Chapter 5

Ultimate Bearing Capacity of Shallow Foundations: Special Cases

Omitted parts:

Sections 5.6, 5.7 , 5.10 , 5.12

Ultimate Bearing Capacity of Shallow Foundations

The ultimate bearing capacity problems described in Chapter 4 assume that :

- **The soil supporting the foundation is homogeneous and extends to a great depth below the bottom of the foundation.**
- **The ground surface is horizontal.**

However, that is not true in all cases:

- **It is possible to encounter a rigid layer at a shallow depth.**
- **The soil may be layered and have different shear strength parameters.**
- **It may be necessary to construct foundations on or near a slope.**
- **It may be required to design a foundation subjected to uplifting load.**

This chapter discusses bearing capacity problems related to these special cases.

Foundation Supported by a Soil with a Rigid Base at Shallow Depth

If a rigid, rough base is located at a depth of *H* < *D* below the bottom of the foundation, full development of the failure surface in soil will be restricted. In such a case, the soil failure zone and the development of slip lines at ultimate load will be as shown in the Figure below.

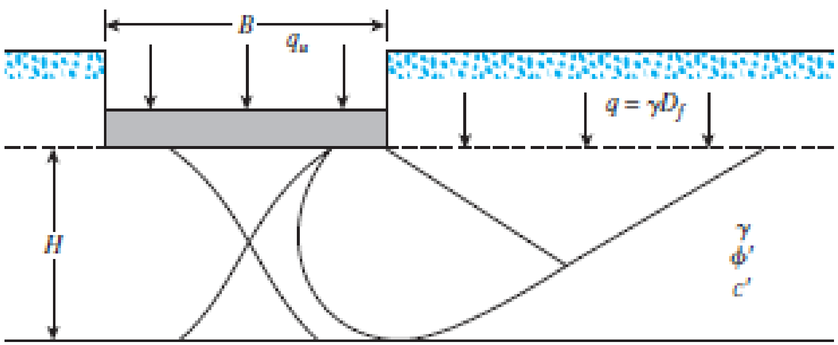


Figure 5.2 Failure surface under a rough, continuous foundation with a rigid, rough base located at a shallow depth

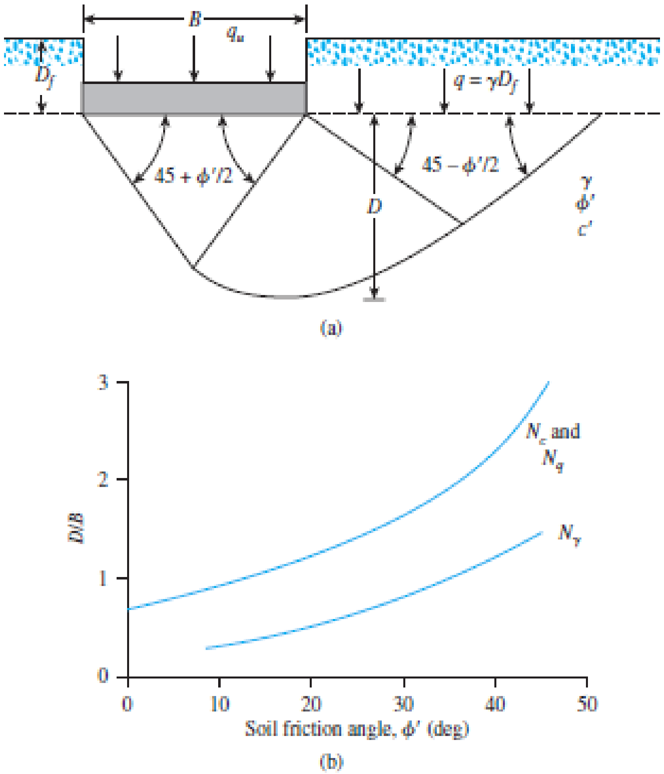


Figure 5.1 (a) Failure surface under a rough continuous foundation; (b) variation of D/B with soil friction angle ϕ'

Foundation Supported by a Soil with a Rigid Base at Shallow Depth

$$q_u = c'N_c^* + qN_q^* + \frac{1}{2}\gamma BN_\gamma^*$$

where

N_c^*, N_q^*, N_γ^* = modified bearing capacity factors
B = width of foundation
γ = unit weight of soil

for $H \geq D$, $N_c^* = N_c$, $N_q^* = N_q$, and $N_\gamma^* = N_\gamma$

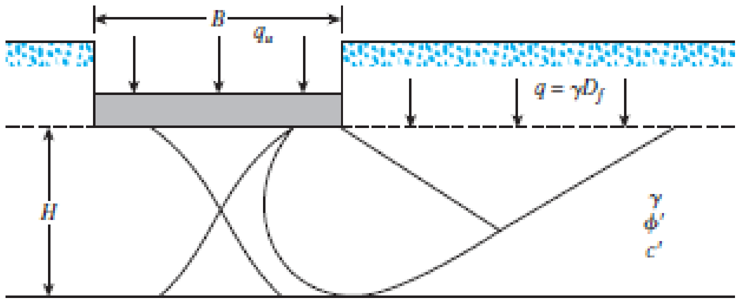
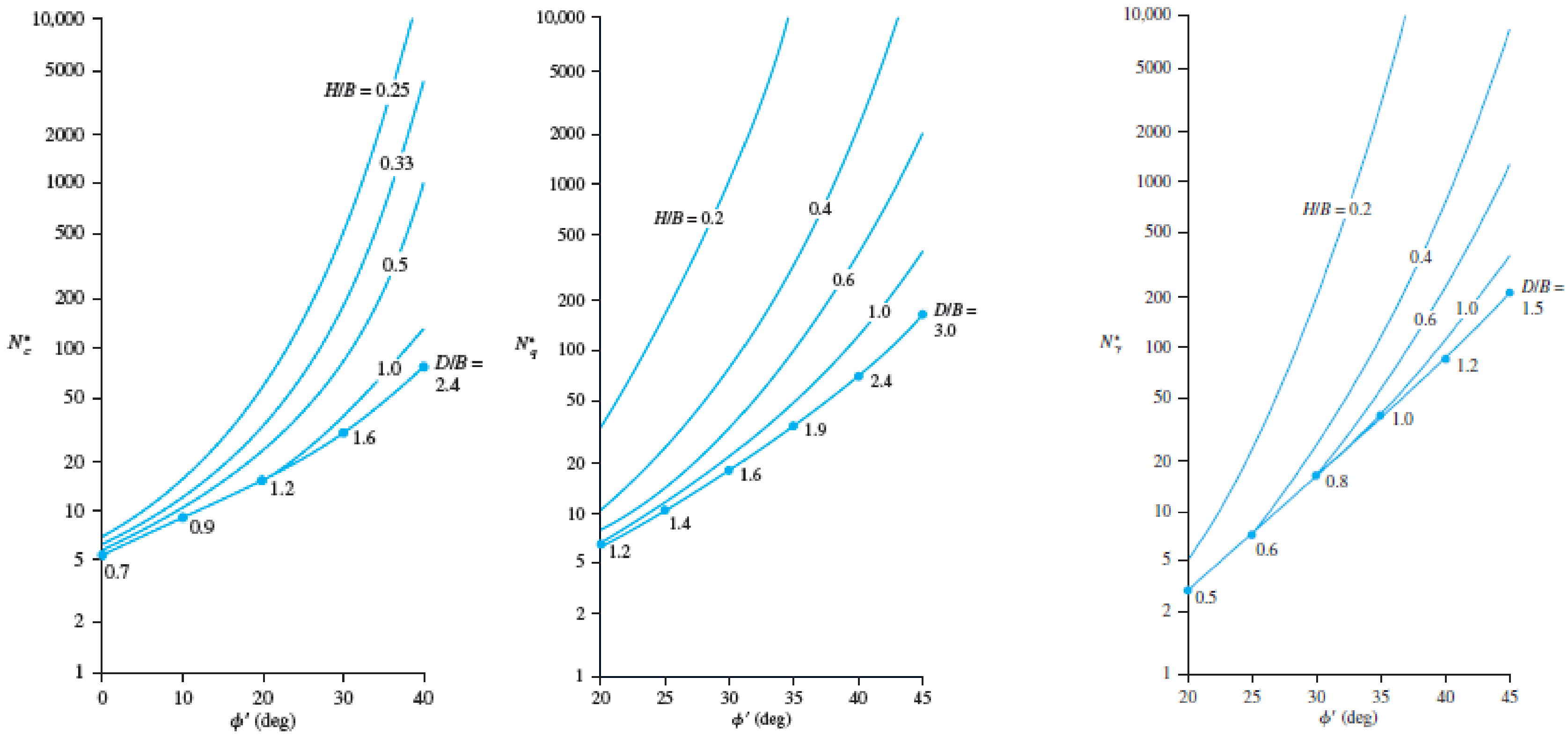


