Chapter 11
INTRODUCTION

ELASTIC SETTLEMENT
Stress distribution in soil masses

CONSOLIDATION SETTLEMENT
• Fundamentals of consolidation
• Calculation of 1-D Consolidation Settlement
• One-dimensional Laboratory Consolidation Test
• Calculation of Settlement from 1-D Primary Consolidation

TIME RATE OF CONSOLIDATION SETTLEMENT
1-D theory of consolidation

SECONDARY CONSOLIDATION SETTLEMENT
Why should soil compressibility be studied?

Ignoring soil compressibility may lead to unfavorable settlement and other engineering problems.

Embankment and building constructed on soft ground (highly compressible soil)

Settlement is one of the aspects that control the design of structures.
Why soils compressed?

• Every material undergoes a certain amount of strain when a stress is applied.

• A steel rod *lengthens* when it is subjected to tensile stress, and a concrete column *shortens* when a compressive load is applied.

• The same thing holds true for soils which undergo compressive strains upon loading. Compressive strains are responsible for settlement of the structure.

• What distinguish soils from other civil engineering materials is the fact that the deformation of soils is largely *unrecoverable* (i.e. permanent). Therefore simple elasticity theory like elasticity cannot be applied to soils.
What makes soil compressed?

In soils voids exist between particles and the voids may be filled with a liquid, usually water, or gas, usually air. As a result, soils are often referred to as a three-phase material or system (solid, liquid and gas).

- Solid (mineral particles)
- Gas (air),
- Liquid (usually water)
Causes of settlement

Settlement of a structure resting on soil may be caused by two distinct kinds of action within the foundation soils:-

I. Settlement Due to Shear Stress (Distortion Settlement)

In the case the applied load caused shearing stresses to develop within the soil mass which are greater than the shear strength of the material, then the soil fails by sliding downward and laterally, and the structure settle and may tip of vertical alignment. This will be discussed in CE483 Foundation Engineering. This is what we referred to as BEARING CAPACITY.

II. Settlement Due to Compressive Stress (Volumetric Settlement)

As a result of the applied load a compressive stress is transmitted to the soil leading to compressive strain. Due to the compressive strain the structure settles. This is important only if the settlement is excessive otherwise it is not dangerous.
• However, in certain structures, like for example foundation for RADAR or telescope, even small settlement is not allowed since this will affect the function of the equipment.

• This type of settlement is what we will consider in this chapter and this course. In the following sections we will discuss its components and ways for their evaluation. We will consider only the simplest case where settlement is one-dimensional and a condition of zero lateral strain is assumed.
Causes of Settlement

- Alien Causes
  - Subsidence
  - Cavities
  - Excavation
  - etc..

- Shear Stresses
  - Bearing Capacity Failure

- Compressive Stresses
  - Immediate
  - Primary
  - Secondary
Compression of soil is due to a number of mechanisms:

- **Deformation** of soil particles or grains
- **Relocations** of soil particles
- **Expulsion** of water or air from the void spaces
Components of settlement

Settlement of a soil layer under applied load is the sum of two broad components or categories:

1. Elastic settlement (or immediate) settlements
2. Consolidation settlement

1. Elastic settlement (or immediate) settlements

Elastic or immediate settlement takes place instantly at the moment of the application of load due to the distortion (but no bearing failure) and bending of soil particles (mainly clay). It is not generally elastic although theory of elasticity is applied for its evaluation. It is predominant in coarse-grained soils.
Consolidation settlement is the sum of two parts or types:

A. Primary consolidation settlement

    In this the compression of clay is due to expulsion of water from pores. The process is referred to as PRIMARY CONSOLIDATION and the associated settlement is termed PRIMARY CONSOLIDATION SETTLEMENT. Commonly they are referred to simply as CONSOLIDATION AND CONSOLIDATION SETTLEMENT.

