

TOPICS

- ❑ Introduction
- ❑ Components of Shear Strength of Soils
- ❑ Normal and Shear Stresses on a Plane
- ❑ **Mohr-Coulomb Failure Criterion**
- ❑ Laboratory Shear Strength Testing
 - Direct Shear Test
 - Triaxial Compression Test
 - Unconfined Compression Test
- ❑ Field Testing (Vane test)

FAILURE CRITERIA FOR SOILS

- There are many ways of defining failure in real materials, or put another way, there are many **failure criteria**.
- Various theories are available for different engineering materials. However, no one is general for all materials
- The one generally accepted and used for soil is **Mohr Theory of Failure**.
- According to Coulomb relation for shear strength

$$\tau_f = C + \sigma_n \tan \phi$$

Where σ_n is the normal stress on the failure plane.

From the previous slides we now know how to estimate σ_n but we still need to know the plane of failure.

FAILURE CRITERIA FOR SOILS

What is failure Plane?

Is it the plane of a maximum shear stress **No**

Is it the plane of a maximum normal stress
(major principal stress) **No**

It is the plane when shear stress reaches some unique function of the normal stress on that plane: $\tau_f = f(\sigma_f)$

It is the plane , where the failure angle measured

From the plane of the major principal stress is

$$\theta_f = 45 + \frac{\phi}{2}$$

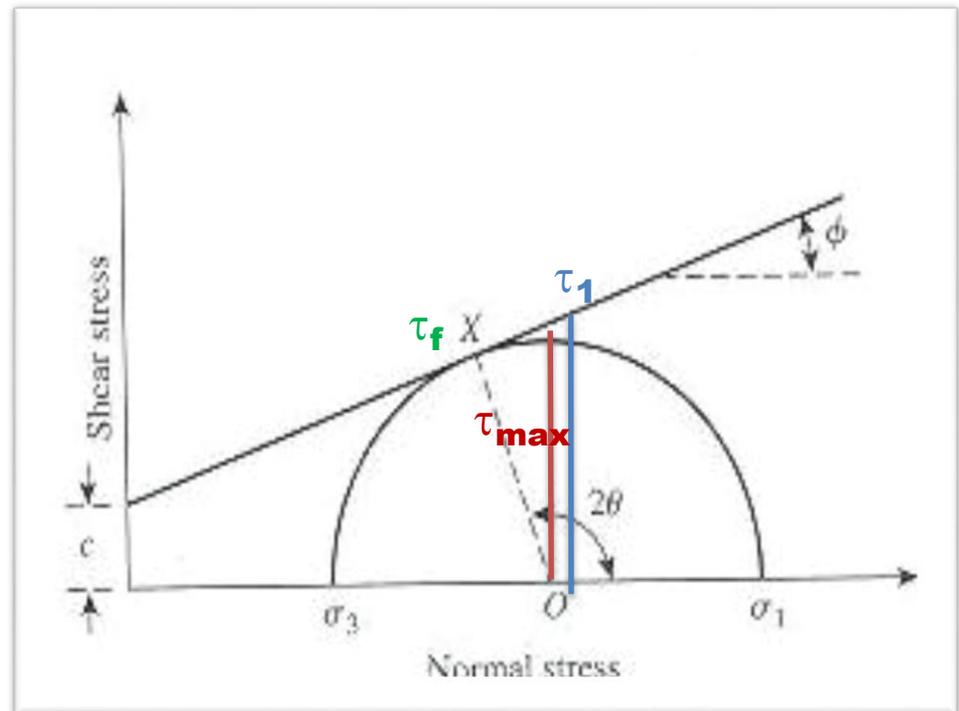
FAILURE CRITERIA FOR SOILS

Inclination of failure Plane due to shear

τ_f is **not** the maximum shear stress Why?

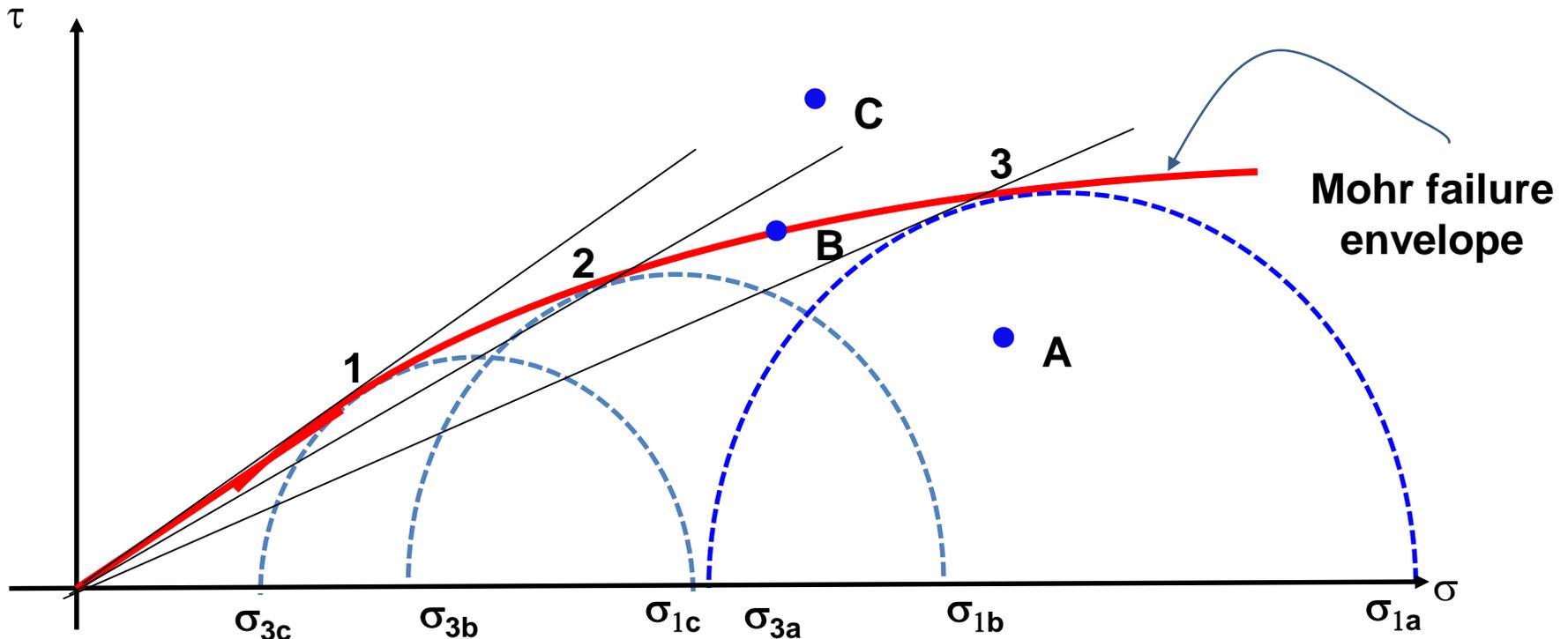
$$F_s = \frac{\tau_{\text{available}}}{\tau_{\text{applied}}}$$

$$F_s = \frac{\tau_1}{\tau_{\text{max}}}$$



Mohr Failure Envelope

- By plotting Mohr's circles for different states of stresses and in each case draw a tangent to each circle from the **origin** we come up with points 1,2,3... etc. If we connect those points we come up with what is called **Mohr's failure envelope**.



Mohr Failure Envelope

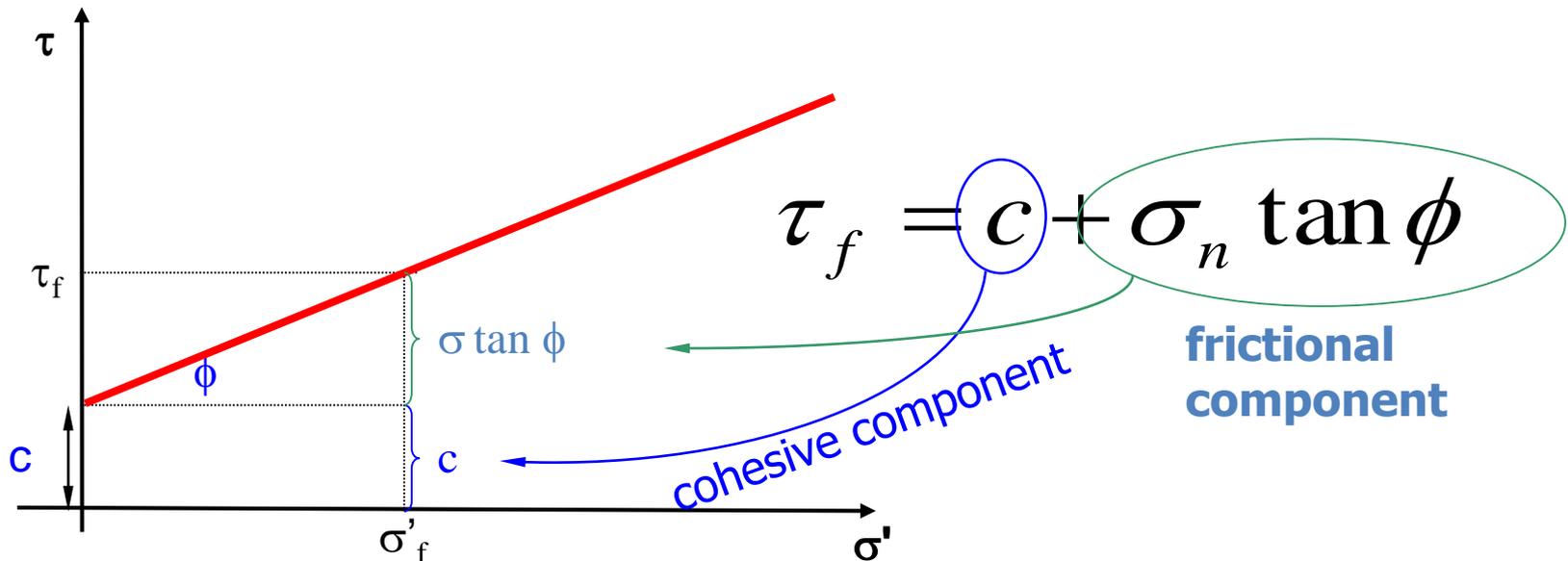
- This envelope separates cases of stresses which cause failure from that which do not.
- For instance if the normal stress and shear stress on a plane in a soil mass are such that they plot as point **A** , shear failure **will not occur** along that plane.
- For point **B** failure takes place, and point **C** cannot exist, since it plots above failure envelope and shear failure in a soil would have occurred already.
- Therefore, failure occurs only when the combination of shear and normal stress is such that the Mohr circle is **TANGENT** to the Mohr failure envelope.
- Then once the point of the tangency is determined the angle of failure plane and the stresses $(\sigma_n, \tau)_f$ can be determined using the **POLE** method.