Figure 5.2 Failure surface under a rough, continuous foundation with a rigid, rough base located at a shallow depth

Foundation Supported by a Soil with a Rigid Base at Shallow Depth



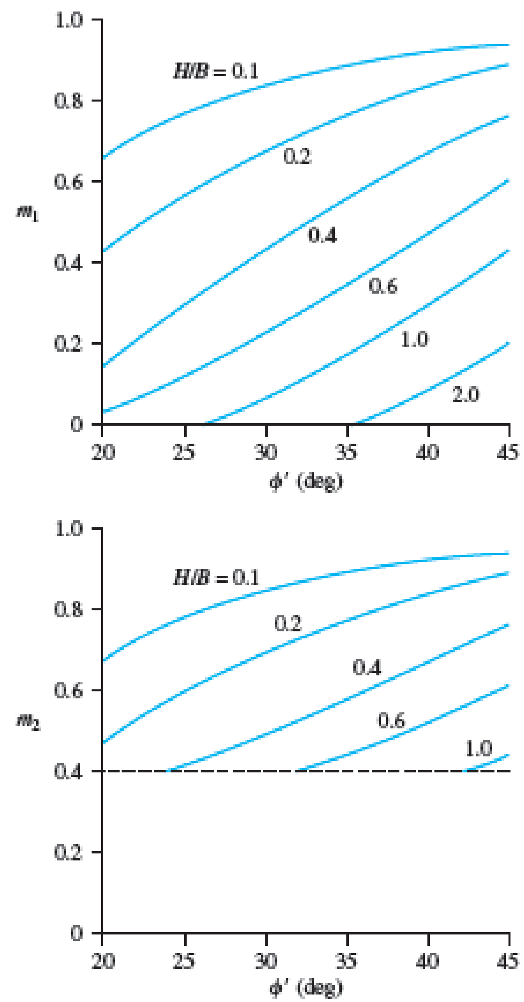
Rectangular Foundation on Granular Soil

$$q_u = qN_q^* F_{qs}^* + \frac{1}{2}\gamma B N_\gamma^* F_{\gamma s}^*$$

where F_{qs}^*, $F_{\gamma s}^*$ = modified shape factors.

$$F_{qs}^* = 1 - m_1\left(\frac{B}{L}\right)$$

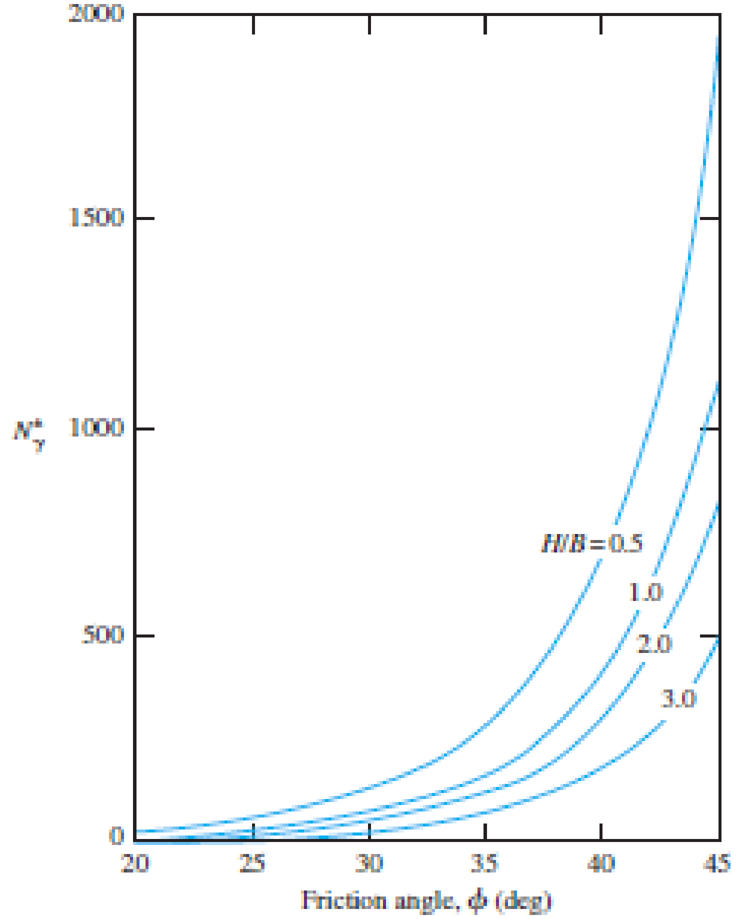
$$F_{\gamma s}^* = 1 - m_2\left(\frac{B}{L}\right)$$



Square and Circular Foundations on Granular Soil

$$q_u = qN_q^* + 0.4\gamma BN_\gamma^* \text{ (square foundation)}$$

$$q_u = qN_q^* + 0.3\gamma BN_\gamma^* \text{ (circular foundation)}$$



Foundations on Saturated Clay

$$q_u = c_u N_c^* + q$$

$$q_{u(\text{square})} = \left(\pi + 2 + \frac{B}{2H} - \frac{\sqrt{2}}{2}\right)c_u + q \qquad \left(\text{for } \frac{B}{2H} - \frac{\sqrt{2}}{2} \geq 0\right)$$

$$q_{u(\text{square})} = \underbrace{5.14\left(1 + \frac{0.5\dfrac{B}{H} - 0.707}{5.14}\right)}_{N_{c(\text{square})}^*}c_u + q$$

Table 5.1 Values of N_c^* for Continuous and Square Foundations ($\phi = 0$)

$\frac{B}{H}$	N_c^*	
	Square[a]	Continuous[b]
2	5.43	5.24
3	5.93	5.71
4	6.44	6.22
5	6.94	6.68
6	7.43	7.20
8	8.43	8.17
10	9.43	9.05

[a]Buisman's analysis (1940)
[b]Mandel and Salencon's analysis (1972)

EXAMPLE 5.1

Example 5.1

A square foundation measuring 1.2 m × 1.2 m is constructed on a layer of sand. We are given that $D_f = 1$ m, $\gamma = 15.5$ kN/m^3, $\phi' = 35°$, and $c' = 0$. A rock layer is located at a depth of 0.48 m below the bottom of the foundation. Using a factor of safety of 4, determine the gross allowable load the foundation can carry.

Solution

From Eq. (5.3),

$$q_u = qN_q^* F_{qs}^* + \frac{1}{2}\gamma B N_\gamma^* F_{\gamma s}^*$$

and we also have

$$q = 15.5 \times 1 = 15.5 \text{ kN/m}^3$$

For $\phi' = 35°$, $H/B = 0.48/1.2 = 0.4$, $N_q^* \approx 336$ (Figure 5.4), and $N_\gamma^* \approx 138$ (Figure 5.5), and we have

$$F_{qs}^* = 1 - m_1\left(\frac{B}{L}\right)$$

From Figure 5.6a for $\phi' = 35°$, $H/B = 0.4$. The value of $m_1 \approx 0.58$, so

$$F_{qs}^* = 1 - (0.58)(1.2/1.2) = 0.42$$

Similarly,

$$F_{\gamma s}^* = 1 - m_2(B/L)$$

From Figure 5.6b, $m_2 = 0.6$, so

$$F_{\gamma s}^* = 1 - (0.6)(1.2/1.2) = 0.4$$

Hence,

$$q_u = (15.5)(336)(0.42) + (1/2)(15.5)(1.2)(138)(0.4) = 2700.72 \text{ kN/m}^2$$

and

$$Q_{\text{all}} = \frac{q_u B^2}{\text{FS}} = \frac{(2700.72)(1.2 \times 1.2)}{4} = \mathbf{972.3 \text{ kN}}$$

EXAMPLE 5.2

Example 5.2

Consider a square foundation 1 m × 1 m in plan located on a saturated clay layer underlain by a layer of rock. Given:

Clay: $c_u = 72$ kN/m^2
Unit weight: $\gamma = 18$ kN/m^3
Distance between the bottom of foundation and the rock layer = 0.25 m
$D_f = 1$ m

Estimate the gross allowable bearing capacity of the foundation. Use FS = 3.

