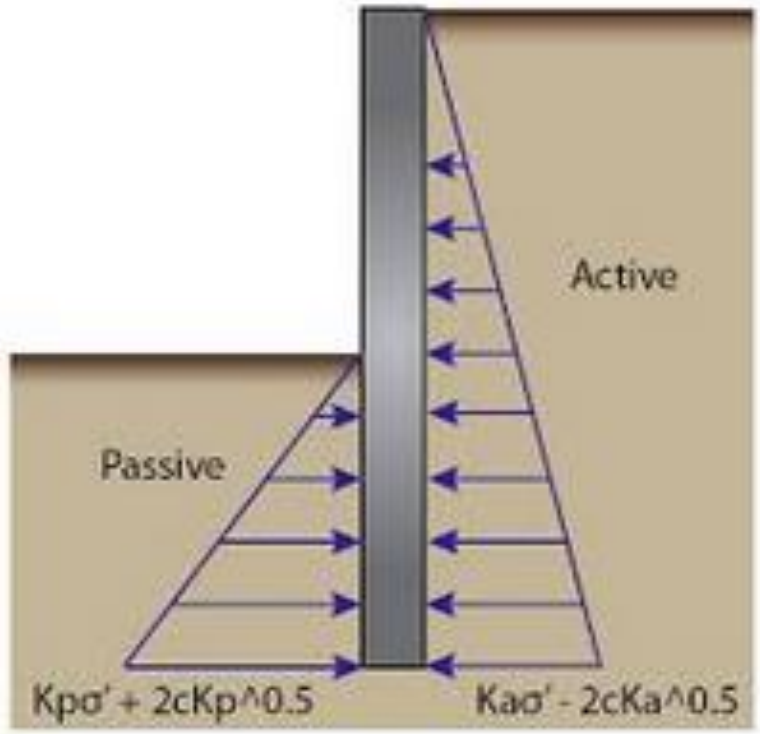
Lateral Earth Pressure

CHAPTER 12



Omitted parts:

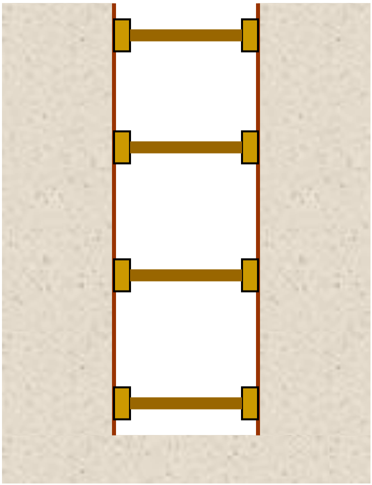
Sections 12.8, 12.9, 12.15

INTRODUCTION

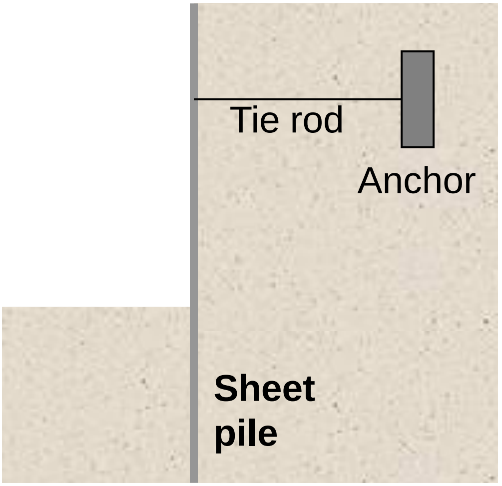
- Proper design and construction of many structures such as:
  - **Retaining walls (basements walls, highways and railroads, platforms, landscaping, and erosion controls)**
  - **Braced excavations**
  - **Anchored bulkheads**
  - **Grain pressure on silo walls and bins**

require a thorough knowledge of the lateral forces that act between the retaining structures and the soil masses being retained.

