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CALCULUS FOR ENGINEERS  
MATH 1110

Evaluate the following iterated integral

1-

$$\int_0^1 \int_0^z \int_0^{x+z} 6xz \, dy \, dx \, dz$$

$$\begin{aligned} \int_0^1 \int_0^z \int_0^{x+z} 6xz \, dy \, dx \, dz &= \int_0^1 \int_0^z [6xyz]_{y=0}^{y=x+z} \, dx \, dz = \int_0^1 \int_0^z 6xz(x+z) \, dx \, dz \\ &= \int_0^1 \left[ 2x^3 z + 3x^2 z^2 \right]_{x=0}^{x=z} \, dz = \int_0^1 (2z^4 + 3z^4) \, dz = \int_0^1 5z^4 \, dz = \left[ z^5 \right]_0^1 = 1 \end{aligned}$$

2-

$$\int_0^1 \int_x^{2x} \int_0^y 2xyz \, dz \, dy \, dx$$

$$\begin{aligned} \int_0^1 \int_x^{2x} \int_0^y 2xyz \, dz \, dy \, dx &= \int_0^1 \int_x^{2x} [xyz^2]_{z=0}^{z=y} \, dy \, dx = \int_0^1 \int_x^{2x} xy^3 \, dy \, dx \\ &= \int_0^1 \left[ \frac{1}{4} xy^4 \right]_{y=x}^{y=2x} \, dx = \int_0^1 \frac{15}{4} x^5 \, dx = \left[ \frac{5}{8} x^6 \right]_0^1 = \frac{5}{8} \end{aligned}$$

3-

$$\int_0^3 \int_0^1 \int_0^{\sqrt{1-z^2}} z e^y dx dz dy$$

$$\begin{aligned} \int_0^3 \int_0^1 \int_0^{\sqrt{1-z^2}} z e^y dx dz dy &= \int_0^3 \int_0^1 \left[ x z e^y \right]_{x=0}^{x=\sqrt{1-z^2}} dz dy = \int_0^3 \int_0^1 z e^y \sqrt{1-z^2} dz dy \\ &= \int_0^3 \left[ -\frac{1}{3} (1-z^2)^{3/2} e^y \right]_{z=0}^{z=1} dy = \int_0^3 \frac{1}{3} e^y dy = \left[ \frac{1}{3} e^y \right]_0^3 = \frac{1}{3} (e^3 - 1) \end{aligned}$$

4-

$$\int_0^1 \int_0^z \int_0^y z e^{-y^2} dx dy dz$$

$$\begin{aligned} \int_0^1 \int_0^z \int_0^y z e^{-y^2} dx dy dz &= \int_0^1 \int_0^z \left[ x z e^{-y^2} \right]_{x=0}^{x=y} dy dz = \int_0^1 \int_0^z y z e^{-y^2} dy dz = \int_0^1 \left[ -\frac{1}{2} z e^{-y^2} \right]_{y=0}^{y=z} dz \\ &= \int_0^1 -\frac{1}{2} z \left( e^{-z^2} - 1 \right) dz = \frac{1}{2} \int_0^1 \left( z - z e^{-z^2} \right) dz \\ &= \frac{1}{2} \left[ \frac{1}{2} z^2 + \frac{1}{2} e^{-z^2} \right]_0^1 = \frac{1}{4} (1 + e^{-1} - 0 - 1) = \frac{1}{4e} \end{aligned}$$

5-

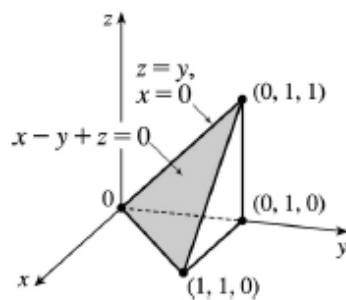
$\iiint_E 6xy dV$ , where  $E$  lies under the plane  $z = 1 + x + y$  and above the region in the  $xy$ -plane bounded by the curves  $y = \sqrt{x}$ ,  $y = 0$ , and  $x = 1$