B. Secondary consolidation settlement

    The compression of clay soil due to plastic readjustment of soil grains and progressive breaking of clayey particles and their interparticles bonds is known as SECONDARY CONSOLIDATION OR SECONDARY COMPRESSION, and the associated settlement is called SECONDARY CONSOLIDATION SETTLEMENT or SECONDARY COMPRESSION.
The total settlement of a foundation can be expressed as:

\[ S_T = S_e + S_c + S_s \]

- \( S_T \) = Total settlement
- \( S_e \) = Elastic or immediate settlement
- \( S_c \) = Primary consolidation settlement
- \( S_s \) = Secondary consolidation settlement

It should be mentioned that \( S_c \) and \( S_s \) overlap each other and impossible to detect which certainly when one type ends and the other begins. However, for simplicity they are treated separately and secondary consolidation is usually assumed to begin at the end of primary consolidation.
The total soil settlement $S_T$ may contain one or more of these types:

- **Immediate settlement**: Due to distortion or elastic deformation with no change in water content. Occurs rapidly during the application of load. Quite small quantity in dense sands, gravels and stiff clays.

- **Primary consolidation settlement**: Decrease in voids volume due to squeeze of pore-water out of the soil. Occurs in saturated fine grained soils (low coefficient of permeability). Time dependent. Only significant in clays and silts.

- **Secondary consolidation or creep**: Due to gradual changes in the particulate structure of the soil. Occurs very slowly, long after the primary consolidation is completed. Time dependent. Most significant in saturated soft clayey and organic soils and peats.
A gradual reduction in volume change of a fully saturated soils of low permeability due to the drainage of pore water from soil voids
# Rates of Drainage

<table>
<thead>
<tr>
<th>soil type</th>
<th>coeff. of permeability (k)</th>
<th>seepage rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gravel</td>
<td>&gt; $10^{-2}$ m/sec</td>
<td>very quick</td>
</tr>
<tr>
<td>Sand</td>
<td>$10^{-2}$ ~ $10^{-5}$</td>
<td>quick</td>
</tr>
<tr>
<td>Silt</td>
<td>$10^{-5}$ ~ $10^{-8}$</td>
<td>slow</td>
</tr>
<tr>
<td>Clay</td>
<td>&lt; $10^{-8}$</td>
<td>very slow</td>
</tr>
</tbody>
</table>

For design purposes it is common to assume:
- Quick drainage in coarse soils (Sand and Gravel)
- Slow drainage in fine soils (Clay and Silt).
Granular soils are freely drained, and thus the settlement is instantaneous.

\[ S_T = S_e + S_c + S_s \]
When a saturated clay is loaded externally, the water is squeezed out of the clay over a long time (due to low permeability of the clay).

This leads to settlements occurring over a long time.....which could be several years.

\[ S_t = S_e + S_c + S_s \]

e negligible
This type of settlement occur **immediately** after the application of load. It is predominant in coarse-grained soil (i.e. gravel, sand). Analytical evaluation of this settlement is a problem which requires satisfaction of the same set of conditions as the determination of stresses in continuous media.

In fact we could view the process as one of:

- Determining the stresses at each point in the medium
- Evaluating the vertical strains
- Integrating these vertical strains over the depth of the material.
- Theory of elasticity is used to determine the immediate settlement. This is to a certain degree reasonable in cohesive soils but not reasonable for cohesionless soils.
Contact pressure and settlement profile

The contact pressure distribution and settlement profile under the foundation will depend on:
- **Flexibility** of the foundation (flexible or rigid).
- **Type** of soil (clay, silt, sand, or gravel).

![Contact pressure distribution and settlement profile diagrams](image)
There are solutions available for different cases depending on the following conditions:

- **Load:**
  - point
  - distributed

- **Loaded area:**
  - Rectangular
  - Square
  - Circular

- **Stiffness:**
  - Flexible
  - Rigid

- **Soil:**
  - Cohesive
  - Cohesionless

- **Medium:**
  - Finite
  - Infinite
  - Layered

- These conditions are the same as those discussed at the time when we presented stresses in soil mass from theory of elasticity in CE 382.