Cantilever retaining wall



Braced excavation



Anchored sheet pile

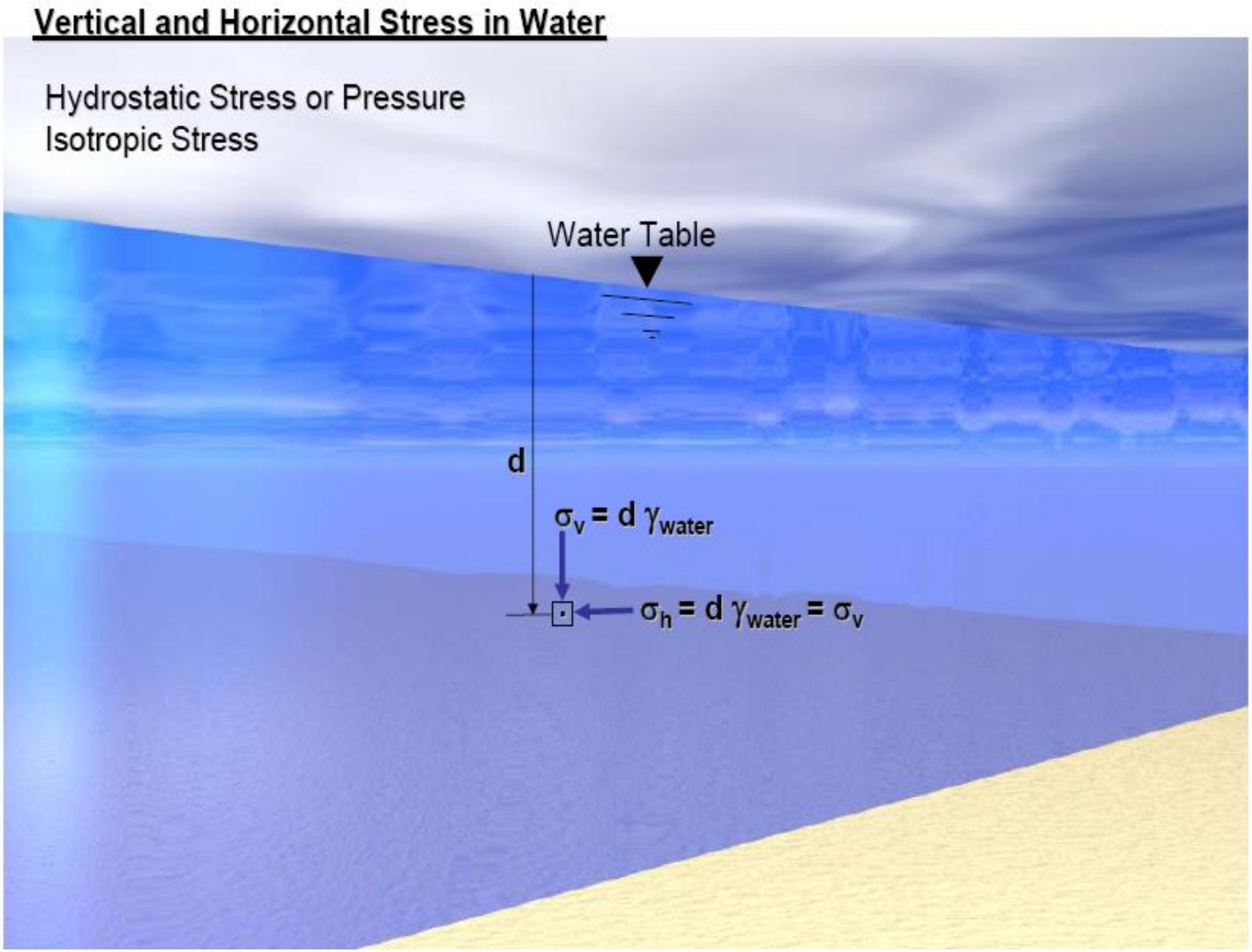
INTRODUCTION

- These lateral forces are caused by lateral earth pressure.
- We have to estimate the lateral soil pressures acting on these structures, to be able to design them.