Here  $E = \{(x, y, z) | 0 \leq x \leq 1, 0 \leq y \leq \sqrt{x}, 0 \leq z \leq 1 + x + y\}$ , so

$$\begin{aligned}
\iiint_E 6xy dV &= \int_0^1 \int_0^{\sqrt{x}} \int_0^{1+x+y} 6xy dz dy dx = \int_0^1 \int_0^{\sqrt{x}} [6xyz]_{z=0}^{z=1+x+y} dy dx \\
&= \int_0^1 \int_0^{\sqrt{x}} 6xy(1+x+y) dy dx = \int_0^1 \left[ 3xy^2 + 3x^2 y^2 + 2xy^3 \right]_{y=0}^{y=\sqrt{x}} dx \\
&= \int_0^1 (3x^2 + 3x^3 + 2x^{5/2}) dx = \left[ x^3 + \frac{3}{4} x^4 + \frac{4}{7} x^{7/2} \right]_0^1 = \frac{65}{28}
\end{aligned}$$

6-

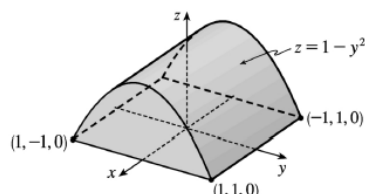
$\iiint_E xz dV$ , where  $E$  is the solid tetrahedron with vertices  $(0, 0, 0)$ ,  $(0, 1, 0)$ ,  $(1, 1, 0)$ , and  $(0, 1, 1)$



$$\begin{aligned}
\int_0^1 \int_0^y \int_0^{y-z} xz dx dz dy &= \int_0^1 \int_0^y \frac{1}{2} (y-z)^2 z dz dy \\
&= \frac{1}{2} \int_0^1 \left[ \frac{1}{2} y^2 z^2 - \frac{2}{3} yz^3 + \frac{1}{4} z^4 \right]_{z=0}^{z=y} dy \\
&= \frac{1}{24} \int_0^1 y^4 dy = \frac{1}{24} \left[ \frac{1}{5} y^5 \right]_0^1 = \frac{1}{120}
\end{aligned}$$

7-

$\iiint_E x^2 e^y dV$ , where  $E$  is bounded by the parabolic cylinder  $z = 1 - y^2$  and the planes  $z = 0$ ,  $x = 1$ , and  $x = -1$

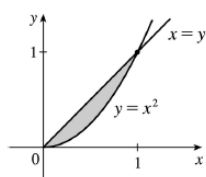


$E$  is the region below the parabolic cylinder  $z=1-y^2$  and above the square  $[-1,1] \times [-1,1]$  in the  $xy$ -plane.

$$\begin{aligned} \iiint_E x^2 e^y dV &= \int_{-1}^1 \int_{-1}^1 \int_0^{1-y^2} x^2 e^y dz dy dx \\ &= \int_{-1}^1 \int_{-1}^1 x^2 e^y (1-y^2) dy dx \\ &= \int_{-1}^1 x^2 dx \int_{-1}^1 (e^y - y^2 e^y) dy \\ &= \left[ \frac{1}{3} x^3 \right]_{-1}^1 \left[ e^y - (y^2 - 2y + 2)e^y \right]_{-1}^1 \\ &= \frac{1}{3} (2) [e - e^{-1} + 5e^{-1}] = \frac{8}{3e} \end{aligned}$$

8-

$\iiint_E (x + 2y) dV$ , where  $E$  is bounded by the parabolic cylinder  $y = x^2$  and the planes  $x = z$ ,  $x = y$ , and  $z = 0$



$E$  is the solid above the region shown in the  $xy$ -plane and below the plane  $z=x$ . Thus,

$$\begin{aligned} \iiint_E (x+2y) dV &= \int_0^1 \int_{x^2}^x \int_0^x (x+2y) dz dy dx \\ &= \int_0^1 \int_{x^2}^x (x^2 + 2xy) dy dx = \int_0^1 \left[ x^2 y + xy^2 \right]_{y=x^2}^{y=x} dx \\ &= \int_0^1 (2x^3 - x^4 - x^5) dx = \left[ \frac{1}{2} x^4 - \frac{1}{5} x^5 - \frac{1}{6} x^6 \right]_0^1 = \frac{2}{15} \end{aligned}$$