Mohr-Coulomb Failure Criterion

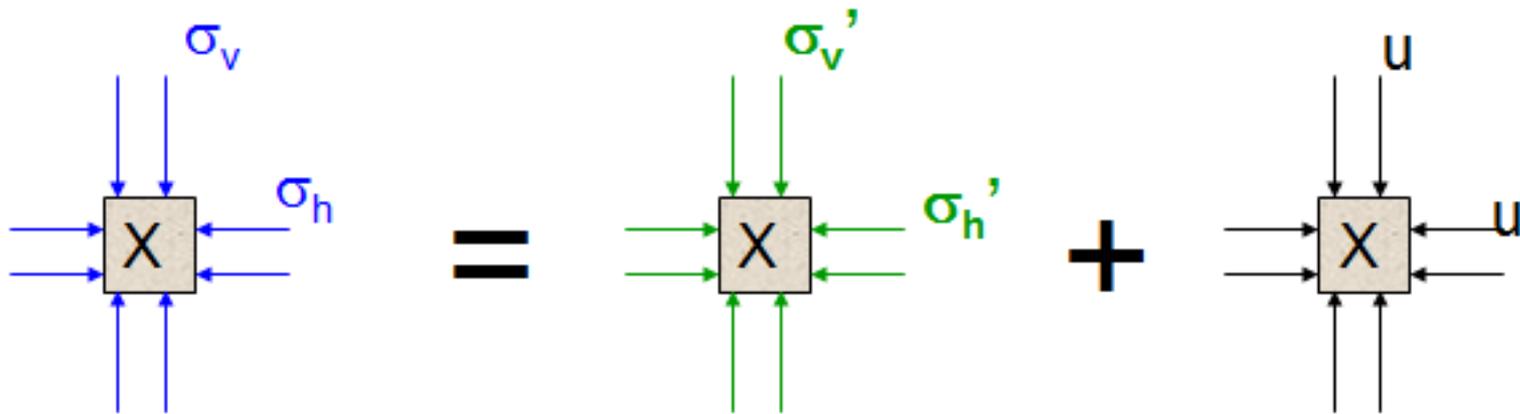
- The disadvantages of **Mohr failure envelope** is that it is a curved line, and needs a lot of tests to construct and difficult to use.
- It was then approximated to be a straight line, and the equation for the line was written in terms of the Coulomb strength parameters C and ϕ , as

$$\tau_f = C + \sigma_n \tan\phi$$

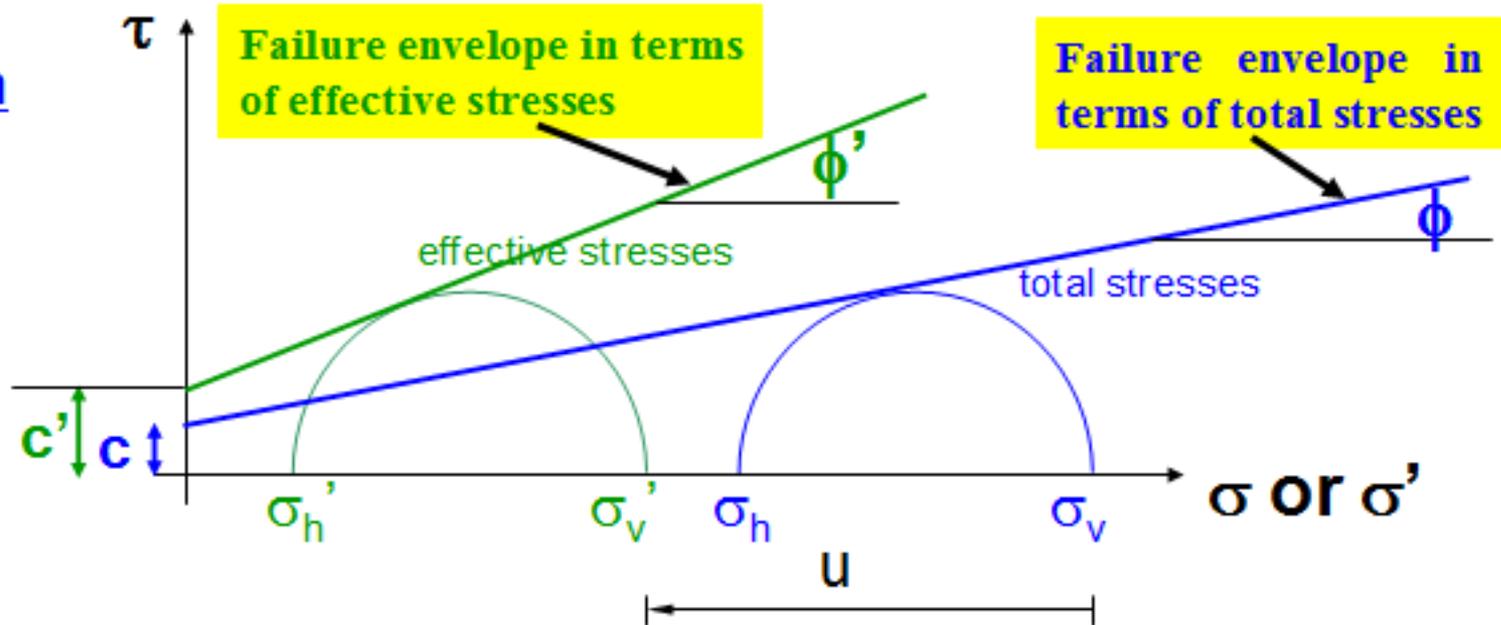
- This gave the birth to **MOHR-COULOMB FAILURE CRITERION**, which is by far the most popular criterion applied to soils.



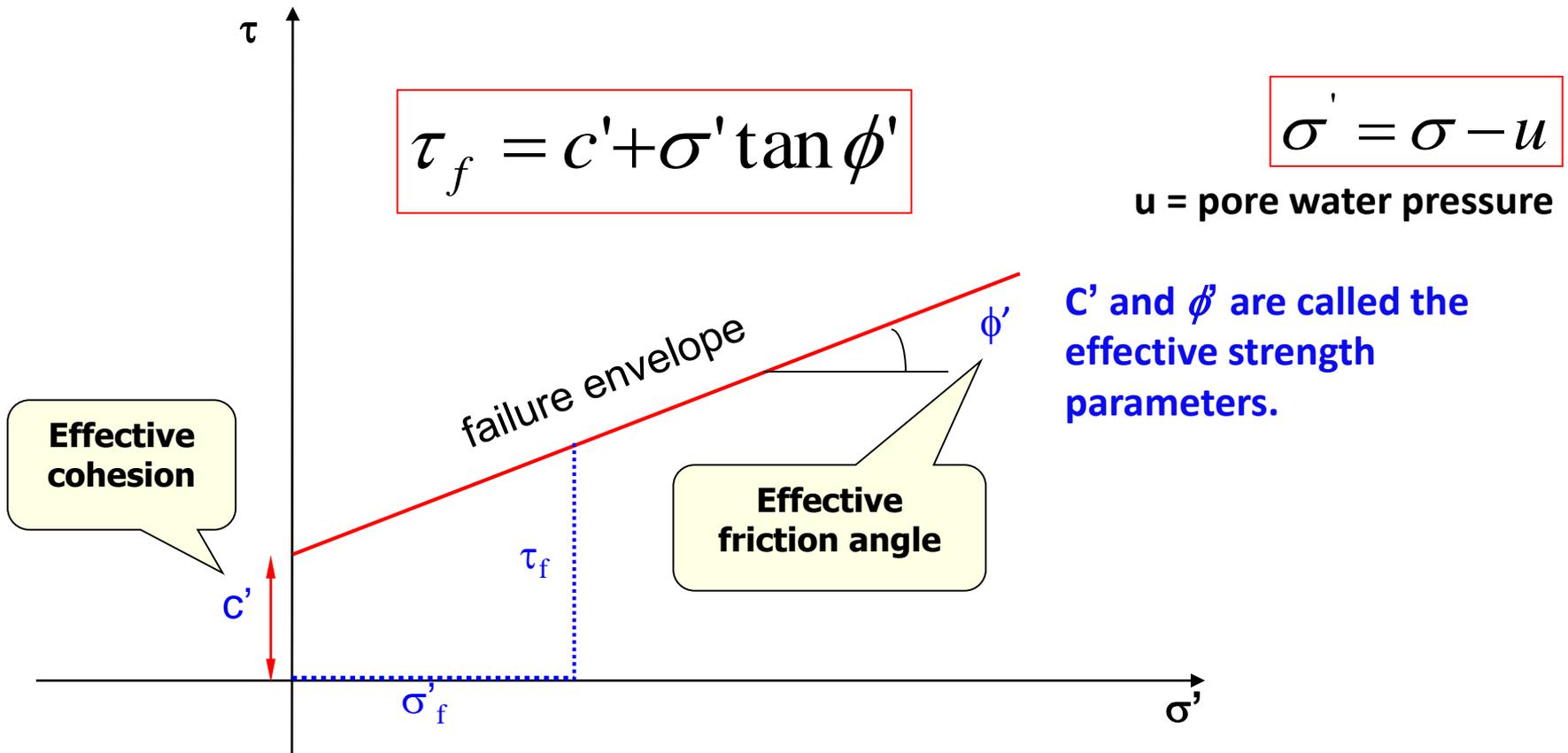
Mohr circles & failure envelope in terms of total and effective stress