- One of the well-known and used formula is that for the vertical settlement of the surface of an elastic half space uniformly loaded.
In CE 382, the relationships for determining the increase in stress (which causes elastic settlement) were based on the following assumptions:

- The load is applied at the ground surface.
- The loaded area is *flexible*.
- The soil medium is homogeneous, elastic, isotropic, and extends to a great depth.
For shallow foundation subjected to a net force per unit area equal to $\Delta \sigma$ and if the foundation is perfectly flexible, the settlement may be expressed as:

$$S_e = \Delta \sigma (\alpha B') \frac{1 - \mu_s^2}{E_s} I_s I_f$$

(\text{flexible})

$E_s = $ Average modulus of elasticity of soil

$\mu_s = $ Poisson’s ratio of soil

$B' = B/2$ center = B corner of foundation

$I_s = $ shape Factor

$I_f = $ depth factor

$\alpha = $ factor depends on location where settlement of foundation is calculated ($\alpha = 4$ center of foundation, $\alpha = 1$ corner of the foundation).

$S_e$ (rigid) = 0.93 $S_e$ (flexible-center)

More details about the calculation are given in Section 11.3 in the textbook.
Settlement Based on the Theory of Elasticity

\[
S_e = q_o(\alpha B') \left( \frac{1 - \mu_s^2}{E_s} \right) I_s I_f
\]

where

- \( q_o \): net applied pressure on the foundation
- \( \mu_s \): Poisson’s ratio of soil
- \( E_s \): average modulus of elasticity of the soil under the foundation, measured from \( z = 0 \) to about \( z = 5B \)
- \( B' \): \( B/2 \) for center of foundation, \( B \) for corner of foundation
- \( I_s \): shape factor (Steinbrenner, 1934)
- \( I_f \): depth factor (Fox, 1948)
- \( \alpha \): a factor that depends on the location on the foundation where settlement is being calculated

The elastic settlement of a rigid foundation can be estimated as

\[
S_e(\text{rigid}) = 0.93S_e(\text{flexible, center})
\]
Elastic Settlement in Granular Soil

To calculate settlement at the center of the foundation, we use

\[ \alpha = 4 \]
\[ m' = \frac{L}{B} \]

and

\[ n' = \frac{H}{\left( \frac{B}{2} \right)} \]

To calculate settlement at a corner of the foundation,

\[ \alpha = 1 \]
\[ m' = \frac{L}{B} \]

and

\[ n' = \frac{H}{B} \]

\[ I_f = F_1 + \frac{1 - 2\mu_s}{1 - \mu_s} F_2 \]

Table 7.2 Variation of \( F_1 \) with \( m' \) and \( n' \)

\[ \begin{array}{ccccccccc}
 m' & 1.0 & 1.2 & 1.4 & 1.6 & 1.8 & 2.0 & 2.5 & 3.0 & 3.5 & 4.0 \\
 n' & & & & & & & & & & \\
 0.25 & 0.014 & 0.013 & 0.012 & 0.011 & 0.011 & 0.010 & 0.010 & 0.010 & 0.010 & 0.010 \\
 0.50 & 0.049 & 0.046 & 0.044 & 0.042 & 0.041 & 0.040 & 0.038 & 0.038 & 0.037 & 0.037 \\
 0.75 & 0.095 & 0.090 & 0.087 & 0.084 & 0.082 & 0.080 & 0.077 & 0.076 & 0.074 & 0.074 \\
 1.00 & 0.142 & 0.138 & 0.134 & 0.130 & 0.127 & 0.125 & 0.121 & 0.118 & 0.116 & 0.115 \\
\end{array} \]