Solution

From Eq. (5.10),

$$q_u = 5.14\left(1 + \frac{0.5\dfrac{B}{H} - 0.707}{5.14}\right)c_u + q$$

For $B/H = 1/0.25 = 4$; $c_u = 72$ kN/m^2; and $q = \gamma D_f = (18)(1) = 18$ kN/m^3.

$$q_u = 5.14\left[1 + \frac{(0.5)(4) - 0.707}{5.14}\right]72 + 18 = 481.2 \text{ kN/m}^2$$

$$q_{\text{all}} = \frac{q_u}{\text{FS}} = \frac{481.2}{3} = \mathbf{160.4\ kN/m^2}$$

Foundations on Layered Clay ($\phi = 0$)

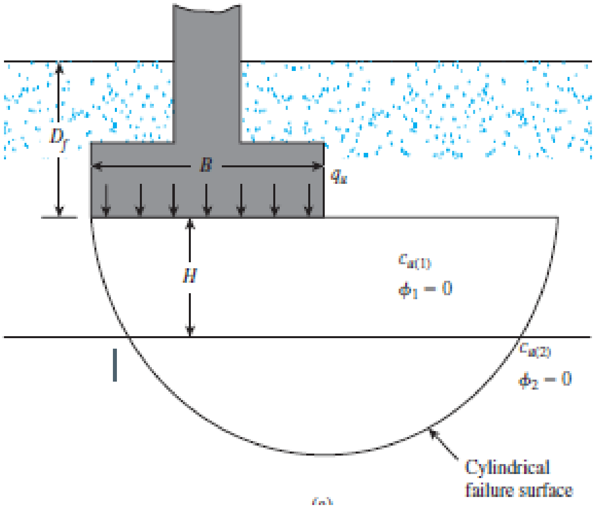
For undrained loading ($\phi = 0$ condition) :

let $c_{u(1)}$ = shear strength of the upper clay layer

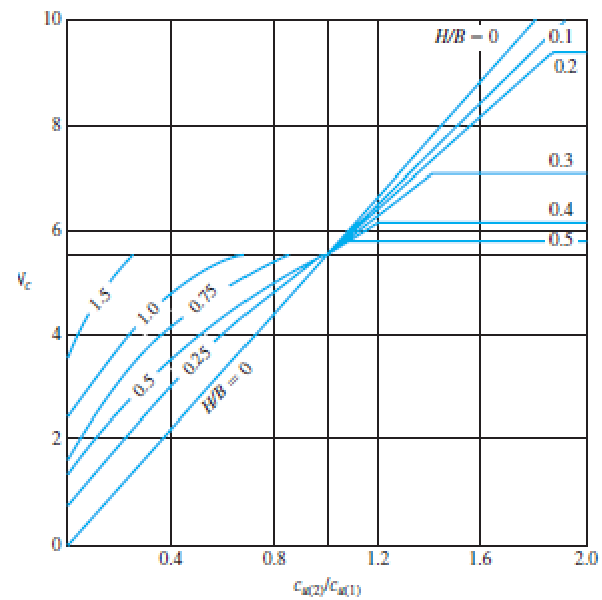
$c_{u(2)}$ = shear strength of the lower clay layer

$$q_u = c_{u(1)} N_c F_{cs} F_{cd} + q$$

The relationships for F_{cs} and F_{cd} given in Table 4.3



The variation of N_c is given in the Figure



If the lower layer of clay is softer than the top one ($c_{u(2)} / c_{u(1)} < 1$), the value of (N_c) is lower than when the soil is not layered ($c_{u(2)} / c_{u(1)} = 1$).

This means that the ultimate bearing capacity is reduced by the presence of a softer clay layer below the top layer.

Weaker Layer underlain by Stronger Layer ($\phi = 0$)

Ultimate bearing capacity of a foundation supported by a weaker clay layer [$c_{u(1)}$] underlain by a stronger clay layer [$c_{u(2)}$] i.e ($c_{u(1)} / c_{u(2)} < 1$) :

$$q_u = c_{u(1)} m N_c F_{cs} F_{cd} + q$$

where

$$N_c = \begin{cases} 5.14 \text{ for continous foundation} \\ 6.17 \text{ for square or circular foundation} \end{cases}$$

F_{cs} = shape factor

F_{cd} = depth factor

$$m = f\left[\frac{c_{u(1)}}{c_{u(2)}}, \frac{H}{B}, \text{and } \frac{B}{L}\right]$$

Table 5.3 Variation of m [Equation (5.12)] for Square Foundation ($B/L = 1$)

	H/B				
$c_{u(1)}/c_{u(2)}$	≥ 0.25	0.125	0.083	0.063	0.05
1	1	1	1	1	1
0.667	1	1.028	1.052	1.075	1.096
0.5	1	1.047	1.091	1.131	1.167
0.333	1	1.075	1.143	1.207	1.267
0.25	1	1.091	1.177	1.256	1.334
0.2	1	1.102	1.199	1.292	1.379
0.1	1	1.128	1.254	1.376	1.494

Based on Vesic (1975)

Table 5.2 Variation of m [Equation (5.12)] for Continuous Foundation ($B/L \leq 0.2$)

	H/B				
$c_{u(1)}/c_{u(2)}$	≥ 0.5	0.25	0.167	0.125	0.1
1	1	1	1	1	1
0.667	1	1.033	1.064	1.088	1.109
0.5	1	1.056	1.107	1.152	1.193
0.333	1	1.088	1.167	1.241	1.311
0.25	1	1.107	1.208	1.302	1.389
0.2	1	1.121	1.235	1.342	1.444
0.1	1	1.154	1.302	1.446	1.584

Based on Vesic (1975)

EXAMPLE 5.3

Example 5.3

Refer to Figure 5.8a. A foundation 1.5 m × 1 m is located at a depth (D_f) of 1 m in a clay. A softer clay layer is located at a depth (H) of 1 m measured from the bottom of the foundation. Given:

For top clay layer,
Undrained shear strength = 120 kN/m²
Unit weight = 16.8 kN/m³

For bottom clay layer,
Undrained shear strength = 48 kN/m²
Unit weight = 16.2 kN/m³

Determine the gross allowable load for the foundation with a factor of safety of 4. Use Eq. (5.11).

Solution

From Eq. (5.11),

$$q_u = c_{u(1)}N_cF_{cs}F_{cd} + q$$

$$c_{u(1)} = 120 \text{ kN/m}^2$$

$$q = \gamma D_f = (16.8)(1) = 16.8 \text{ kN/m}^2$$

$$\frac{c_{u(2)}}{c_{u(1)}} = \frac{48}{120} = 0.4; \frac{H}{B} = \frac{1}{1} = 1$$

From Figure 5.8b, for $H/B = 1$ and $c_{u(2)}/c_{u(1)} = 0.4$, the value of N_c is equal to 4.6. From Table 4.3,

$$F_{cs} = 1 + \left(\frac{B}{L}\right)\left(\frac{N_q}{N_c}\right) = 1 + \left(\frac{1}{1.5}\right)\left(\frac{1}{4.6}\right) = 1.145$$

$$F_{cd} = 1 + 0.4\frac{D_f}{B} = 1 + 0.4\left(\frac{1}{1}\right) = 1.4$$

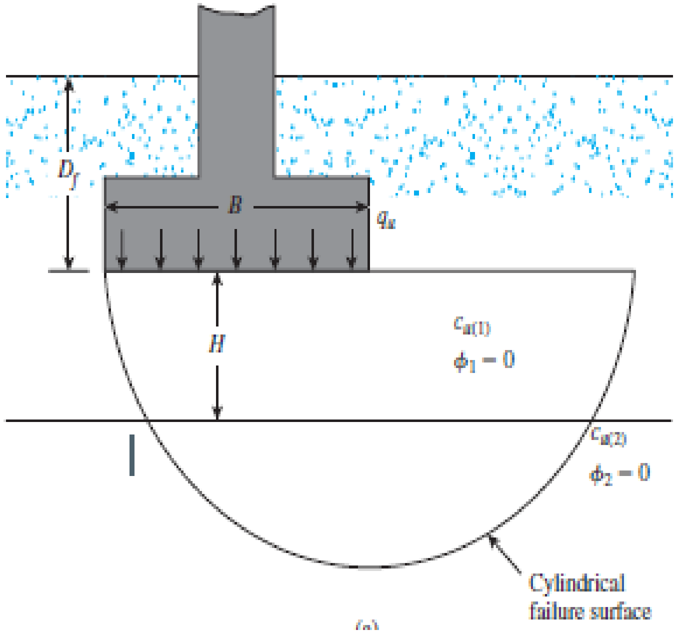
Thus,

$$q_u = (120)(4.6)(1.145)(1.4) + 16.8 = 884.8 + 16.8 = 901.6 \text{ kN/m}^2$$

So

$$q_{all} = \frac{q_u}{FS} = \frac{901.6}{4} = 225.4 \text{ kN/m}^2$$

Total allowable load $= (q_{all})(B \times L) = (225.4)(1 \times 1.5) =$ **338.1 kN** ■