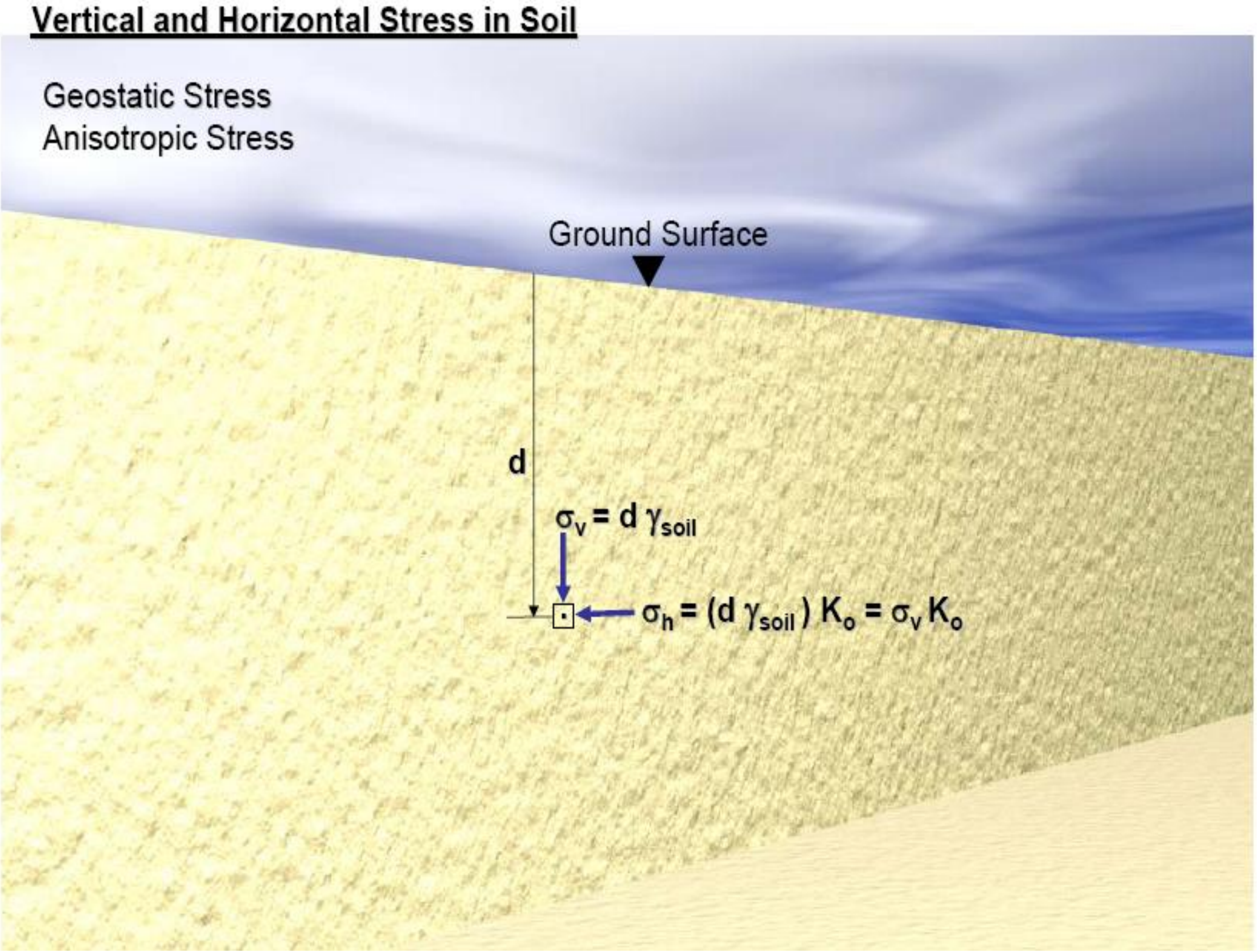
The magnitude and distribution of lateral earth pressure depends on many factors, such as:

- The shear strength parameters of the soil being retained,
- The inclination of the surface of the backfill,
- The height and inclination of the retaining wall at the wall–backfill interface,
- The nature of wall movement under lateral pressure,
- The adhesion and friction angle at the wall–backfill interface.

Vertical and Horizontal Stress in Water

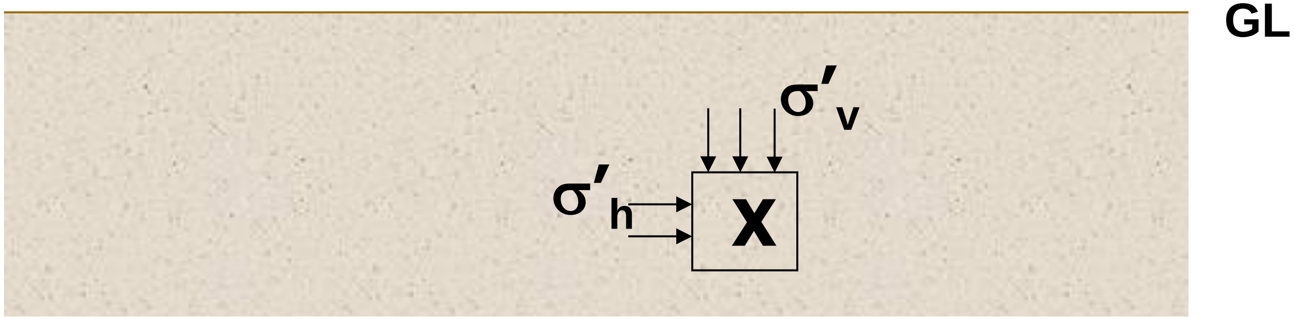


Vertical and Horizontal Stress in Soil



Coefficient of Lateral Earth Pressure

In a homogeneous natural soil deposit,



The ratio σ_h'/σ_v' is a constant known as coefficient of lateral earth pressure.