If X is on failure

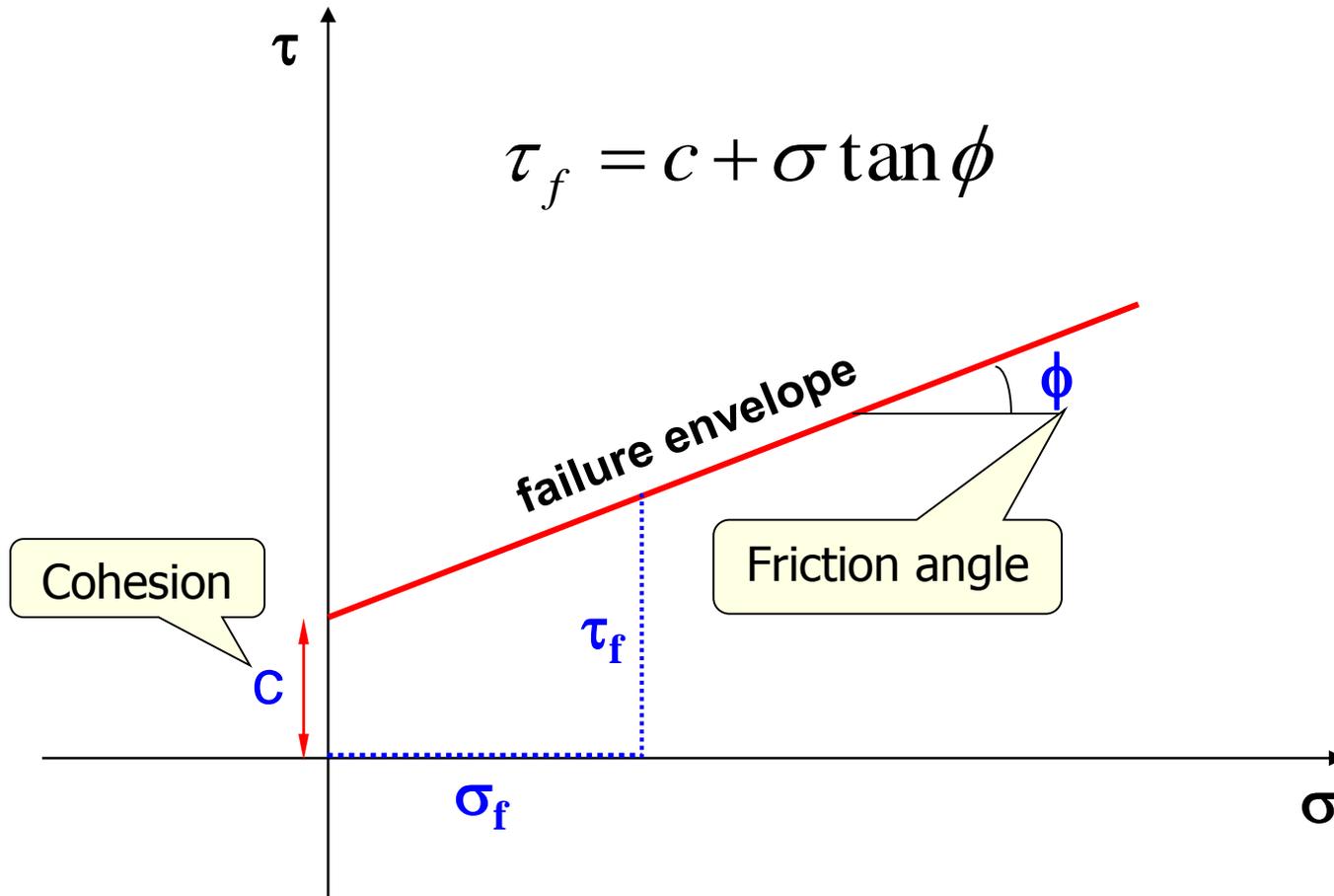


Mohr-Coulomb Failure Criterion in terms of effective stress



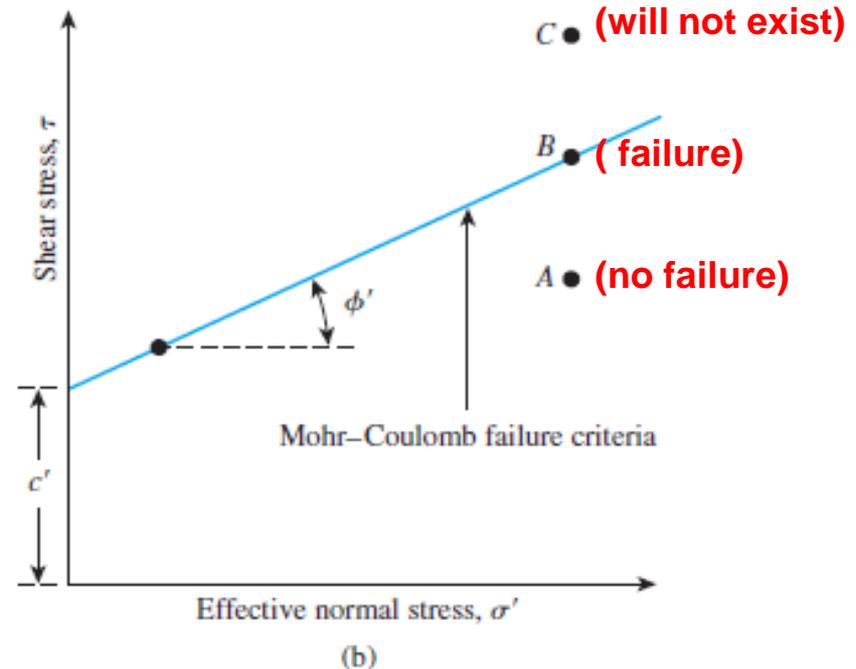
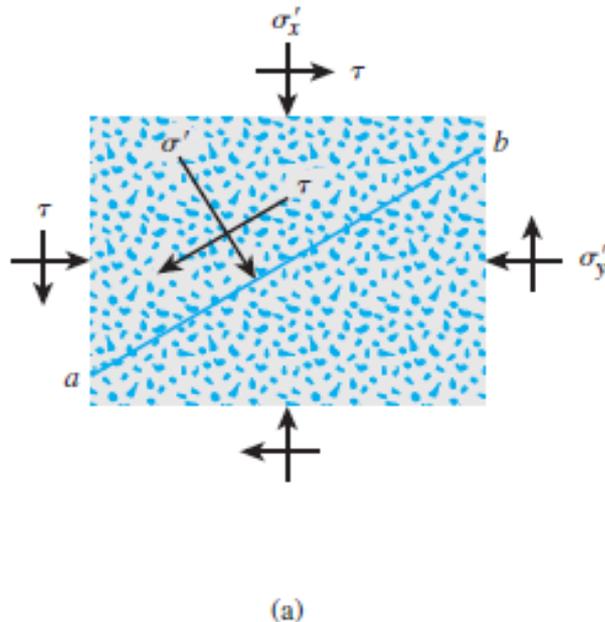
In this case, soil behavior is controlled by effective stresses, and the effective strength parameters are the fundamental strength parameters.

Mohr-Coulomb Failure Criterion in terms of total stress



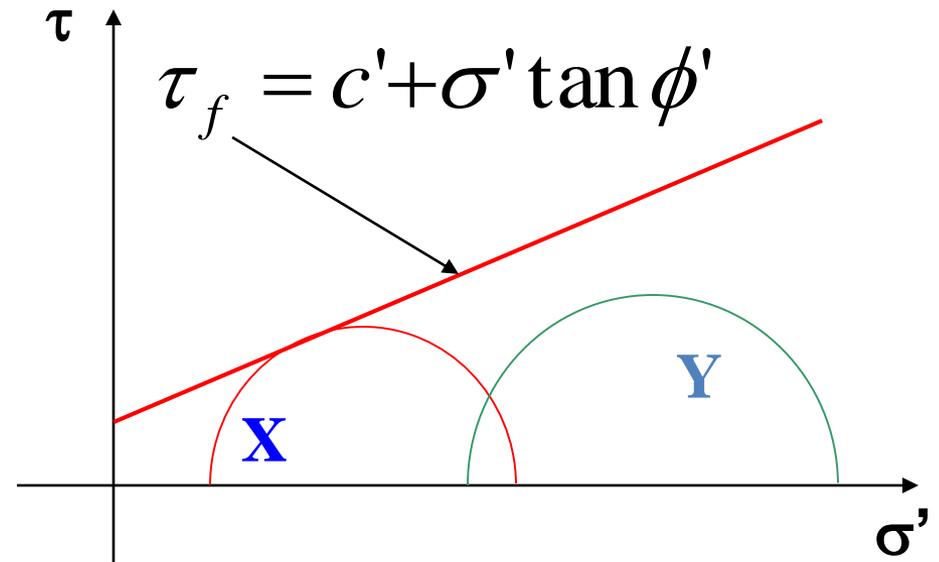
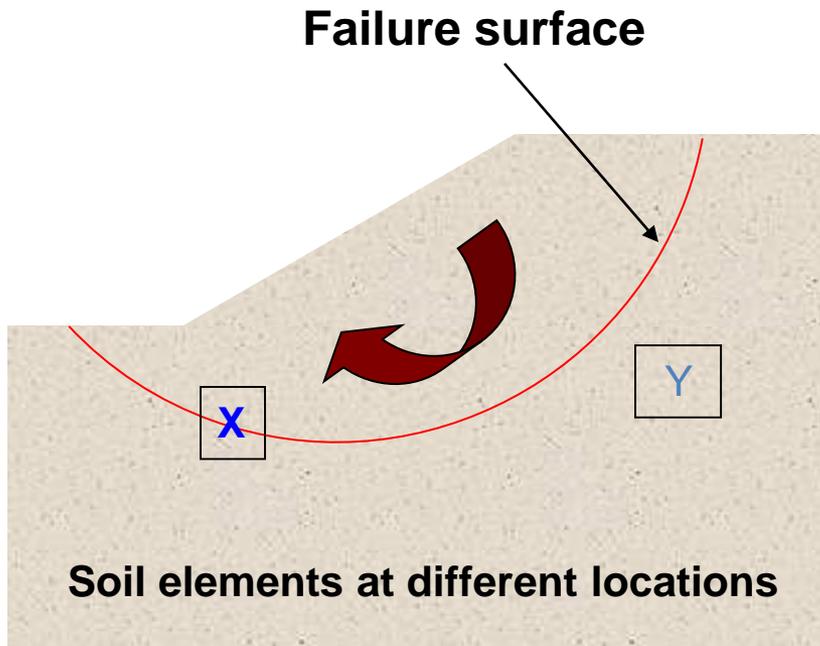
τ_f is the maximum shear stress the soil can take without failure, under normal stress of σ .

What does the Failure Envelope Signify?



- If the magnitudes of σ and τ on plane ab are such that they plot as point **A** shear failure will not occur along the plane.
- If the effective normal stress and the shear stress on plane ab plot as point **B** (which falls on the failure envelope), shear failure will occur along that plane.
- A state of stress on a plane represented by point **C** cannot exist, because it plots above the failure envelope, and shear failure in a soil would have occurred already.