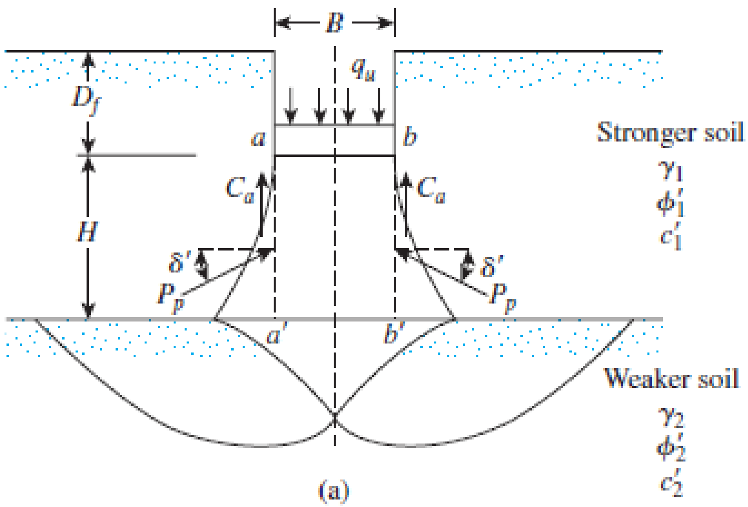


Stronger Layer underlain by Weaker Layer (c'- ϕ' soil)

$$q_u = q_b + \frac{2(C_a + P_p \sin\delta')}{B} - \gamma_1 H$$

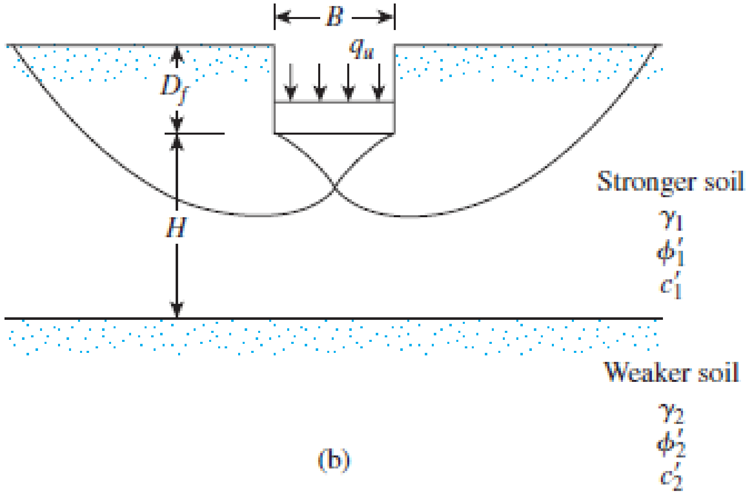
where

B = width of the foundation
C_a = adhesive force
P_p = passive force per unit length of the faces aa' and bb'
q_b = bearing capacity of the bottom soil layer
δ' = inclination of the passive force P_p with the horizontal



(a)

If the depth H is relatively small compared with the foundation width B, a punching shear failure will occur in the top soil layer, followed by a general shear failure In the bottom soil layer (Figure a).



(b)

If the depth H is relatively large, then the failure surface will be completely located in the top soil layer, which is the upper limit for the ultimate bearing capacity (Figure b).

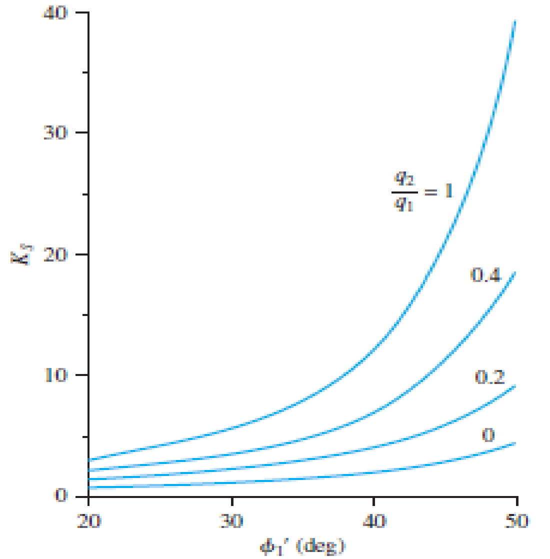
Stronger Layer underlain by Weaker Layer (c'- ϕ' soil)

$$q_u = q_b + \frac{2c_a'H}{B} + \gamma_1 H^2\left(1 + \frac{2D_f}{H}\right)\frac{K_s \tan\phi_1'}{B} - \gamma_1 H \quad \textbf{(a)}$$

$$K_s = f\left(\frac{q_2}{q_1}, \phi_1'\right)$$

$$q_1 = c_1' N_{c(1)} + \tfrac{1}{2}\gamma_1 B N_{\gamma(1)}$$

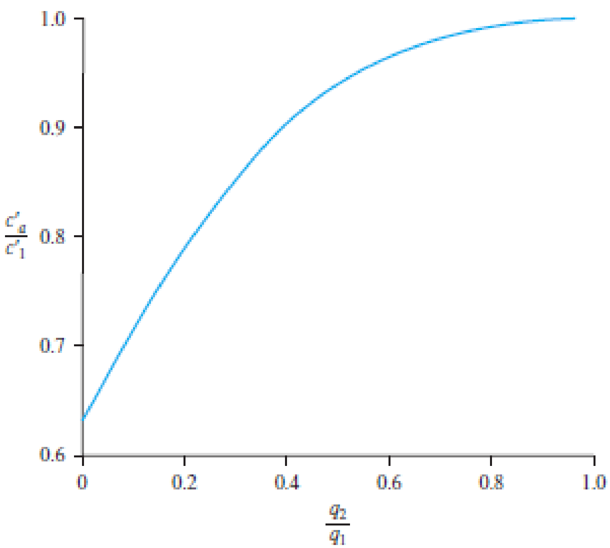
$$q_2 = c_2' N_{c(2)} + \tfrac{1}{2}\gamma_2 B N_{\gamma(2)}$$



The variation of K_s with q_2/q_1 and ϕ_1 is shown in Figure.
The variation of c_a/c_1 with q_2/q_1 is shown in Figure .

