Chapter 6&7:

1. The Point Estimator for the Population Parameters:

	Population Parameter	Point Estimator
Mean	μ	$\bar{X}$
Variance	σ^2	S^2
Standard Deviation	σ	S
Proportion	p	$\hat{p}$
The Difference between two Means	$\mu_1 - \mu_2$	$\bar{X}_1 - \bar{X}_2$ or $\bar{X} - \bar{Y}$
The Difference between two Proportions	$p_1 - p_2$	$\hat{p}_1 - \hat{p}_2$

2. The Confidence Intervals for the Population Parameters:

<u>**Case 1:**</u> **The Confidence Interval for the Population Mean:**

The 100 (1-α) %

Confidence Interval for the Population Mean μ

- σ^2 *is known*

$$\bar{X} \pm Z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$$

- σ^2 *is unknown*
- $n > 30$

$$\bar{X} \pm Z_{\frac{\alpha}{2}} \frac{S}{\sqrt{n}}$$

- σ^2 *is unknown*
- Normal distribution.
- $n < 30$

$$\bar{X} \pm t_{\frac{\alpha}{2}} \frac{S}{\sqrt{n}}$$

<u>Case 2:</u> The Confidence Interval for the Difference between two Population Means $\mu_1 - \mu_2$:

The 100 (1-α) %
Confidence Interval for the difference between two Population Means μ_1-μ_2

- $\sigma_1^2 \& \sigma_2^2$ known

$$(\bar{X}-\bar{Y}) \pm Z_{\frac{\alpha}{2}}\sqrt{\frac{\sigma_1^2}{m}+\frac{\sigma_2^2}{n}}$$

- $\sigma_1^2 \& \sigma_2^2$ unknown
- $m, n > 30$

$$(\bar{X}-\bar{Y}) \pm Z_{\frac{\alpha}{2}}\sqrt{\frac{S_1^2}{m}+\frac{S_2^2}{n}}$$

- $\sigma_1^2 = \sigma_2^2 = \sigma$ Unknown but equal
- Normal distribution
- $m, n < 30$

$$(\bar{X}-\bar{Y}) \pm t_{\frac{\alpha}{2}} S_p \sqrt{\frac{1}{m}+\frac{1}{n}}$$

Where, $S_p = \sqrt{\frac{(m-1)S_1^2+(n-1)S_2^2}{m+n-2}}$ & $df = v = m+n-2$.

<u>Case 3:</u> The Confidence Interval for the Population Proportion p:

When $n > 30,\ np > 5,\ nq > 5$ and $\hat{p} = \frac{X}{n}$
Then the $100(1-\alpha)\%$ confidence interval for p is

$$\hat{p} \pm Z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

<u>Case 4:</u> The Confidence Interval for the Difference between two Population Proportions:

When $m > 30, n > 30,\ mp_1 > 5,\ np_2 > 5,\ mq_1 > 5,\ nq_2 > 5$ and $\hat{p}_1 = \frac{X_1}{m}, \hat{p}_2 = \frac{X_2}{n}$.
Then the $100(1-\alpha)\%$ confidence interval for $p_1 - p_2$ is

$$(\hat{p}_1 - \hat{p}_2) \pm Z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}_1\hat{q}_1}{m}+\frac{\hat{p}_2\hat{q}_2}{n}}$$

<u>Case 5:</u> The Confidence Interval for Paired Data:

In this case, we are interested in comparing the means of two related (non-independent/dependent) normal populations $X_1, \dots, X_N$ and $Y_1, \dots, Y_N$.

Examples of related populations are:

1. Height of the father and height of his son.
2. Mark of the student in MATH and his mark in STAT.
3. Hemoglobin level of the patient before and after the medical treatment.

   Calculate:

   - $\mu_D = \mu_1 - \mu_2$
   - $D_i's$ where $D_i = X_i - Y_i$ (as it explained in the table below)
   - $\overline{D} = \frac{\sum_{i=1}^{n} D_i}{n}$
   - $S_D = \sqrt{\frac{\sum_{i=1}^{n}(D_i - \overline{D})^2}{n-1}}$

X_i	Y_i	D_i
X_1	Y_1	$D_1 = X_1 - Y_1$
$\vdots$	$\vdots$	$\vdots$
X_n	Y_n	$D_n = X_n - Y_n$
		$\overline{D} = \frac{\sum_{i=1}^{n} D_i}{n}$

Thus, $(1-\alpha)100\%$ confidence interval for $\mu_D = \mu_1 - \mu_2$ is:

$$\overline{D} \pm t_{\frac{\alpha}{2}} \frac{S_D}{\sqrt{n}}$$

<u>Case 6:</u> The Confidence Interval for the Variance/Standard Deviation:

The $100(1-\alpha)\%$ confidence interval for σ^2 is:

$$\frac{(n-1)s^2}{\chi^2_{\frac{\alpha}{2},n-1}} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_{1-\frac{\alpha}{2},n-1}}$$

Thus, The $100(1-\alpha)\%$ confidence interval for σ is:

$$\sqrt{\frac{(n-1)s^2}{\chi^2_{\frac{\alpha}{2},n-1}}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi^2_{1-\frac{\alpha}{2},n-1}}}$$

<u>Case 7:</u> The Confidence Interval for two Population Variances:

The $100(1-\alpha)\%$ confidence interval for σ_1^2/σ_2^2 is:

$$\frac{s_1^2/s_2^2}{F_{\frac{\alpha}{2},m-1,n-1}} < \frac{\sigma_1^2}{\sigma_2^2} < \frac{s_1^2/s_2^2}{F_{1-\frac{\alpha}{2},m-1,n-1}}$$