Mohr Circles & Failure Envelope

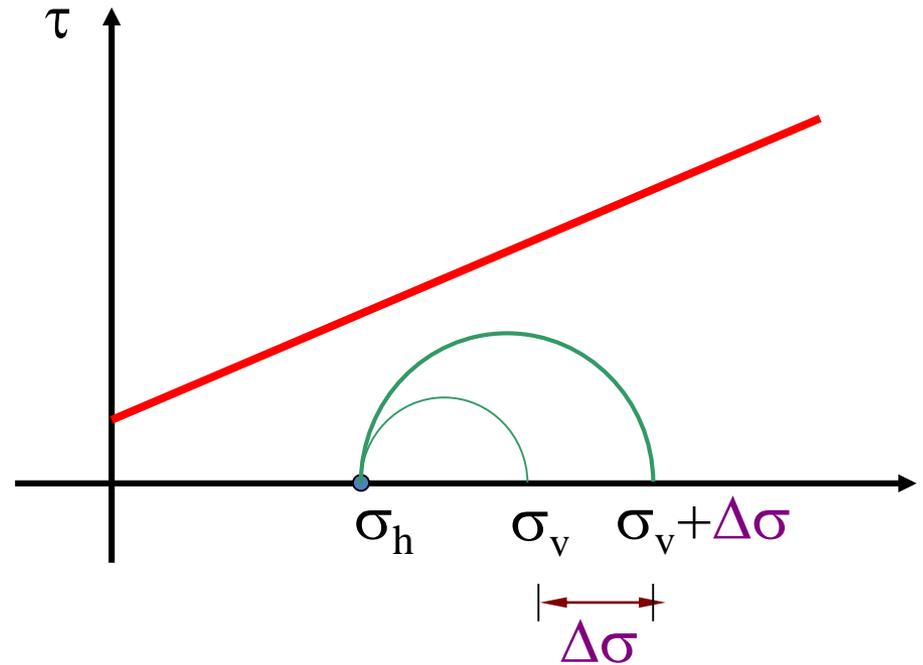
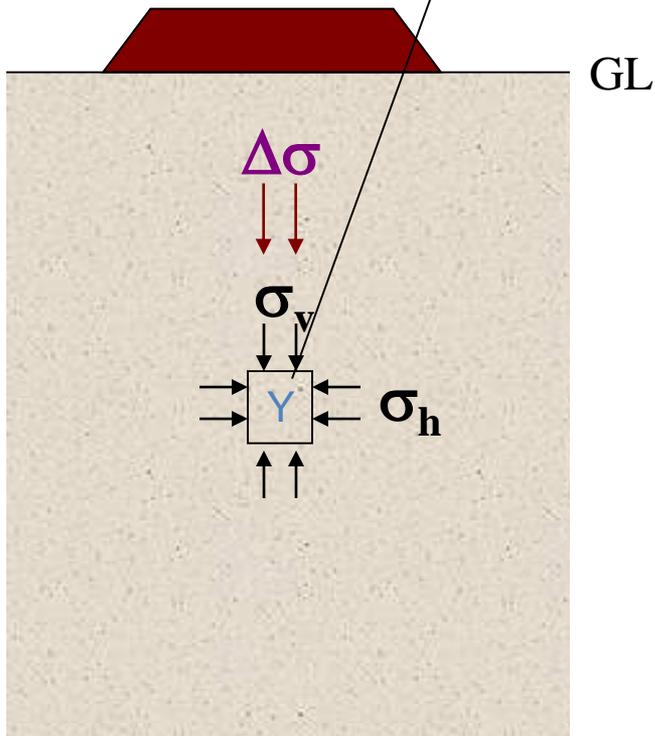


Y \sim stable

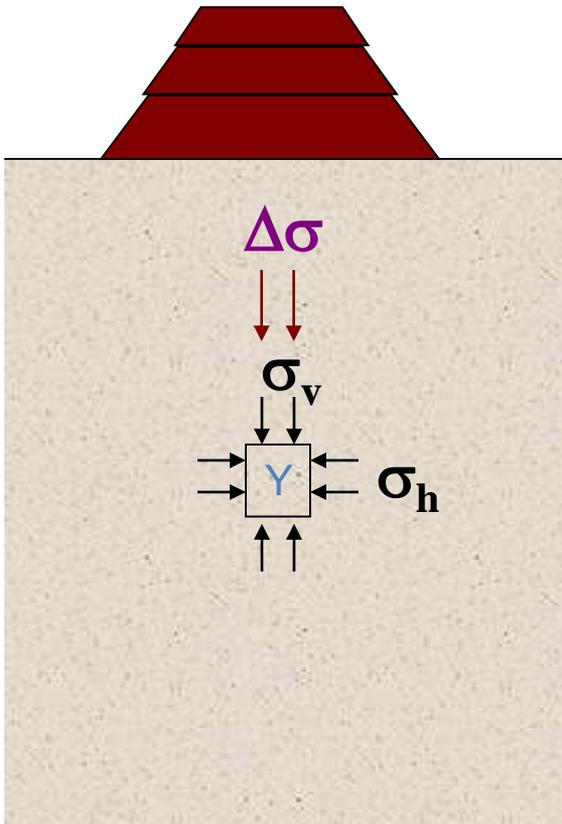
X \sim failure

Mohr Circles & Failure Envelope

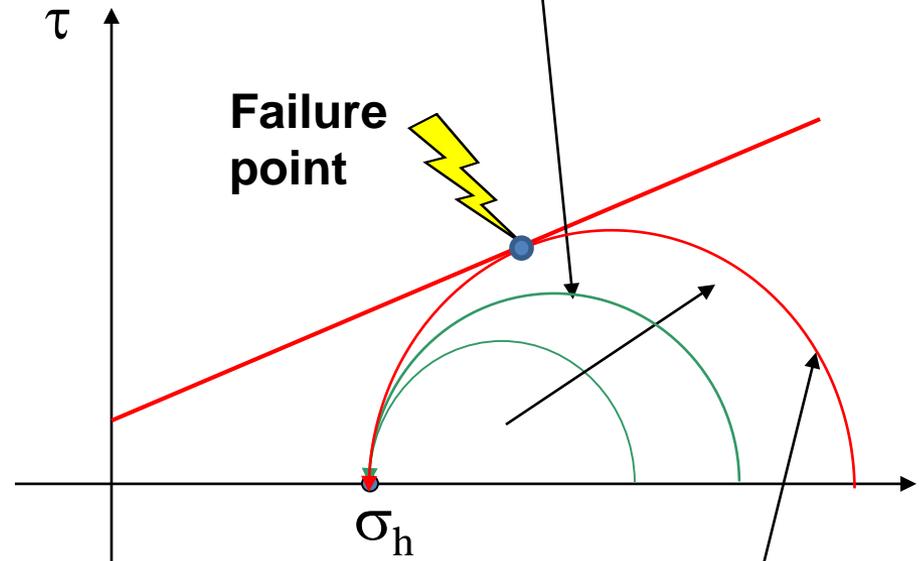
The soil element does not fail if the Mohr circle is contained within the envelope



Mohr Circles & Failure Envelope

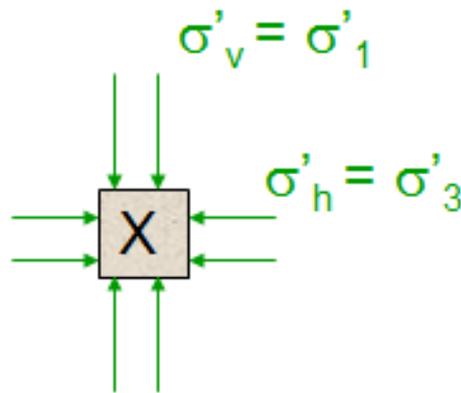


As loading progresses, Mohr circle becomes larger...

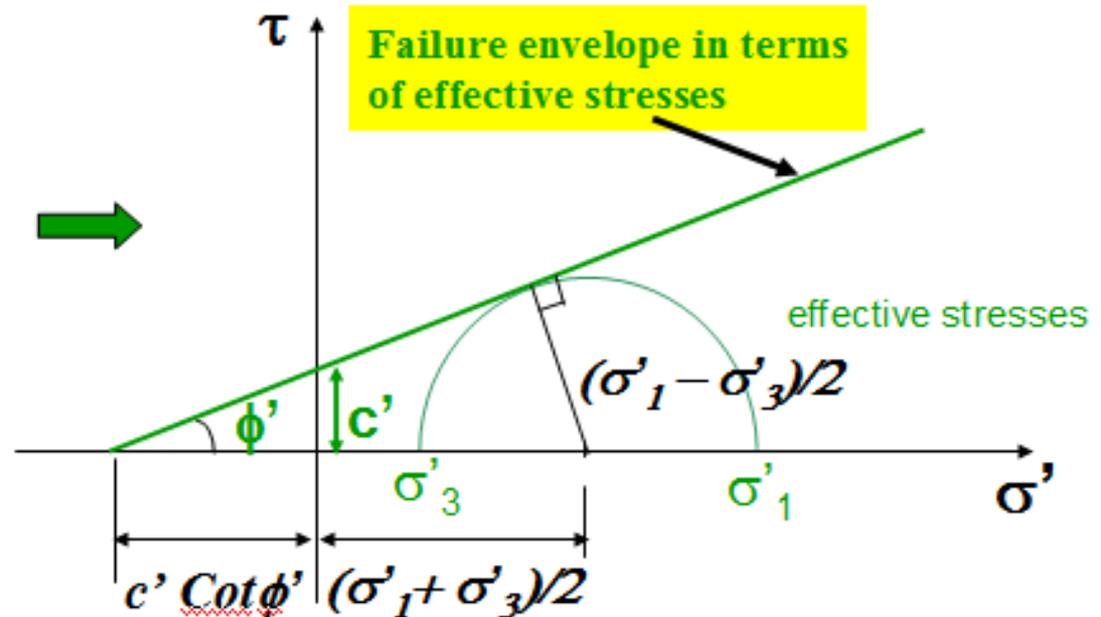


.. and finally failure occurs when Mohr circle touches the envelope

Relationship between principle stresses and shear strength parameters



X is on failure



Therefore,

$$\left[c' \cot \phi' + \left(\frac{\sigma'_1 + \sigma'_3}{2} \right) \right] \sin \phi' = \left(\frac{\sigma'_1 - \sigma'_3}{2} \right)$$

Relationship between principle stresses and shear strength parameters

$$\left[c' \cot \phi' + \left(\frac{\sigma_1' + \sigma_3'}{2} \right) \right] \sin \phi' = \left(\frac{\sigma_1' - \sigma_3'}{2} \right)$$

$$(\sigma_1' - \sigma_3') = (\sigma_1' + \sigma_3') \sin \phi' + 2c' \cos \phi'$$

$$\sigma_1'(1 - \sin \phi') = \sigma_3'(1 + \sin \phi') + 2c' \cos \phi'$$

$$\sigma_1' = \sigma_3' \frac{(1 + \sin \phi')}{(1 - \sin \phi')} + 2c' \frac{\cos \phi'}{(1 - \sin \phi')}$$

$$\sigma_1' = \sigma_3' \tan^2 \left(45 + \frac{\phi'}{2} \right) + 2c' \tan \left(45 + \frac{\phi'}{2} \right)$$

REMARKS

- C and ϕ are measures of shear strength and are called shear strength parameters.
- The parameters C , ϕ are in general not soil constants. They depend on the initial state of the soil (e.g. density, water content), and type of loading (drained or undrained).
- In case of using effective stress, C' and ϕ' are called effective shear strength parameters.
- The value of C' for sand and inorganic silt is 0. For **normally** consolidated clays, C' can be approximated at 0. **Overconsolidated** clays have values of C' that are greater than 0.

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Laboratory Shear Strength Testing

- The purpose of laboratory testing is to determine the shear strength parameters of soil (C , ϕ or C' , ϕ') through the determination of failure envelope.
- The shear strength parameters for a particular soil can be determined by means of laboratory tests on specimens taken from representative samples of the in-situ soil.
- There are **several** laboratory methods available to determine the shear strength parameters (i.e. C , ϕ , C' , ϕ'). Some of the tests are rather complicated .
- For further details you should consult manuals and books on laboratory testing, especially those by the **ASTM**.