If the height H is relatively large, then the failure surface in soil will be completely located in the stronger upper-soil layer.
For this case,

$$q_u = q_t = c_1' N_{c(1)} + q N_{q(1)} + \frac{1}{2}\gamma_1 B N_{\gamma(1)}. \quad \textbf{(b)}$$



Combining Eqs. (a) and (b) yields

$$q_u = q_b + \frac{2c_a'H}{B} + \gamma_1 H^2\left(1 + \frac{2D_f}{H}\right)\frac{K_s \tan\phi_1'}{\mathrm{B}} - \gamma_1 H \leq q_t$$

Stronger Layer underlain by Weaker Layer (c'- ϕ' soil)

For rectangular foundations

$$q_u = q_b + \left(1 + \frac{B}{L}\right)\left(\frac{2c_a'H}{B}\right) + \gamma_1 H^2\left(1 + \frac{B}{L}\right)\left(1 + \frac{2D_f}{H}\right)\left(\frac{K_s \tan \phi_1'}{B}\right) - \gamma_1 H \leqslant q_t$$

where

$$q_b = c_2' N_{c(2)} F_{cs(2)} + \gamma_1 (D_f + H) N_{q(2)} F_{qs(2)} + \frac{1}{2}\gamma_2 B N_{\gamma(2)} F_{\gamma s(2)}$$

and

$$q_t = c_1' N_{c(1)} F_{cs(1)} + \gamma_1 D_f N_{q(1)} F_{qs(1)} + \frac{1}{2}\gamma_1 B N_{\gamma(1)} F_{\gamma s(1)}$$

in which

$F_{cs(1)}, F_{qs(1)}, F_{\gamma s(1)}$ = shape factors with respect to top soil layer (Table 4.3)
$F_{cs(2)}, F_{qs(2)}, F_{\gamma s(2)}$ = shape factors with respect to bottom soil layer (Table 4.3)

Stronger Layer underlain by Weaker Layer (c'- ϕ' soil)

Top layer is strong sand and bottom layer is saturated soft clay $\phi_2 = 0$

$$q_b = \left(1 + 0.2\frac{B}{L}\right)5.14c_{u(2)} + \gamma_1(D_f + H)$$

and

$$q_t = \gamma_1 D_f N_{q(1)} F_{qs(1)} + \tfrac{1}{2}\gamma_1 B N_{\gamma(1)} F_{\gamma s(1)}$$

Hence,

$$q_u = \left(1 + 0.2\frac{B}{L}\right)5.14c_{u(2)} + \gamma_1 H^2\left(1 + \frac{B}{L}\right)\left(1 + \frac{2D_f}{H}\right)\frac{K_s \tan\phi_1'}{B} + \gamma_1 D_f \leqslant \gamma_1 D_f N_{q(1)} F_{qs(1)} + \frac{1}{2}\gamma_1 B N_{\gamma(1)} F_{\gamma s(1)}$$

where $c_{u(2)}$ = undrained cohesion.

For a determination of K_s from Figure 5.10,

$$\frac{q_2}{q_1} = \frac{c_{u(2)} N_{c(2)}}{\frac{1}{2}\gamma_1 B N_{\gamma(1)}} = \frac{5.14c_{u(2)}}{0.5\gamma_1 B N_{\gamma(1)}}$$

Stronger Layer underlain by Weaker Layer (c'- ϕ' soil)

Top layer is stronger sand and bottom layer is weaker sand $(c_1' = 0, c_2' = 0)$.

$$q_u = \left[\gamma_1(D_f + H)N_{q(2)}F_{qs(2)} + \frac{1}{2}\gamma_2 B N_{\gamma(2)}F_{\gamma s(2)}\right] + \gamma_1 H^2\left(1 + \frac{B}{L}\right)\left(1 + \frac{2D_f}{H}\right)\frac{K_s \tan\phi_1'}{B} - \gamma_1 H \leq q_t$$

where

$$q_t = \gamma_1 D_f N_{q(1)}F_{qs(1)} + \frac{1}{2}\gamma_1 B N_{\gamma(1)}F_{\gamma s(1)}$$

Then

$$\frac{q_2}{q_1} = \frac{\frac{1}{2}\gamma_2 B N_{\gamma(2)}}{\frac{1}{2}\gamma_1 B N_{\gamma(1)}} = \frac{\gamma_2 N_{\gamma(2)}}{\gamma_1 N_{\gamma(1)}}$$

Stronger Layer underlain by Weaker Layer (c'- ϕ' soil)

Top layer is stronger saturated clay and bottom layer is weaker saturated clay ($\phi_1 = \phi_2 = 0$)

$$q_u = \left(1 + 0.2\frac{B}{L}\right)5.14c_{u(2)} + \left(1 + \frac{B}{L}\right)\left(\frac{2c_a H}{B}\right) + \gamma_1 D_f \leqslant q_t$$

where

$$q_t = \left(1 + 0.2\frac{B}{L}\right)5.14c_{u(1)} + \gamma_1 D_f$$

and $c_{u(1)}$ and $c_{u(2)}$ are undrained cohesions. For this case,

$$\frac{q_2}{q_1} = \frac{5.14c_{u(2)}}{5.14c_{u(1)}} = \frac{c_{u(2)}}{c_{u(1)}}$$

EXAMPLE 5.4

Example 5.4

Refer to Figure 5.9a and consider the case of a continuous foundation with $B = 2$ m, $D_f = 1.2$ m, and $H = 1.5$ m. The following are given for the two soil layers:

Top sand layer:

Unit weight $\gamma_1 = 17.5$ kN/m³

$\phi'_1 = 40°$

$c'_1 = 0$

Bottom clay layer:

Unit weight $\gamma_2 = 16.5$ kN/m³

$\phi'_2 = 0$

$c_{u(2)} = 30$ kN/m²

Determine the gross ultimate load per unit length of the foundation.

Solution

For this case, Eqs. (5.27) and (5.28) apply. For $\phi'_1 = 40°$, from Table 4.2, $N_\gamma = 109.41$ and

$$\frac{q_2}{q_1} = \frac{c_{u(2)}N_{c(2)}}{0.5\gamma_1 BN_{\gamma(1)}} = \frac{(30)(5.14)}{(0.5)(17.5)(2)(109.41)} = 0.081$$

From Figure 5.10, for $c_{u(2)}N_{c(2)}/0.5\gamma_1 BN_{\gamma(1)} = 0.081$ and $\phi'_1 = 40°$, the value of $K_s \approx 2.5$. Equation (5.27) then gives

$$q_u = \left[1 + (0.2)\left(\frac{B}{L}\right)\right]5.14c_{u(2)} + \left(1 + \frac{B}{L}\right)\gamma_1 H^2\left(1 + \frac{2D_f}{H}\right)K_s\frac{\tan\phi'_1}{\mathrm{B}} + \gamma_1 D_f$$

$$= [1 + (0.2)(0)](5.14)(30) + (1 + 0)(17.5)(1.5)^2$$

$$\times \left[1 + \frac{(2)(1.2)}{1.5}\right](2.5)\frac{\tan 40}{2.0} + (17.5)(1.2)$$

$$= 154.2 + 107.4 + 21 = 282.6 \text{ kN/m}^2$$

Again, from Eq. (5.26),

$$q_t = \gamma_1 D_f N_{q(1)}F_{qs(1)} + \frac{1}{2}\gamma_1 BN_{\gamma(1)}F_{\gamma s(1)}$$

From Table 4.2, for $\phi'_1 = 40°$, $N_\gamma = 109.4$ and $N_q = 64.20$.
From Table 4.3,

$$F_{qs(1)} = 1 + \left(\frac{B}{L}\right)\tan\phi'_1 = 1 + (0)\tan 40 = 1$$

and

$$F_{\gamma s(1)} = 1 - 0.4\frac{B}{L} = 1 - (0.4)(0) = 1$$

so that

$$q_t = (17.5)(1.2)(64.20)(1) + \left(\frac{1}{2}\right)(17.5)(2)(109.4)(1) = 3262.7 \text{ kN/m}^2$$

Hence,

$$q_u = 282.6 \text{ kN/m}^2$$

$$Q_u = (282.6)(B) = (282.6)(2) = \mathbf{565.2\ kN/m}$$

EXAMPLE 5.5

Example 5.5

A foundation 1.5 m × 1 m is located at a depth, D_f, of 1 m in a stronger clay. A softer clay layer is located at a depth, H, of 1 m measured from the bottom of the foundation. For the top clay layer,

$$\text{Undrained shear strength} = 120 \text{ kN/m}^2$$
$$\text{Unit weight} = 16.8 \text{ kN/m}^3$$

and for the bottom clay layer,

$$\text{Undrained shear strength} = 48 \text{ kN/m}^2$$
$$\text{Unit weight} = 16.2 \text{ kN/m}^3$$

Determine the gross allowable load for the foundation with an FS of 4. Use Eqs. (5.32), (5.33), and (5.34).

