Application of Integration
(Arc Length and Surface of Revolution)

Bander Almutairi
King Saud University
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1. Arc Length

2. Surface of Revolution
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**Theorem**

The arc length of the graph of $f$ from $(a, c)$ to $(b, d)$ is

$$L_{b-a} = \int_{a}^{b} \sqrt{1 + [f'(x)]^2} \, dx$$

or

$$L_{d-c} = \int_{c}^{d} \sqrt{1 + [g'(y)]^2} \, dy$$
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Examples (Swokowski, 335)

[1] If \( f(x) = 3x^{2/3} - 10 \), find the arc length of the graph of \( f \) from the point \( A(8, 2) \) to \( B(27, 17) \). (answer approx. 2.4)

[2] Set up an integral for finding the arc length of the graph of the equation \( y^3 - y - x = 0 \) from \( A(O, -1) \) to \( B(6, 2) \).
Definition

Suppose $f(x)$ is a non negative function on $[a, b]$. 

If the graph of $f(x)$ is revolved about the x-axis, a surface of revolution is generated and the area is given by the formula:

$$S = \int_{a}^{b} 2\pi f(x) \sqrt{1 + \left(f'(x)\right)^2} \, dx.$$
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If the graph of $g$ is revolved about the $y$-axis from $(a, c)$ to $(b, d)$, a surface of revolution is generated and the area is given by the formula:

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Likewise if \( g(y) \) is a non negative function on \([c, d]\).

If the graph of \( g \) is revolved about the \( y \)-axis from \((a, c)\) to \((b, d)\), a surface of revolution is generated and the area is given by the formula:

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Example (1, Swokowski, 340)

The graph of $y = \sqrt{x}$ from (1, 1) to (4, 2) is revolved about the x-axis. Find the area of the resulting surface.
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The graph of $y = \sqrt{x}$ from $(1, 1)$ to $(4, 2)$ is revolved about the $x$-axis. Find the area of the resulting surface.

Example (2, Swokowskki, exercises 342)

The graph of the equation from $A$ to $B$ is revolved about the $x$-axis. Find the area of the resulting surface.

a $4x = y^2; A(0, 0), B(1, 2)$.

b $y = x^3; A(1, 1), B(2, 8)$.

c $y = 2x + 1; A(0, 2), B(3, 4)$.
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Example (3, Swokowsoki, exercises 342)

The graph of the equation from \(A\) to \(B\) is revolved about the y-axis. Find the area of the resulting surface.

\( a \) \( y = 2\sqrt[3]{x}; A(1, 2), B(8, 4). \)

\( b \) \( x = 4\sqrt{y}; A(4, 1), B(12, 9). \)