Determination of shear strength parameters of soils

Determination of shear strength parameters of soils (c , ϕ or c' , ϕ')

Laboratory Tests

Most common laboratory tests to determine the shear strength parameters are,

1. Direct shear test
2. Triaxial shear test

Other laboratory tests include:

- Direct simple shear test
- Torsional or ring shear test
- Hollow cylinder test
- Plane strain triaxial test
- Laboratory vane shear test
- Laboratory fall cone test.

Field Tests

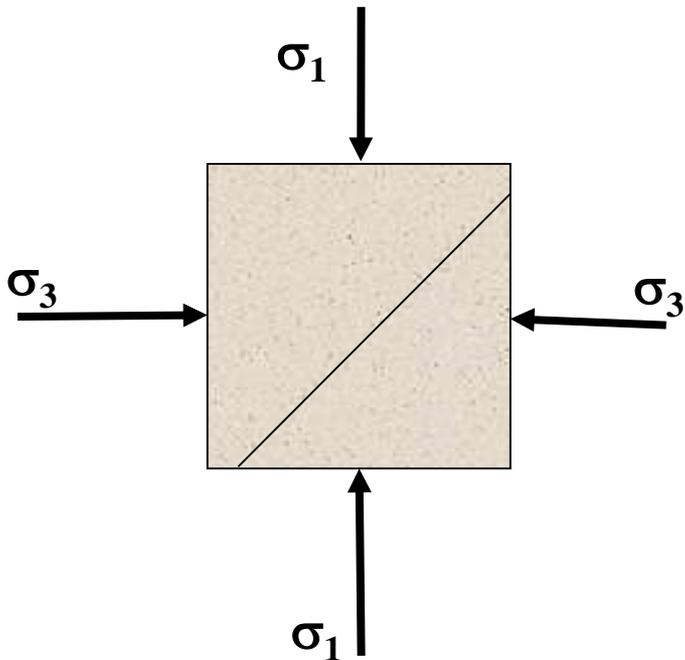
1. Standard penetration test
Pressuremeter
2. Vane shear test
3. Pocket penetrometer
4. Static cone penetrometer

Field test equipment and test methods are described in most textbooks on foundation engineering.

Laboratory Shear Strength Testing

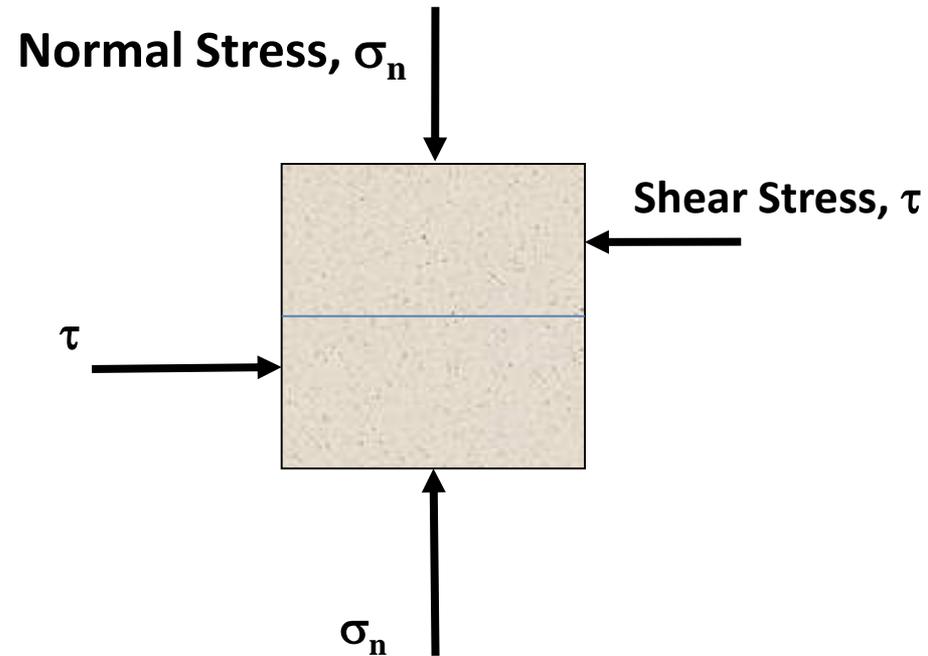
- In laboratory, a soil sample can fail in two ways:

Way 1: Increase normal stress (σ_1) to failure with confining stress (σ_3) constant



Triaxial Shear Test

Way 2: Normal stress is applied and held constant then shear stress is applied to failure

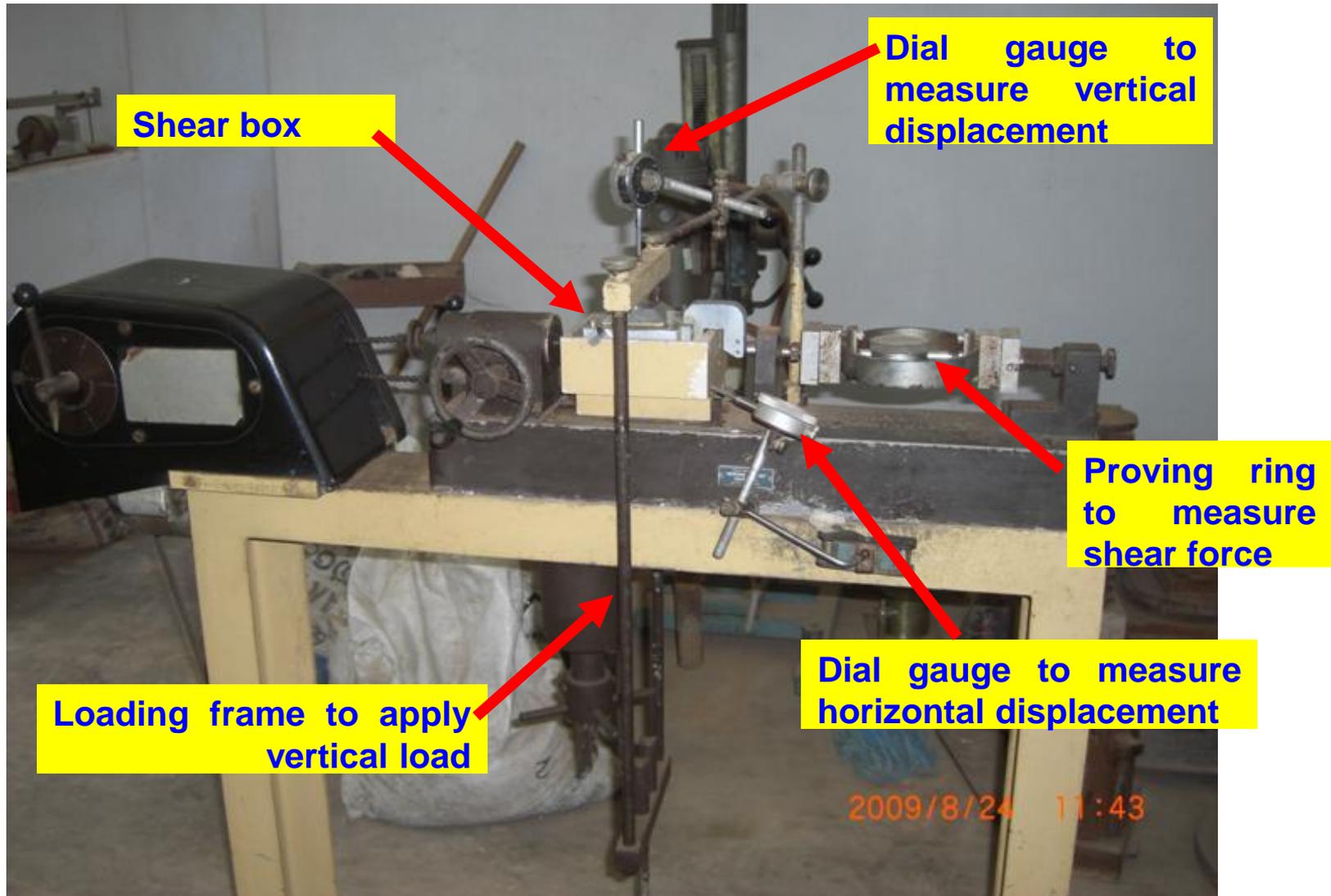


Direct Shear Test

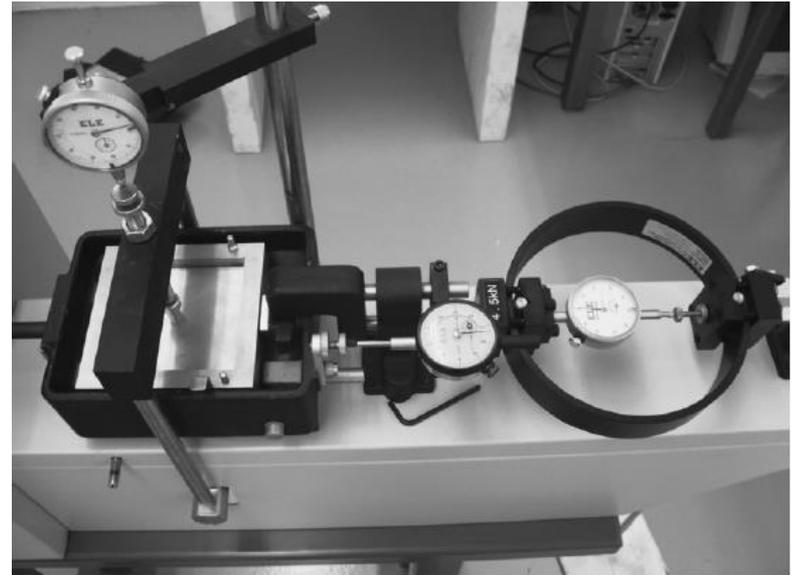
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Direct Shear Test

