 = \frac{7+5}{4} = 3$ ↯

$\cos\theta = 1/2$ so $\theta = \pm \pi/3 + 2k\pi$ with $k \in \mathbb{Z}$

ii) If $z_1 = 2e^{i\frac{\pi}{4}}$ and $z_2 = 5e^{i\frac{\pi}{3}}$, find z_1z_2 and $\frac{z_1}{z_2}$.

• $z_1 z_2 = \left(2e^{i\pi/4}\right)\left(5e^{i\pi/3}\right) = 10\, e^{i\left(\frac{\pi}{4}+\frac{\pi}{3}\right)} = 10\, e^{i\frac{7\pi}{12}}$

2 • $\frac{z_1}{z_2} = \frac{2e^{i\pi/4}}{5e^{i\pi/3}} = \frac{2}{5}\, e^{i\left(\pi/4 - \frac{\pi}{3}\right)} = \frac{2}{5}\, e^{i\left(\frac{-\pi}{12}\right)}$

B) i) Find the rectangular coordinates for the point has polar coordinates $(1, \frac{3\pi}{4})$.

$r = 1$ and $\theta = \frac{3\pi}{4}$

1 • $x = r\cos\theta = \cos\left(\frac{3\pi}{4}\right) = -\sqrt{2}/2$

• $y = r\sin\theta = \sin\left(\frac{3\pi}{4}\right) = \sqrt{2}/2$

ii) Find the polar coordinates for the point has rectangular coordinates $(-2, 2\sqrt{3})$.

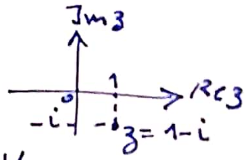
$x = -2$; $y = 2\sqrt{3}$

1 then • $r = \sqrt{x^2+y^2} = \sqrt{4+12} = \sqrt{16} = 4$

$\cos\theta = x/r = -2/4 = -1/2$; $\sin\theta = y/r = \frac{2\sqrt{3}}{4} = \frac{\sqrt{3}}{2}$

So • $\theta = 2\pi/3$

iii) Write the complex number $z = 1 - i$ in polar form and find $(1-i)^8$.



• $z = 1 - i = \sqrt{2}\, e^{-i\pi/4}$

2 • $z^8 = (1-i)^8 = \left(2^{1/2} e^{-i\pi/4}\right)^8 = 2^4 e^{-i2\pi} = 2^4 = 16$

C) Show that there is a root of the equation $x^3 - 2x^2 + x - 1 = 0$ between 1 and 2.

$f(x) = x^3 - 2x^2 + x - 1$ is a polynomial function

It is continuous on the bounded closed inteval $[1,2]$

2 As $f(1) = -1$ and $f(2) = 1$, then $f(1)\,f(2) < 0$

By Mean Value theorem there is $x_0 \in (1,2)$ such $f(x_0) =$

D) Differentiate the following functions:

i) $f(x) = 3x^5 - 5x^2 + \sqrt{x}$.

1 $$f'(x) = 15x^4 - 10x + \frac{1}{2\sqrt{x}}.$$

ii) $g(x) = xe^{-3x}$.

1 $$g'(x) = e^{-3x} + x(-3)e^{-3x} = (1-3x)e^{-3x}.$$

iii) $h(x) = \frac{\cos x}{2+\sin x}$.

1 $$h'(x) = \frac{(-\sin x)(2+\sin x) - (\cos x)(\cos x)}{(2+\sin x)^2} = \frac{-1-2\sin x}{(2+\sin x)^2}.$$

iv) $k(x) = [\ln(1+x^3)]^4$.

1 $$k'(x) = 4\left(\ln(1+x^3)\right)^3 \frac{d}{dx}\ln(1+x^3)$$
$$= 4\left(\ln(1+x^3)\right)^3 \cdot \frac{3x^2}{1+x^3} = \frac{12x^2}{1+x^3}\left(\ln(1+x^3)\right)^3.$$

F) Find an equation of tangent line for the curve $y = 5^x$ at the point $P(0,1)$.

$y = f(x) = 5^x$; $x_0 = 0$; $f(x_0) = f(0) = 1$

$f'(x) = (\ln 5)\, 5^x$; $f'(0) = \ln 5$

1 The equation of tangent line at $P(x_0, f(x_0))$ is

$$T_P : \quad y - f(x_0) = f'(x_0)(x - x_0)$$
$$y - 1 = (\ln 5)(x - 0)$$
$$\boxed{y = (\ln 5)x + 1}$$

End of exam