Solution

For this problem, Eqs. (5.32), (5.33), and (5.34) will apply, or

$$q_u = \left(1 + 0.2\frac{B}{L}\right)5.14c_{u(2)} + \left(1 + \frac{B}{L}\right)\left(\frac{2c_aH}{B}\right) + \gamma_1 D_f$$

$$\leq \left(1 + 0.2\frac{B}{L}\right)5.14c_{u(1)} + \gamma_1 D_f$$

Given:

$B = 1$ m $H = 1$ m $D_f = 1$ m
$L = 1.5$ m $\gamma_1 = 16.8$ kN/m^3

From Figure 5.11, $c_{u(2)}/c_{u(1)} = 48/120 = 0.4$, the value of $c_a/c_{u(1)} \approx 0.9$, so

$$c_a = (0.9)(120) = 108 \text{ kN/m}^2$$

$$q_u = \left[1 + (0.2)\left(\frac{1}{1.5}\right)\right](5.14)(48) + \left(1 + \frac{1}{1.5}\right)\left[\frac{(2)(108)(1)}{1}\right] + (16.8)(1)$$

$$= 279.6 + 360 + 16.8 = 656.4 \text{ kN/m}^2$$

Check: From Eq. (5.33),

$$q_t = \left[1 + (0.2)\left(\frac{1}{1.5}\right)\right](5.14)(120) + (16.8)(1)$$

$$= 699 + 16.8 = 715.8 \text{ kN/m}^2$$

Thus $q_u = 656.4$ kN/m^2 (that is, the smaller of the two values calculated above) and

$$q_{all} = \frac{q_u}{FS} = \frac{656.4}{4} = 164.1 \text{ kN/m}^2$$

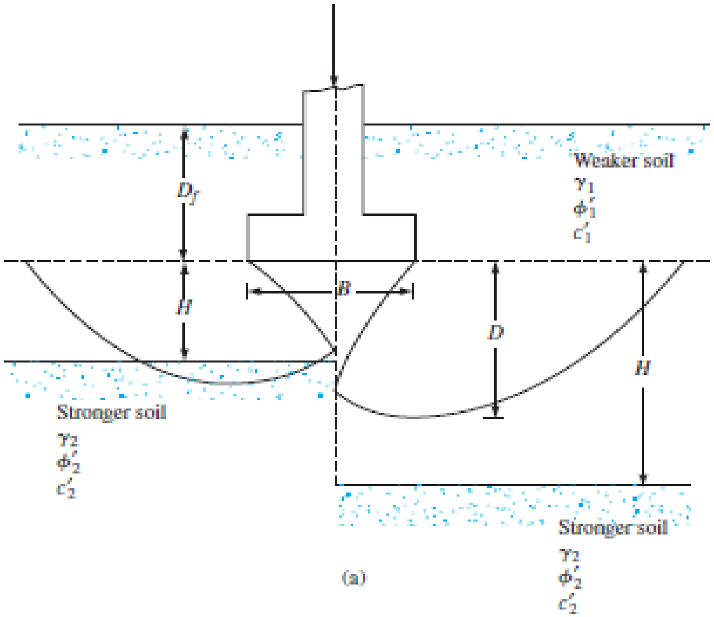
The total allowable load is

$$(q_{all})(1 \times 1.5) = \mathbf{246.15\ kN}$$

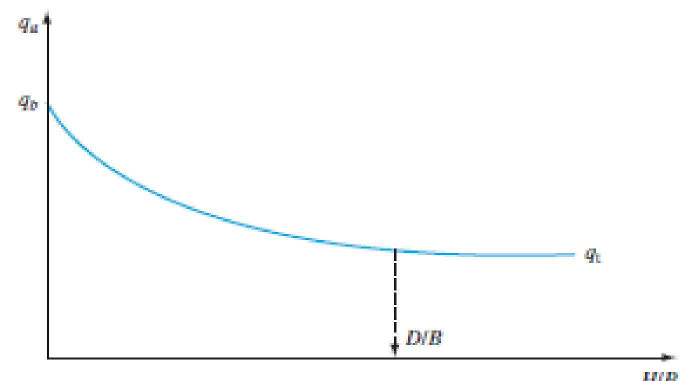
Note: This is the same problem as in Example 5.3. The allowable load is about 40% lower than that calculated in Example 5.3. This is due to the failure surface in the soil assumed at the ultimate load. ■

Weaker Layer underlain by Stronger Layer (c'- ϕ' soil)

When a foundation is supported by a weaker soil layer underlain by a stronger layer, the ratio of q_2/q_1 will be greater than one.
If H/B is relatively small, the failure surface in soil at ultimate load will pass through both soil layers.
However, for larger H/B ratios, the failure surface will be fully located in the top, weaker soil layer.



(a)



(b)

Weaker Layer underlain by Stronger Layer (c'- ϕ' soil)

The ultimate bearing capacity:

$$q_u = q_t + (q_b - q_t)\left(\frac{H}{D}\right)^2 \geq q_t \qquad (5.35)$$

where

D = depth of failure surface beneath the foundation in the thick bed of the upper weaker soil layer
q_t = ultimate bearing capacity in a thick bed of the upper soil layer
q_b = ultimate bearing capacity in a thick bed of the lower soil layer

So

$$q_t = c_1' N_{c(1)} F_{cs(1)} + \gamma_1 D_f N_{q(1)} F_{qs(1)} + \frac{1}{2}\gamma_1 B N_{\gamma(1)} F_{\gamma s(1)} \qquad (5.36)$$

and

$$q_b = c_2' N_{c(2)} F_{cs(2)} + \gamma_2 D_f N_{q(2)} F_{qs(2)} + \frac{1}{2}\gamma_2 B N_{\gamma(2)} F_{\gamma s(2)} \qquad (5.37)$$

where

$N_{c(1)}, N_{q(1)}, N_{\gamma(1)}$ = bearing capacity factors corresponding to the soil friction angle ϕ_1'
$N_{c(2)}, N_{q(2)}, N_{\gamma(2)}$ = bearing capacity factors corresponding to the soil friction angle ϕ_2'
$F_{cs(1)}, F_{qs(1)}, F_{\gamma s(1)}$ = shape factors corresponding to the soil friction angle ϕ_1'
$F_{cs(2)}, F_{qs(2)}, F_{\gamma s(2)}$ = shape factors corresponding to the soil friction angle ϕ_2'

Meyerhof and Hanna (1978) suggested that

- $D \approx B$ for loose sand and clay
- $D \approx 2B$ for dense sand

EXAMPLE 5.6

Example 5.6

Refer to Figure 5.12a. For a layered saturated-clay profile, given: $L = 6$ ft, $B = 4$ ft, $D_f = 3$ ft, $H = 2$ ft, $\gamma_1 = 110$ lb/ft^3, $\phi_1 = 0$, $c_{u(1)} = 1200$ lb/ft^2, $\gamma_2 = 125$ lb/ft^3, $\phi_2 = 0$, and $c_{u(2)} = 2500$ lb/ft^2. Determine the ultimate bearing capacity of the foundation.

Solution

From Eqs. (5.18) and (5.19),

$$\frac{q_2}{q_1} = \frac{c_{u(2)}N_c}{c_{u(1)}N_c} = \frac{c_{u(2)}}{c_{u(1)}} = \frac{2500}{1200} = 2.08 > 1$$

So, Eq. (5.35) will apply.

From Eqs. (5.36) and (5.37) with $\phi_1 = \phi_2 = 0$,

$$q_t = \left(1 + 0.2\frac{B}{L}\right)N_c c_{u(1)} + \gamma_1 D_f$$

$$= \left[1 + (0.2)\left(\frac{4}{6}\right)\right](5.14)(1200) + (3)(110) = 6990.4 + 330 = 7320.4 \text{ lb/ft}^2$$

and

$$q_b = \left(1 + 0.2\frac{B}{L}\right)N_c c_{u(2)} + \gamma_2 D_f$$

$$= \left[1 + (0.2)\left(\frac{4}{6}\right)\right](5.14)(2500) + (3)(125)$$

$$= 14{,}563.3 + 375 = 14{,}938.3 \text{ lb/ft}^2$$

From Eq. (5.35),

$$q_u = q_t + (q_b - q_t)\left(\frac{H}{D}\right)^2$$

$$D \approx B$$

$$q_u = 7320.4 + (14{,}938.3 - 7320.4)\left(\frac{2}{4}\right)^2 \approx 9225 \text{ lb/ft}^2 > q_t$$

Hence,

$$q_u = \mathbf{9225\ lb/ft^2}$$

Bearing Capacity of Foundations on Top of a Slope

The ultimate bearing capacity for *continuous foundations:*

$$q_u = c'N_{cq} + \frac{1}{2}\gamma B N_{\gamma q}$$

For purely granular soil, $c' = 0$, thus,

$$q_u = \frac{1}{2}\gamma B N_{\gamma q}$$

Again, for purely cohesive soil, $\phi = 0$ (the undrained condition); hence,

$$q_u = c_u N_{cq}$$

where c_u = undrained cohesion.