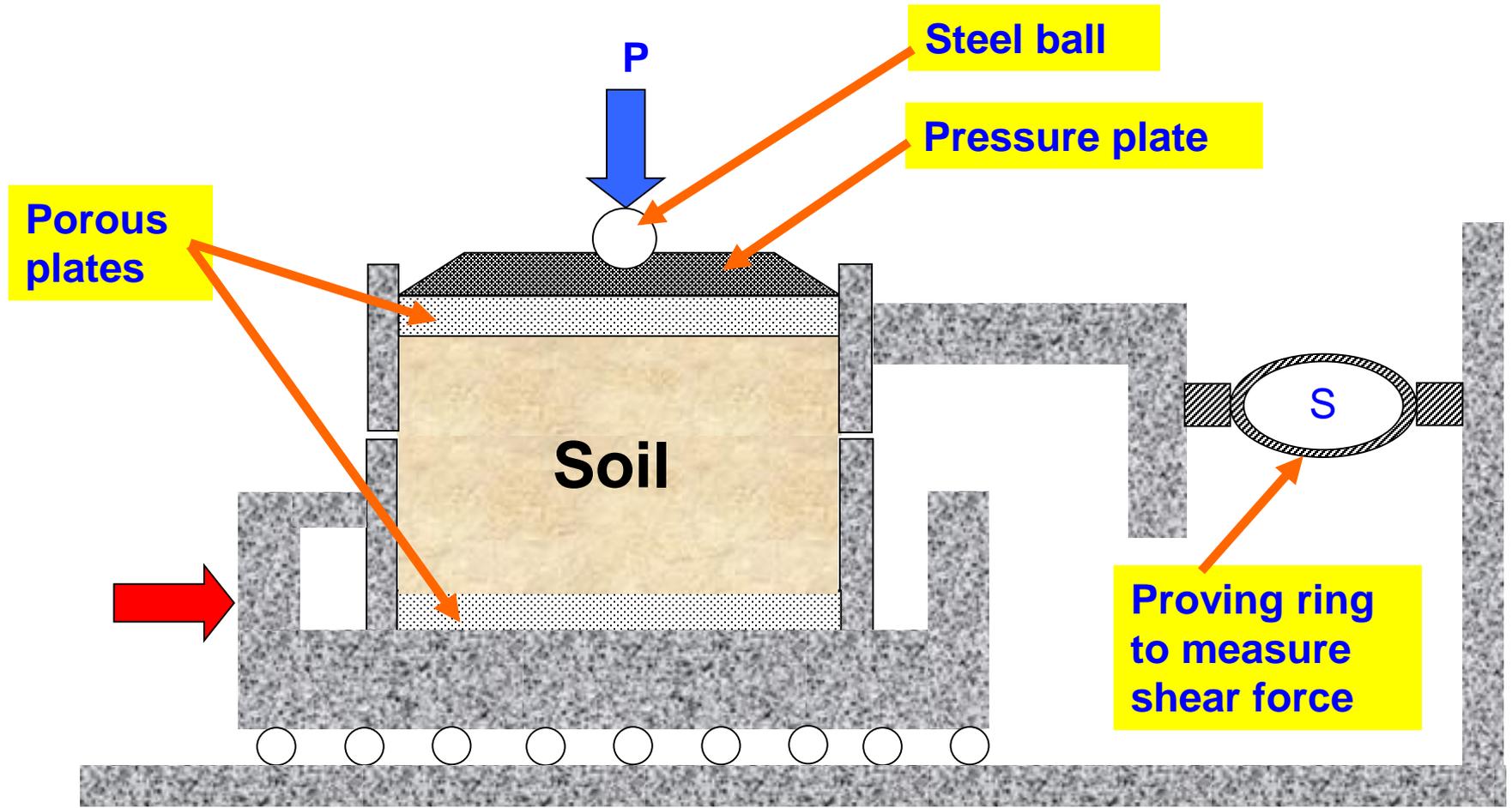


Direct Shear Test

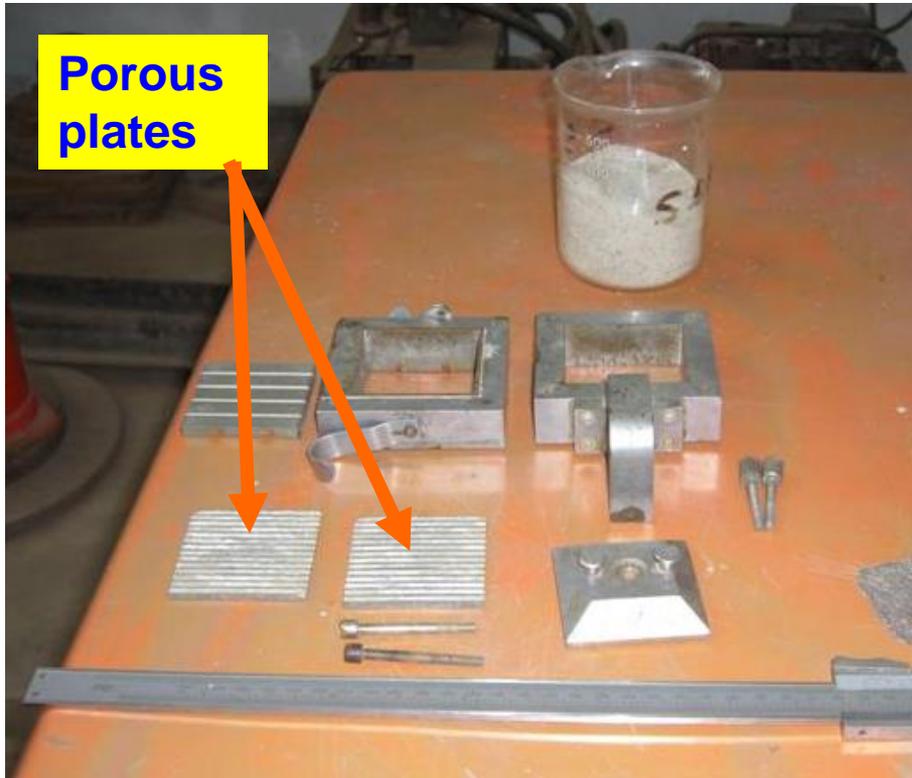


- The advantage of the **strain-controlled** tests is that in the case of dense sand, peak shear resistance (that is, at failure) as well as lesser shear resistance (that is, at a point after failure called ultimate strength) can be observed and plotted.
- Compared with strain-controlled tests, stress-controlled tests probably model real field situations better.

Direct Shear Test



Preparation of a sand specimen



Components of the shear box

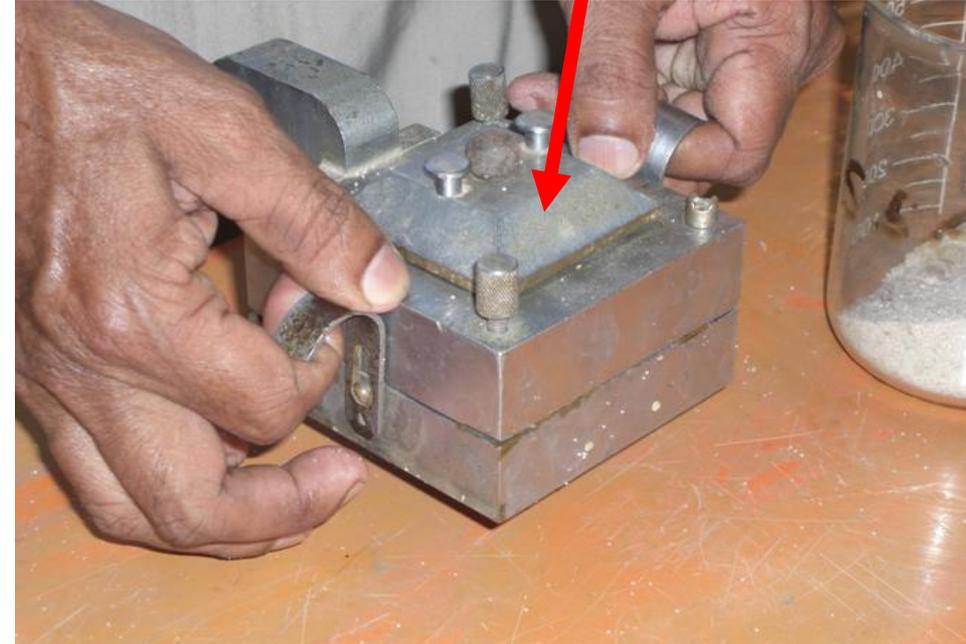


Preparation of a sand specimen

Preparation of a sand specimen



Leveling the top surface of specimen



Pressure plate

Specimen preparation completed

Test Procedure

- A **constant** vertical force (normal stress) is applied through a metal platen.
- Shear force is applied by **moving** one half of the box **relative** to the other and increased to cause failure in the soil sample.
- The tests are **repeated** on similar specimens at **various** normal stresses
- The normal stresses and the corresponding values of τ_f obtained from a number of tests are **plotted** on a graph from which the **shear strength parameters** are determined.

TEST RESULTS

Normal Load : _____ kg

Area of Sample: _____ cm²

Horizontal Dial Reading (mm)	Vertical Dial Reading (mm)	Horizontal Shear Force (N)	Shear Stress (kPa)

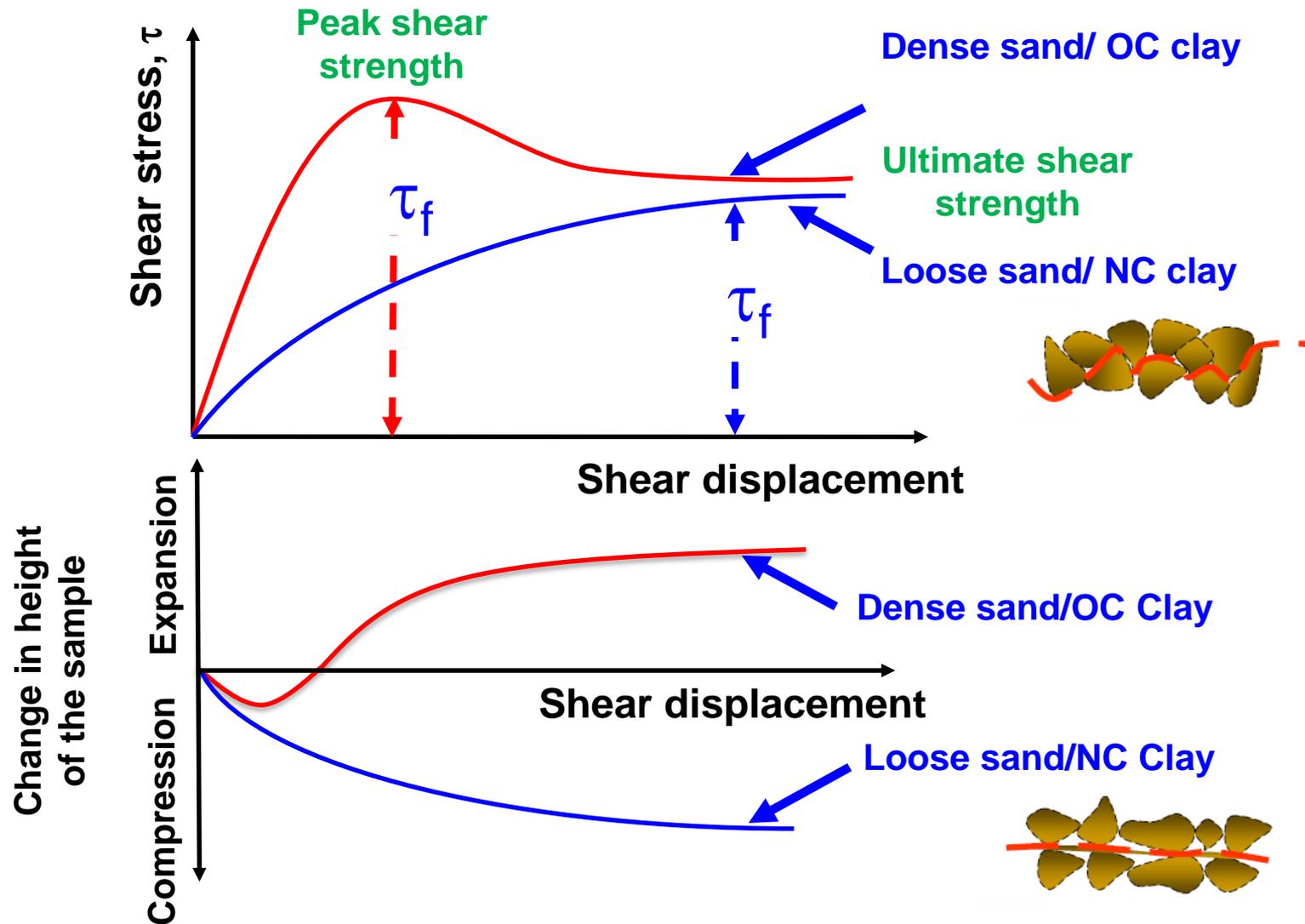
For a given test, the normal stress can be calculated as

$$\sigma = \text{Normal stress} = \frac{\text{Normal force}}{\text{Cross-sectional area of the specimen}}$$