Table 7.3 Variation of \( F_2 \) with \( m' \) and \( n' \)

\[ \begin{array}{ccccccccc}
 m' & 1.0 & 1.2 & 1.4 & 1.6 & 1.8 & 2.0 & 2.5 & 3.0 & 3.5 & 4.0 \\
 n' & & & & & & & & & & \\
 0.25 & 0.049 & 0.050 & 0.051 & 0.051 & 0.051 & 0.052 & 0.052 & 0.052 & 0.052 & 0.052 \\
 0.50 & 0.074 & 0.077 & 0.080 & 0.081 & 0.081 & 0.083 & 0.084 & 0.086 & 0.086 & 0.0878 & 0.0878 \\
 0.75 & 0.083 & 0.089 & 0.093 & 0.097 & 0.099 & 0.101 & 0.104 & 0.106 & 0.107 & 0.107 & 0.108 \\
 1.00 & 0.083 & 0.091 & 0.098 & 0.102 & 0.106 & 0.109 & 0.114 & 0.117 & 0.119 & 0.120 & 0.120 \\
\end{array} \]

Table 7.4 Variation of \( I_f \) with \( D_y/B \), \( B/L \), and \( \mu_s \)

\[ \begin{array}{cccccc}
 \mu_s & D_y/B & 0.2 & 0.5 & 1.0 \\
 0.3 & 0.2 & 0.95 & 0.93 & 0.90 \\
 0.4 & 0.2 & 0.97 & 0.96 & 0.93 \\
 0.5 & 0.2 & 0.99 & 0.98 & 0.96 \\
\end{array} \]

where

\[ E_{s0} = \frac{\Sigma E_{s0} \Delta z}{\bar{z}} \]

\[ \bar{z} = H \text{ or } 5B, \text{ whichever is smaller} \]
Due to the nonhomogeneous nature of soil deposits, the magnitude of $E_s$ may vary with depth. For that reason, Bowles (1987) recommended using a weighted average value of $E_s$.

$$E_s = \frac{\sum E_{s(i)} \Delta z}{\bar{z}}$$

where:

$E_{s(i)}$ soil modulus of elasticity within a depth $\Delta z$.

$$\bar{z} = H \text{ or } 5B,$$

whichever is smaller.
Example 11.1

A rigid shallow foundation 1 m \( \times \) 1 m in plan is shown in Figure 11.4. Calculate the elastic settlement at the center of the foundation.

**Solution**

Given: \( B = 1 \) m and \( L = 1 \) m. Note that \( Z = 5 \) m = 5B. From Eq. (11.10),

\[
E_s = \frac{\sum E_{soj}\Delta z}{Z} = \frac{(8000)(2) + (6000)(1) + (10,000)(2)}{5} = 8400 \text{ kN/m}^2
\]

For the center of the foundation,

\[
\alpha = \frac{4}{m' = \frac{L}{B} = \frac{1}{1} = 1}
\]

From Tables 11.1 and 11.2, \( F_1 = 0.498 \) and \( F_2 = 0.016 \). From Eq. (11.2),

\[
I_s = F_1 + \frac{1 - 2\mu_s}{1 - \mu_s} F_2 = 0.498 + \frac{1 - 0.6}{1 - 0.3}(0.016) = 0.507
\]

Again, \( \frac{D_f}{B} = \frac{1}{1} = 1, \frac{L}{B} = 1, \mu_s = 0.3 \). From Table 11.3, \( I_f = 0.65 \). Hence,

\[
S_{e(\text{flexible})} = \Delta \sigma (\alpha B') - 1 - \mu^2 \frac{I_s I_f}{E_s} = (2000) \left( \frac{4 \times 1}{2} \right) \left( 1 - 0.3^2 \right) (0.507)(0.65) = 0.0143 \text{ m} = 14.3 \text{ mm}
\]

Since the foundation is rigid, from Eq. (11.9),

\[
S_e(\text{rigid}) = (0.93)(14.3) = 13.3 \text{ mm}
\]