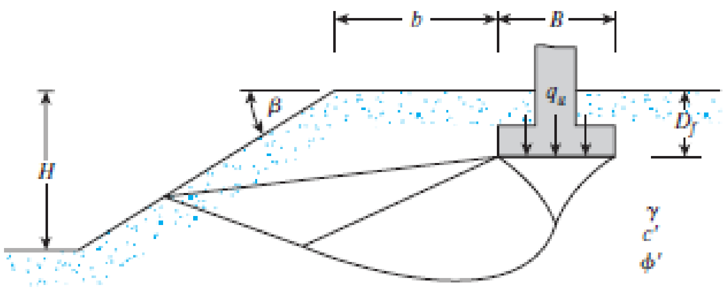


Figure 5.19 Shallow foundation on top of a slope

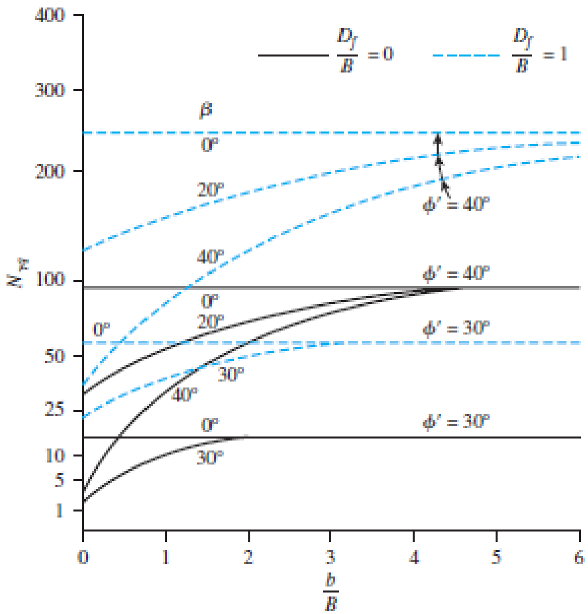


Figure 5.20 Meyerhof's bearing capacity factor $N_{\gamma q}$ for granular soil ($c' = 0$)

$N_{\gamma q}$

Weaker Layer underlain by Stronger Layer (c'- ϕ' soil)

The following points need to be kept in mind in determining N_{cq} :

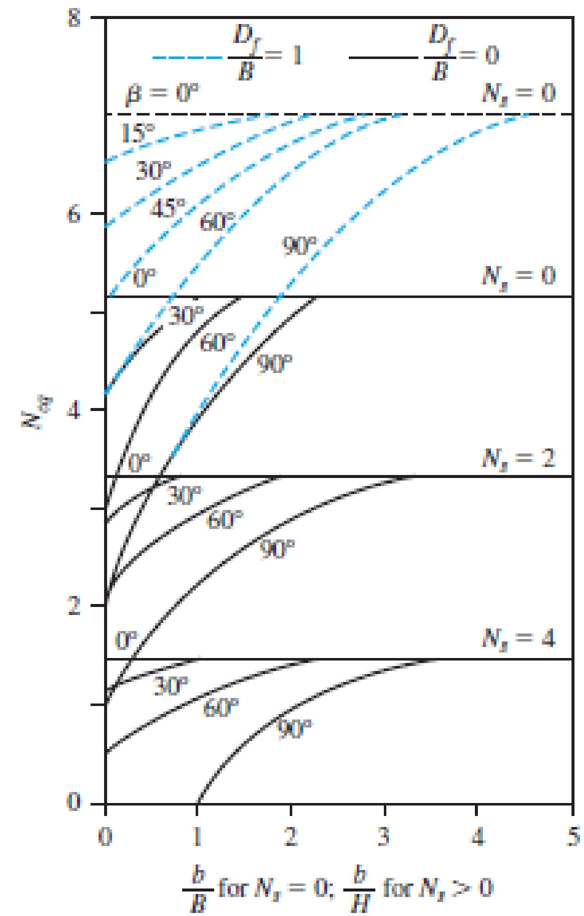
1. The term

$$N_s = \frac{\gamma H}{c_u}$$

is defined as the stability number.

2. If B<H, use the curves for $N_s = 0$.

3. If B>=H, use the curves for the calculated stability number N_s .



N_{cq}

EXAMPLE 5.8

Example 5.8

In Figure 5.19, for a shallow continuous foundation in a clay, the following data are given: $B = 1.2$ m; $D_f = 1.2$ m; $b = 0.8$ m; $H = 6.2$ m; $\beta = 30°$; unit weight of soil $= 17.5$ kN/m^3; $\phi = 0$; and $c_u = 50$ kN/m^2. Determine the gross allowable bearing capacity with a factor of safety FS = 4.

Solution

Since $B < H$, we will assume the stability number $N_s = 0$. From Eq. (5.43),

$$q_u = c_u N_{cq}$$

We are given that

$$\frac{D_f}{B} = \frac{1.2}{1.2} = 1$$

and

$$\frac{b}{B} = \frac{0.8}{1.2} = 0.67$$

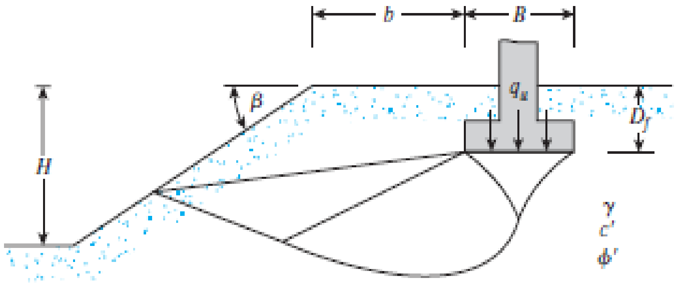


Figure 5.19 Shallow foundation on top of a slope

For $\beta = 30°$, $D_f/B = 1$ and $b/B = 0.67$, Figure 5.21 gives $N_{cq} = 6.3$. Hence,

$$q_u = (50)(6.3) = 315 \text{ kN/m}^2$$

and

$$q_{all} = \frac{q_u}{FS} = \frac{315}{4} = \mathbf{78.8\ kN/m^2}$$

EXAMPLE 5.9

Example 5.9

Figure 5.22 shows a continuous foundation on a slope of a granular soil. Estimate the ultimate bearing capacity.

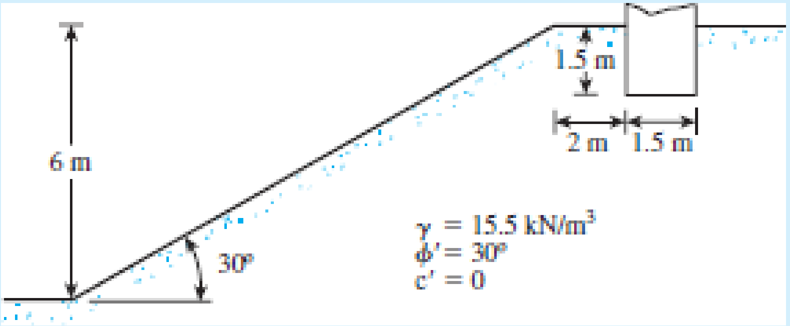


Figure 5.22 Foundation on a granular slope

Solution

For granular soil ($c' = 0$), from Eq. (5.42),

$$q_u = \frac{1}{2}\gamma B N_{\gamma q}$$

We are given that $b/B = 2/1.5 = 1.33$, $D_f/B = 1.5/1.5 = 1$, $\phi' = 30°$, and $\beta = 30°$.

From Figure 5.20, $N_{\gamma q} \approx 41$. So,

$$q_u = \frac{1}{2}(15.5)(1.5)(41) = \mathbf{476.6\ kN/m^2}$$ ■

EXAMPLE 5.10

Example 5.10

Refer to Figure 5.19. For a shallow continuous foundation in a clay, the following are give: $B = 1.2$ m, $D_f = 1.2$ m, $b = 0.8$ m, $H = 6.2$ m, $\beta = 30°$, unit weight of soil $= 17.5$ kN/m^3, $\phi = 0$, and $c_u = 50$ kN/m^2. Determine the gross allowable bearing capacity with a factor of safety $FS = 4$.