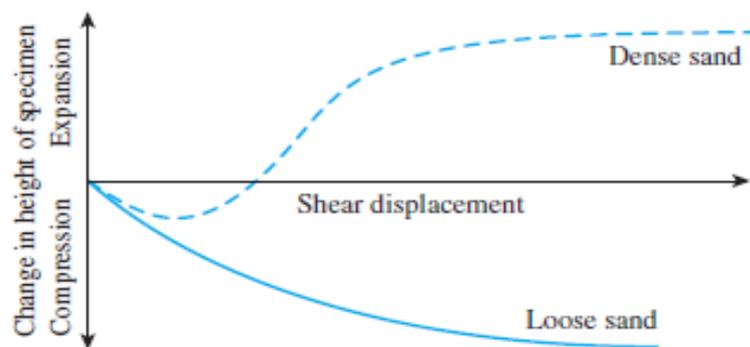
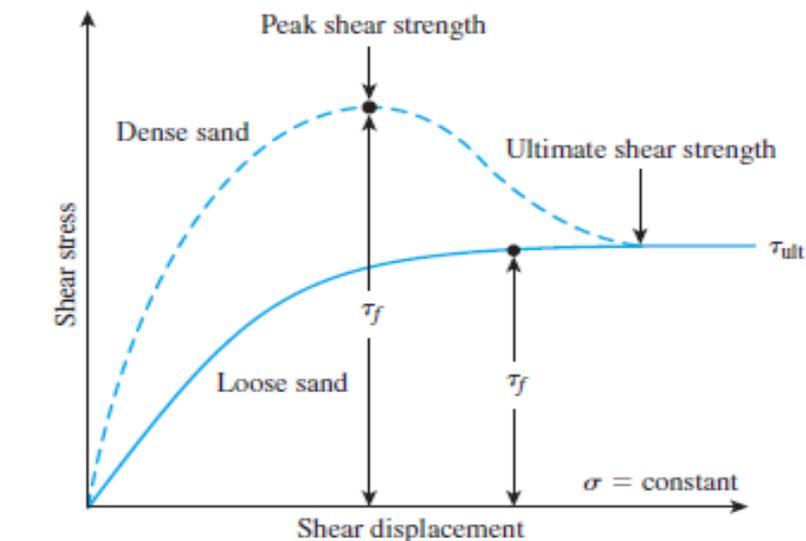
The resisting shear stress for any shear displacement can be calculated as

$$\tau = \text{Shear stress} = \frac{\text{Resisting shear force}}{\text{Cross-sectional area of the specimen}}$$

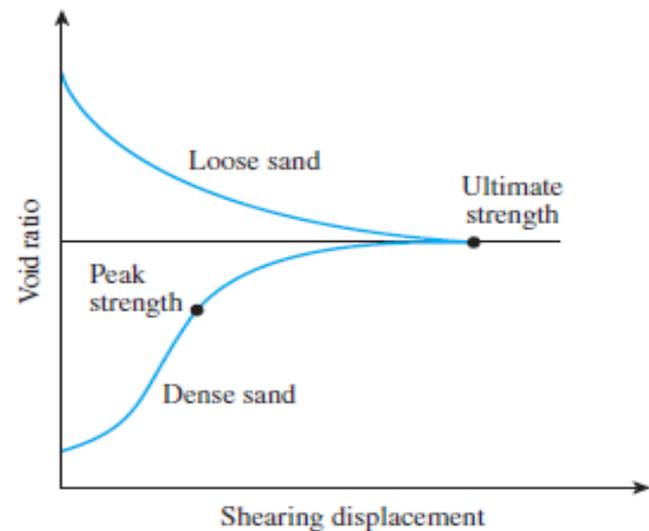
Stress-Strain Relationship



Stress-Strain Relationship



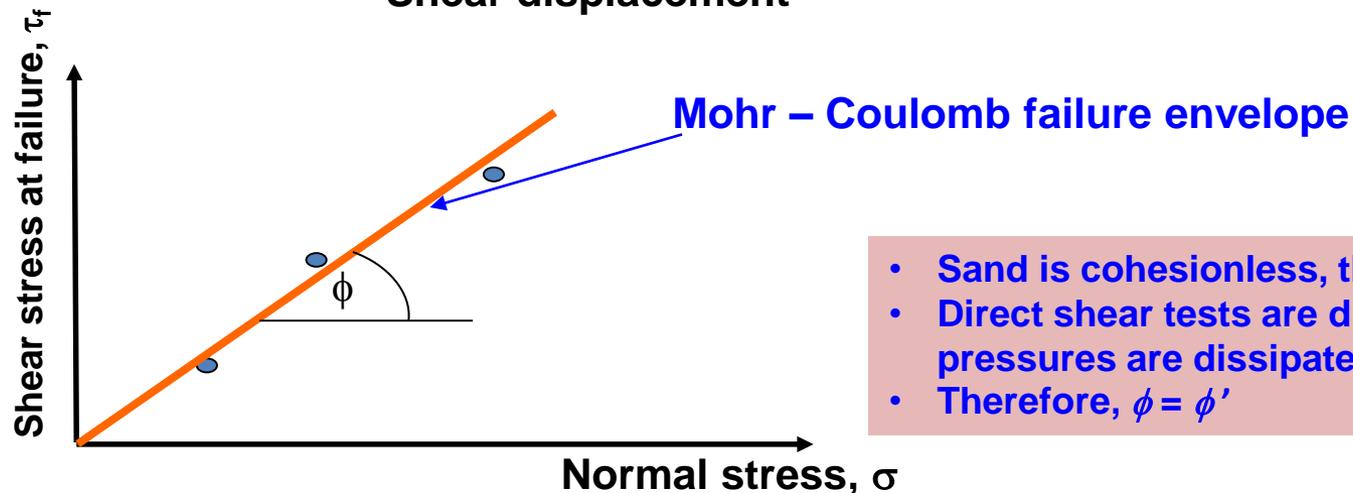
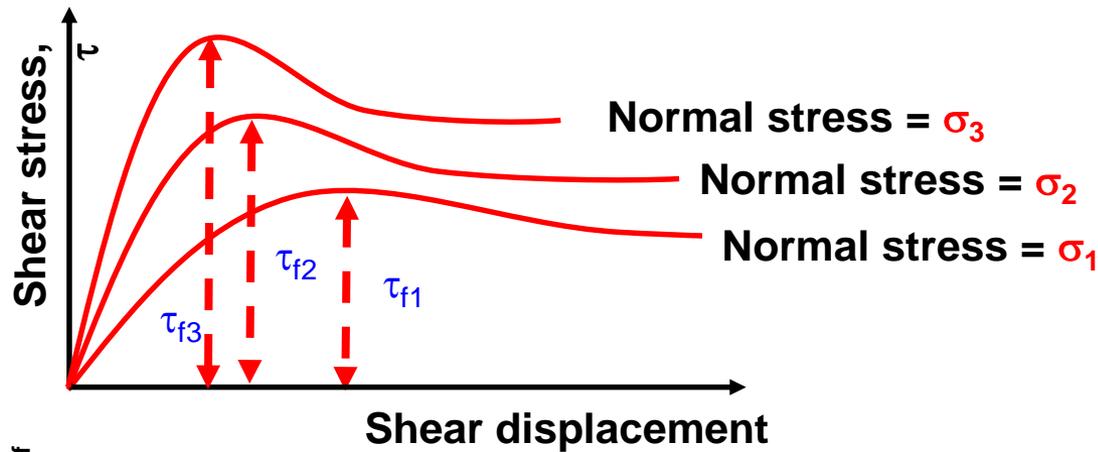
Plot of shear stress and change in height of specimen against shear displacement for loose and dense dry sand (direct shear test)



Nature of variation of void ratio with shearing displacement

Determining strength parameters C and ϕ

An example of testing **three** samples of a sand at the same relative density just before shearing.

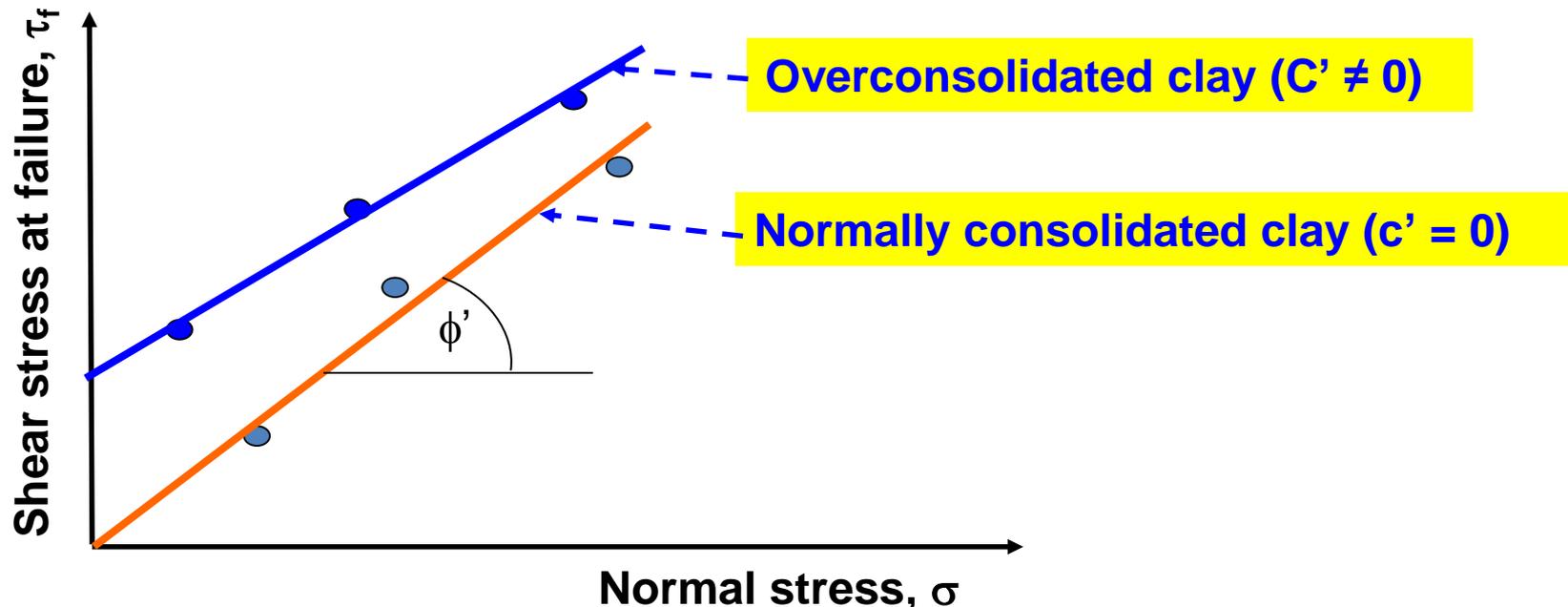


- Sand is cohesionless, then $C = 0$
- Direct shear tests are drained and pore water pressures are dissipated, then $u = 0$
- Therefore, $\phi = \phi'$