Figure 11.4
The improved formula takes into account
• the rigidity of the foundation,
• the depth of embedment of the foundation,
• the increase in the modulus of elasticity of the soil with depth, and
• the location of rigid layers at a limited depth

\[
S_e = \frac{q_o B e I_G I_F I_E}{E_o} \left(1 - \mu_s^2\right)
\]

where

- \( I_G \) = influence factor for the variation of \( E_s \) with depth
  \[
  = f \left( B = \frac{E_o}{kB_e} \frac{H}{B_e} \right)
  \]

- \( I_F \) = foundation rigidity correction factor

- \( I_E \) = foundation embedment correction factor

Equivalent diameter \( B_e \) of

- Rectangular foundation
  \[ B_e = \sqrt{\frac{4BL}{\pi}} \]

- Circular foundation
  \[ B_e = B \]

\[ E_s = E_o + kZ \]
Improved Equation for Elastic Settlement

\[
K_F = \left( \frac{E_f}{E_r + \frac{B_r}{2} k} \right) \left( \frac{2t}{B_c} \right)^3
\]

- Flexibility factor

\[
\beta = \frac{E_i}{kB_c}
\]

\[
H/B_c = \frac{1}{2}
\]
Example 11.2

Refer to Figure 11.5. For a shallow foundation supported by a silty clay, the following are given:

- Length, $L = 1.5$ m
- Width, $B = 1$ m
- Depth of foundation, $D_f = 1$ m
- Thickness of foundation, $t = 0.23$ m
- Load per unit area, $\Delta \sigma = 190$ kN/m$^2$
- $E_t = 15 \times 10^6$ kN/m$^2$

The silty clay soil had the following properties:

- $h = 2$ m
- $\mu_s = 0.3$
- $E_w = 9000$ kN/m$^2$
- $k = 500$ kN/m$^2$/m

Estimate the elastic settlement of the foundation.

Solution

From Eq. (11.11), the equivalent diameter is

$$B_e = \sqrt{\frac{4BL}{\pi}} = \sqrt{\frac{4(1.5)(1)}{\pi}} \approx 1.38 \text{ m}$$

$$\Delta \sigma = 190 \text{ kN/m}^2$$

$$\beta = \frac{E_w}{kB_e} = \frac{9000}{500(1.38)} = 13.04$$

$$\frac{h}{B_e} = \frac{2}{1.38} = 1.45$$

From Figure 11.6, for $\beta = 13.04$ and $h/B_e = 1.45$, the value of $I_e = 0.74$. Thus, from Eq. (11.15),

$$I_e = \frac{\pi}{4} + \frac{1}{4.6 + 10 \left( \frac{E_t}{E_o + \frac{B_e^2}{2k}} \right)^{\frac{2t}{B_e}}}^{\frac{2t}{B_e}}$$

$$= \frac{\pi}{4} + \frac{1}{4.6 + 10 \left[ \frac{15 \times 10^6}{9000 + \frac{(1.38)^2}{2}(300)} \right]^{\frac{0.23}{1.38}}} = 0.787$$

From Eq. (11.16),

$$I_e = 1 - \frac{1}{3.5 \exp(1.22\mu_s - 0.4) \left( \frac{B_e}{D_f} + 1.6 \right)}$$

$$= 1 - \frac{1}{3.5 \exp(1.22(0.3) - 0.4) \left( \frac{1.38}{1} + 1.6 \right)} = 0.907$$

From Eq. (11.14),

$$S_e = \frac{\Delta \sigma B_e I_e I_x I_p}{E_o} (1 - \mu_s^2) = \frac{(190)(1.38)(0.74)(0.787)(0.907)(1 - 0.3^2)}{9000}$$