Solution

Since $B < H$, we will assume the stability number $N_s = 0$. From Eq. (5.43).

$$q_u = c_u N_{cq}$$

Given:

$$\frac{D_f}{B} = \frac{1.2}{1.2} = 1$$

$$\frac{b}{B} = \frac{0.8}{1.2} = 0.75$$

For $\beta = 30°$, $D_f/B = 1$ and $b/B = 0.75$, Figure 5.21 given $N_{cq} = 6.3$. Hence,

$$q_u = (50)(6.3) = 315 \text{ kN/m}^2$$

$$q_{all} = \frac{q_u}{FS} = \frac{315}{4} = \mathbf{78.8\ kN/m^2}$$

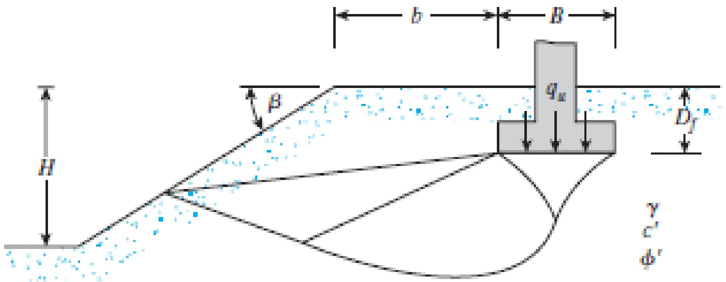
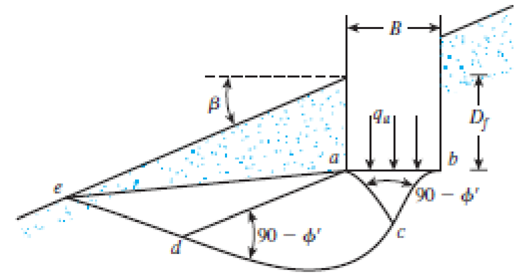


Figure 5.19 Shallow foundation on top of a slope

Bearing Capacity of Foundations on a Slope

$q_u = c_u N_{cq s}$ (for purely cohesive soil, that is, $\phi = 0$)

$q_u = \frac{1}{2}\gamma B N_{\gamma q s}$ (for granular soil, that is $c' = 0$)



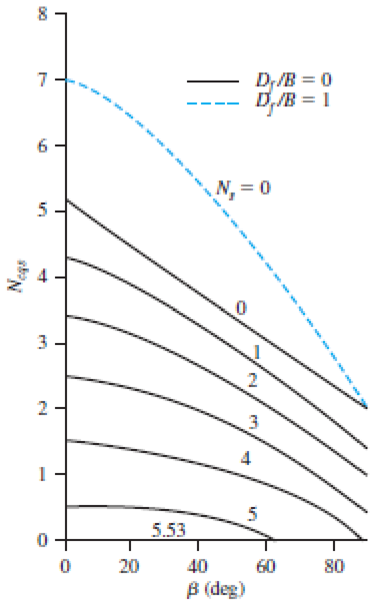
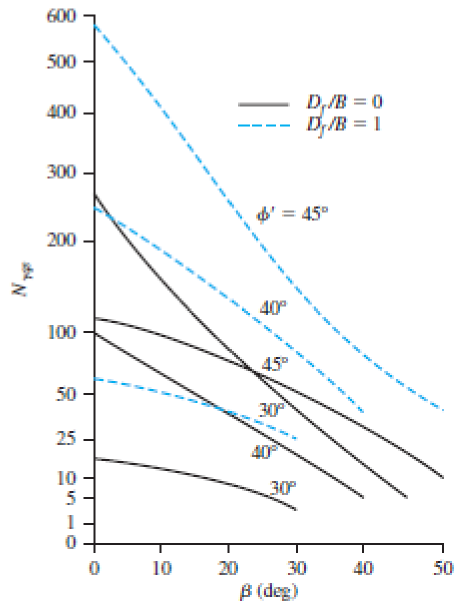


Figure 5.24 Variation of $N_{cq s}$ with β.
(*Note:* $N_s = \gamma H / c_u$)



Foundations on Rock

$q_u = c' N_c + q N_q + 0.5 \gamma B N_\gamma$

$$N_c = 5 \tan^4\left(45 + \frac{\phi'}{2}\right)$$

$$N_q = \tan^6\left(45 + \frac{\phi'}{2}\right)$$

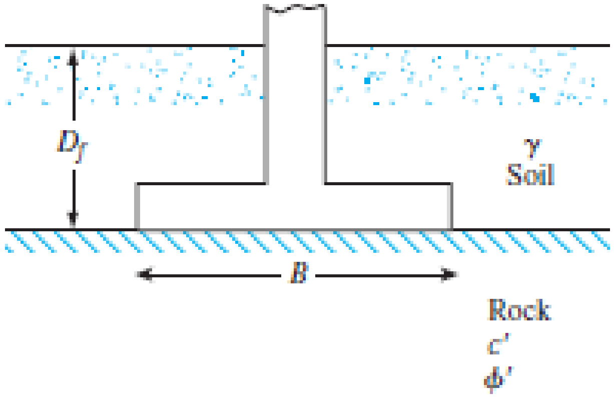
$$N_\gamma = N_q + 1$$

$$q_{uc} = 2c' \tan\left(45 + \frac{\phi'}{2}\right)$$

where

q_{uc} = unconfined compression strength of rock
ϕ' = angle of friction

$$q_{u(\mathrm{modified})} = q_u (\mathrm{RQD})^2$$



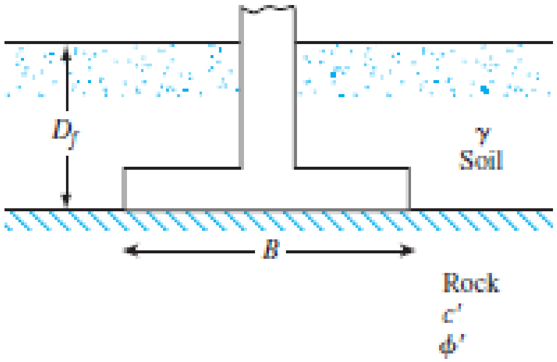
EXAMPLE 5.13

Example 5.13

Refer to Figure 5.32. A square column foundation is to be constructed over siltstone. Given:

Foundation: $B \times B = 2.5 \text{ m} \times 2.5 \text{ m}$
$D_f = 2 \text{ m}$
Soil: $\gamma = 17 \text{ kN/m}^3$
Siltstone: $c' = 32 \text{ MN/m}^2$
$\phi' = 31°$
$\gamma = 25 \text{ kN/m}^3$
$\text{RDQ} = 50\%$

Estimate the allowable load-bearing capacity. Use FS = 4. Also, for concrete, use $f_c' = 30 \text{ MN/m}^2$.



Solution

From Eq. (4.17),

$$q_u = 1.3c'N_c + qN_q + 0.4\gamma BN_\gamma$$

$$N_c = 5\tan^4\left(45 + \frac{\phi'}{2}\right) = 5\tan^4\left(45 + \frac{31}{2}\right) = 48.8$$

$$N_q = \tan^6\left(45 + \frac{\phi'}{2}\right) = \tan^6\left(45 + \frac{31}{2}\right) = 30.5$$

$$N_\gamma = N_q + 1 = 30.5 + 1 = 31.5$$

Hence,

$$\begin{aligned} q_u &= (1.3)(32 \times 10^3 \text{ kN/m}^2)(48.8) + (17 \times 2)(30.5) + (0.4)(25)(2.5)(31.5) \\ &= 2030.08 \times 10^3 + 1.037 \times 10^3 + 0.788 \times 10^3 \\ &= 2031.9 \times 10^3 \text{ kN/m}^2 \approx 2032 \text{ MN/m}^2 \end{aligned}$$

$$q_{u(\text{modified})} = q_u(\text{RQD})^2 = (2032)(0.5)^2 = 508 \text{ MN/m}^2$$

$$q_{\text{all}} = \frac{508}{4} = 127 \text{ MN/m}^2$$

Since 127 MN/m^2 is greater than f_c', use $q_{\text{all}} = \mathbf{30\ MN/m^2}$. ■