Determining strength parameters C and ϕ

In case of **clay**, horizontal displacement should be applied at a **very slow rate** to allow dissipation of pore water pressure (therefore, one test would take **several days** to finish)

Failure envelopes for clay from drained direct shear tests



Notes on Direct Shear Test

- This test is probably the **oldest** strength test because **Coulomb** used a type of shear box test more than two centuries ago to determine the necessary parameters for his strength equation.
- The test is quick and inexpensive and common in practice.
- Used to determine the shear strength of both cohesive as well as non-cohesive soils.
- The test equipment consists of a metal box in which the soil specimen is placed. The box is split horizontally into two halves.
- The shear test can be either stress controlled or strain controlled.
- Tests on sands and gravels are usually performed dry. Water does not significantly affect the (drained) strength.

Notes on Direct Shear Test

- Usually only relatively slow drained tests are performed in shear box apparatus. For clays rate of shearing must be chosen to prevent excess pore pressures building up. For sands and gravels tests can be performed quickly.
- If there are no excess pore pressures and as the pore pressure is approximately zero the total and effective stresses will be identical.
- The failure stresses thus define an effective stress failure envelope from which the effective (drained) strength parameters C' , ϕ' can be determined.
- Normally consolidated clays ($OCR = 1$) and loose sands do not show separate peak and ultimate failure loci, and for soils in these states $C' = 0$.
- Overconsolidated clays and dense sands have peak strengths with $C' > 0$.

Advantages of direct shear test

- Inexpensive, fast, and simple, especially for granular materials.
- Easiness of sample preparation in case of sand.
- Due to the smaller thickness of the sample, rapid drainage can be achieved
- Large deformations can be achieved by reversing shear direction. This is useful for determining the residual strength of a soil.
- Samples may be sheared along predetermined planes. This is useful when the shear strengths along fissures or an interface is required.

Disadvantages of direct shear test

- Failure occurs along a **predetermined failure plane** which may **not be the weakest plane**.
- **Non-uniform of shear stresses** along failure surface in the specimen. There are rather stress concentrations at the sample boundaries, which lead to highly nonuniform stress conditions within the test specimen itself.
- There is no means of estimating pore pressures, so **effective stresses cannot be determined** and only the total normal stress can be determined.
- It is very difficult if not impossible to **control drainage**, especially for fine-grained soils. Consequently, the test is not suitable for other than completely drained conditions.

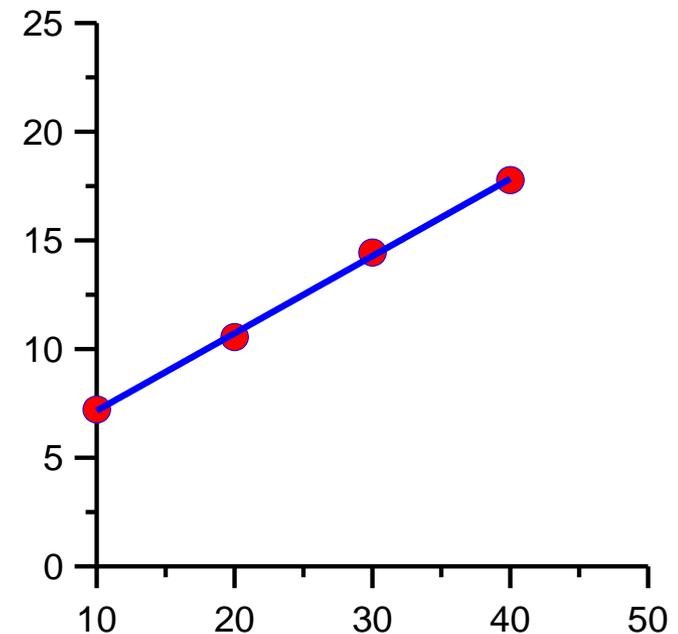
Example

A direct shear test was performed on a **clay** sample. The cross section area of the device is 6 cm x 6 cm.

Normal Load (kg)	Shear Load (kg)
360	260
720	380
1080	520
1440	640

Determine the shear envelope and shear strength parameters for the clay.

Normal Load (kg)	Normal Stress (kg/cm ²)	Shear Load at failure (kg)	Shear Stress at failure (kg/cm ²)
360	10	260	7.22
720	20	380	10.56
1080	30	520	14.44
1440	40	640	17.78



Example 12.1

Example 12.1

Direct shear tests were performed on a dry, sandy soil. The size of the specimen was 2 in. \times 2 in. \times 0.75 in. Test results are as follows:

Test no.	Normal force (lb)	Normal ^a stress $\sigma = \sigma'$ (lb/ft ²)	Shear force at failure (lb)	Shear stress ^b at failure τ_f (lb/ft ²)
1	20	720	12.0	432.0
2	30	1080	18.3	658.8
3	70	2520	42.1	1515.6
4	100	3600	60.1	2163.6

$${}^a\sigma' = \frac{\text{normal force}}{\text{area of specimen}} = \frac{(\text{normal force}) \times 144}{(2 \text{ in.})(2 \text{ in.})}$$

$${}^b\tau_f = \frac{\text{shear force}}{\text{area of specimen}} = \frac{(\text{shear force}) \times 144}{(2 \text{ in.})(2 \text{ in.})}$$

Find the shear stress parameters.

Solution

The shear stresses, τ_f , obtained from the tests are plotted against the normal stresses in Figure 12.18, from which $c' = 0$, $\phi' = 32^\circ$.

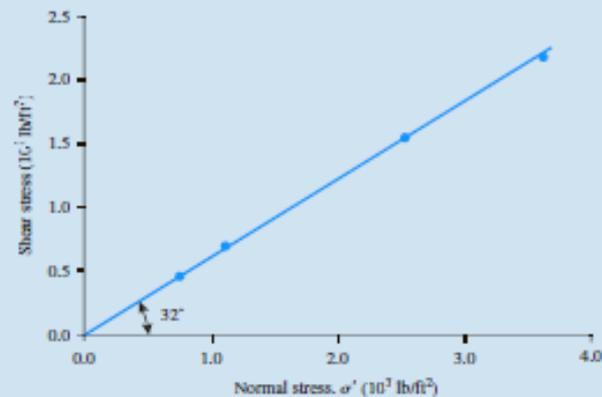


Figure 12.18

Example 12.2

Example 12.2

Following are the results of four drained direct shear tests on an *overconsolidated clay*:

Diameter of specimen = 50 mm

Height of specimen = 25 mm

Test no.	Normal force, N (N)	Shear force at failure, S_{peak} (N)	Residual shear force, $S_{residual}$ (N)
1	150	157.5	44.2
2	250	199.9	56.6
3	350	257.6	102.9
4	550	363.4	144.5

Determine the relationships for *peak shear strength* (τ_f) and *residual shear strength* (τ_r).

Solution

Area of the specimen (A) = $(\pi/4) \left(\frac{50}{1000} \right)^2 = 0.0019634 \text{ m}^2$. Now the following table can be prepared.

Test no.	Normal force, N (N)	Normal stress, σ' (kN/m ²)	Peak shear force, S_{peak} (N)	$\tau_f = \frac{S_{peak}}{A}$ (kN/m ²)	Residual shear force, $S_{residual}$ (N)	$\tau_r = \frac{S_{residual}}{A}$ (kN/m ²)
1	150	76.4	157.5	80.2	44.2	22.5
2	250	127.3	199.9	101.8	56.6	28.8
3	350	178.3	257.6	131.2	102.9	52.4
4	550	280.1	363.4	185.1	144.5	73.6

Example 12.2

The variations of τ_p and τ_r with σ' are plotted in Figure 12.19. From the plots, we find that

$$\text{Peak strength: } \tau_p(\text{kN/m}^2) = 40 + \sigma' \tan 27$$

$$\text{Residual strength: } \tau_r(\text{kN/m}^2) = \sigma' \tan 14.6$$

(Note: For all *overconsolidated clays*, the residual shear strength can be expressed as

$$\tau_r = \sigma' \tan \phi'_r$$

where ϕ'_r = effective residual friction angle.)

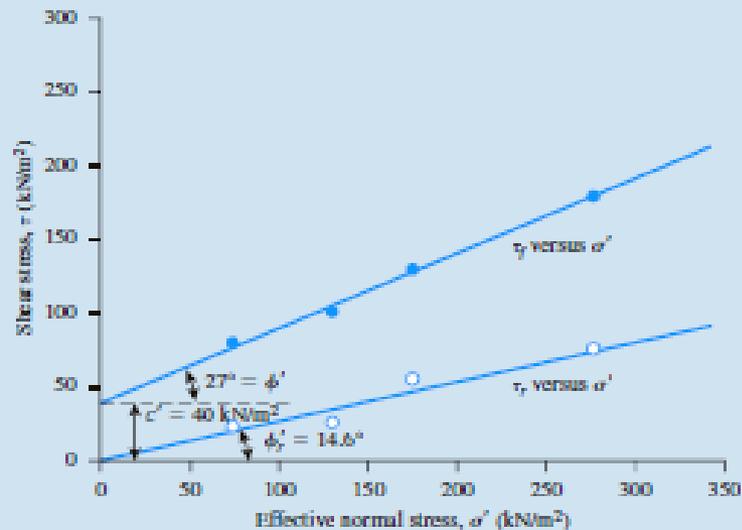


Figure 12.19 Variations of τ_p and τ_r with σ'

Example

For a dry sand specimen in a direct shear test box, the following are given:

- Size of specimen: 63.5 mm 63.5 mm 31.75 mm (height)
- Angle of friction: 33°
- Normal stress: 193 kN/m^2

Determine the shear force required to cause failure.

$$\tau = 193 \tan (33) = 125.33 \text{ kPa}$$

$$\text{Shear force} = \tau * \text{area}$$

$$\text{Shear force} = 125.33 \times 0.0635 \times 0.0635 = 0.50538 \text{ kN} = 505.38 \text{ N}$$