$$= 0.014 \text{ m} \approx 14 \text{ mm}$$
## Settlement calculation

### Depth factor \( I_f \)

<table>
<thead>
<tr>
<th>( L/B )</th>
<th>( D_f/B )</th>
<th>( \mu_s = 0.3 )</th>
<th>( \mu_s = 0.4 )</th>
<th>( \mu_s = 0.5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.5</td>
<td>0.77</td>
<td>0.82</td>
<td>0.85</td>
</tr>
<tr>
<td></td>
<td>0.75</td>
<td>0.69</td>
<td>0.74</td>
<td>0.77</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>0.65</td>
<td>0.69</td>
<td>0.72</td>
</tr>
<tr>
<td>2</td>
<td>0.5</td>
<td>0.82</td>
<td>0.86</td>
<td>0.89</td>
</tr>
<tr>
<td></td>
<td>0.75</td>
<td>0.75</td>
<td>0.79</td>
<td>0.83</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>0.71</td>
<td>0.75</td>
<td>0.79</td>
</tr>
<tr>
<td>5</td>
<td>0.5</td>
<td>0.87</td>
<td>0.91</td>
<td>0.93</td>
</tr>
<tr>
<td></td>
<td>0.75</td>
<td>0.81</td>
<td>0.86</td>
<td>0.89</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>0.78</td>
<td>0.82</td>
<td>0.85</td>
</tr>
</tbody>
</table>

### Poisson’s ratio of soil \( \mu_s \)

<table>
<thead>
<tr>
<th>Type of soil</th>
<th>Poisson’s ratio, ( \mu_s )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loose sand</td>
<td>0.2–0.4</td>
</tr>
<tr>
<td>Medium sand</td>
<td>0.25–0.4</td>
</tr>
<tr>
<td>Dense sand</td>
<td>0.3–0.45</td>
</tr>
<tr>
<td>Silty sand</td>
<td>0.2–0.4</td>
</tr>
<tr>
<td>Soft clay</td>
<td>0.15–0.25</td>
</tr>
<tr>
<td>Medium clay</td>
<td>0.2–0.5</td>
</tr>
</tbody>
</table>

### Average modulus of elasticity of soil \( E_s \)

<table>
<thead>
<tr>
<th>Soil type</th>
<th>( E_s ) (kN/m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Soft clay</td>
<td>1800–3500</td>
</tr>
<tr>
<td>Hard clay</td>
<td>6000–14,000</td>
</tr>
<tr>
<td>Loose sand</td>
<td>10,000–28,000</td>
</tr>
<tr>
<td>Dense sand</td>
<td>35,000–70,000</td>
</tr>
</tbody>
</table>
Stresses Distribution in Soils

Geostatic Stresses
- Total Stress
- Effective Stress
- Pore Water Pressure

\[ \sigma_{\text{total}} = \sigma_{\text{eff}} + u \]

Added Stresses (Point, line, strip, triangular, circular, rectangular)

Westergaard's Method (For Pavement)

Bossiniseque Equations
1. Point Load
2. Line Load
3. Strip Load
4. Triangular Load
5. Circular Load
6. Rectangular Load

Approximate Method
- 1:2 Method

Influence Charts

\[ \Delta \sigma_y = \tau_y \cdot q \]

Stress Bulbs

Newmark Charts
I. Stresses from approximate methods

2:1 Method

- In this method it is assumed that the STRESSED AREA is larger than the corresponding dimension of the loaded area by an amount equal to the depth of the subsurface area.

\[ \sigma_z = \frac{P}{(B+z)(L+z)} \]
Settlement is caused by stress increase, therefore for settlement calculations, we first need vertical stress increase, $\Delta \sigma$, in soil mass imposed by a net load, $q$, applied at the foundation level.

**CE 382** and Chapter 10 in the textbook present many methods based on Theory of Elasticity to estimate the stress in soil imposed by foundation loadings.

Since we consider only vertical settlement we limit ourselves to vertical stress distribution.

Since mostly we have distributed load we will not consider point or line load.
For **wide uniformly distributed load**, such as for very wide embankment fill, the stress increase at any depth, \( z \), can be given as:

\[
\Delta \sigma_z = q
\]
II. Stresses from theory of elasticity

- There are a number of solutions which are based on the theory of elasticity. Most of them assume the following assumptions:
  - The soil is homogeneous
  - The soil is isotropic
  - The soil is perfectly elastic infinite or semi-finite medium

- Tens of solutions for different problems are now available in the literature. It is enough to say that a whole book (Poulos and Davis) is now available for the elastic solutions of various problems.

The book contains a comprehensive collection of graphs, tables and explicit solutions of problems in elasticity relevant to soil and rock mechanics.
Vertical Stress Below the Center of a Uniformly Loaded Circular Area

\[ \Delta \sigma_z = q \left( 1 - \frac{1}{[(R/z)^2 + 1]^{3/2}} \right) \]

**Table 10.7 Variation of \( \Delta \sigma_z / q \) with \( z/R \) [Eq. (10.27)]**

<table>
<thead>
<tr>
<th>( z/R )</th>
<th>( \Delta \sigma_z / q )</th>
<th>( z/R )</th>
<th>( \Delta \sigma_z / q )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>1.0</td>
<td>0.6465</td>
</tr>
<tr>
<td>0.02</td>
<td>0.9999</td>
<td>1.5</td>
<td>0.4240</td>
</tr>
<tr>
<td>0.05</td>
<td>0.9998</td>
<td>2.0</td>
<td>0.2845</td>
</tr>
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<td>0.10</td>
<td>0.9990</td>
<td>2.5</td>
<td>0.1996</td>
</tr>
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<td>0.2</td>
<td>0.9925</td>
<td>3.0</td>
<td>0.1436</td>
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<tr>
<td>0.4</td>
<td>0.9488</td>
<td>4.0</td>
<td>0.0869</td>
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<tr>
<td>0.5</td>
<td>0.9106</td>
<td>5.0</td>
<td>0.0571</td>
</tr>
<tr>
<td>0.8</td>
<td>0.7562</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Vertical Stress Below any point of a Uniformly Loaded Circular Area

\[ \Delta \sigma_z = q(A^- + B^-) \]

Tables 10.8 & 10.9
Vertical Stress Below the Corner of a Uniformly Loaded Rectangular Area

\[ \Delta \sigma_z = \int d\sigma_z = \int_{y=0}^{B} \int_{x=0}^{L} \frac{3qz^3(dx \, dy)}{2\pi(x^2 + y^2 + z^2)^{5/2}} = qI_3 \]

\[ I_3 = \frac{1}{4\pi} \left[ \frac{2mn\sqrt{m^2 + n^2 + 1}}{m^2 + n^2 + m^2n^2 + 1} \left( \frac{m^2 + n^2 + 2}{m^2 + n^2 + 1} \right) \right. \\ + \left. \tan^{-1} \left( \frac{2mn\sqrt{m^2 + n^2 + 1}}{m^2 + n^2 - m^2n^2 + 1} \right) \right] \]

\[ m = \frac{B}{z} \quad n = \frac{L}{z} \]

\( I_3 \) is a dimensionless factor and represents the influence of a surcharge covering a rectangular area on the vertical stress at a point located at a depth \( z \) below one of its corner.
Table 10.10 Variation of $h_i$ with $m$ and $n$ [Eq. (10.32)]

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Newmark’s Influence Chart

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Step 1. Determine the depth $z$ below the uniformly loaded area at which the stress increase is required.

Step 2. Plot the plan of the loaded area with a scale of $z$ equal to the unit length of the chart ($AB$).

Step 3. Place the plan (plotted in step 2) on the influence chart in such a way that the point below which the stress is to be determined is located at the center of the chart.

Step 4. Count the number of elements $(M)$ of the chart enclosed by the plan of the loaded area.

$$\Delta \sigma_z = (IV)qM$$
Components of Settlement

\[ S_t = S_e + S_c + S_s \]

- \( S_t \) = Total settlement
- \( S_e \) = elastic (immediate) settlement
- \( S_c \) = Primary consolidation settlement
- \( S_s \) = Secondary consolidation settlement