In other words, it is the ratio of the effective horizontal stress (σ_h') to the effective vertical stress (σ_v'); then

$$\boxed{K = \frac{\sigma_h'}{\sigma_v'}}$$

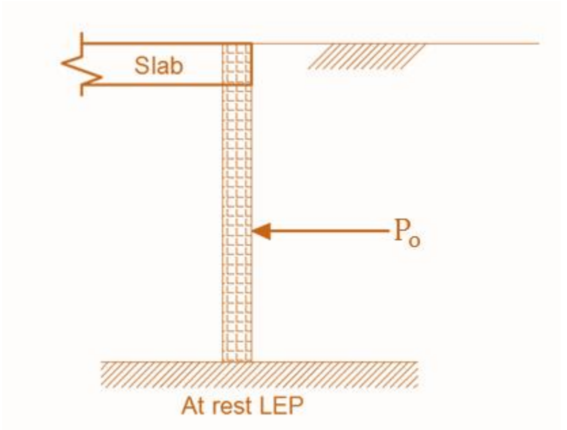
Or in terms of total stresses

$$\boxed{K = \frac{\sigma_h}{\sigma_v}}$$

Types of Lateral Earth Pressures

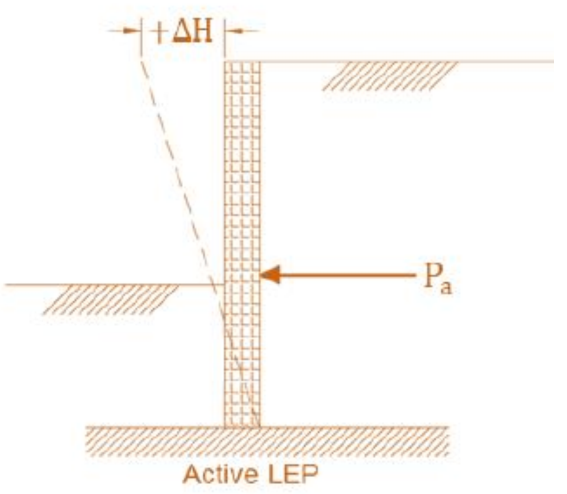
1. At Rest Lateral Earth Pressure:

The wall may be restrained from moving, for example; basement wall is restrained to move due to slab of the basement and the lateral earth force in this case can be termed as" P_o".



2. Active Lateral Earth Pressure:

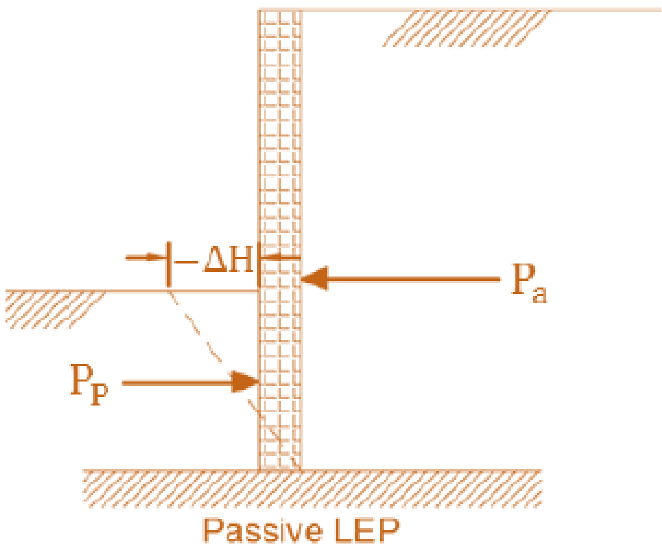
In case of the wall is free from its upper edge (retaining wall), the wall may move away from the soil that is retained with distance "+ΔH " (i.e. the soil pushes the wall away) this means the soil is active and the force of this pushing is called active force and termed by " P_a".



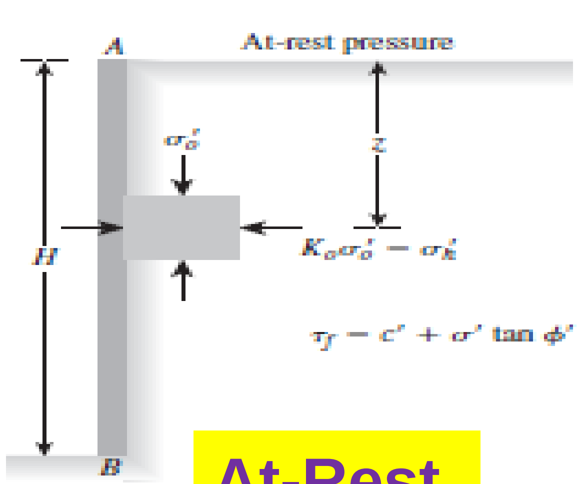
Types of Lateral Earth Pressures

3. Passive Lateral Earth Pressure:

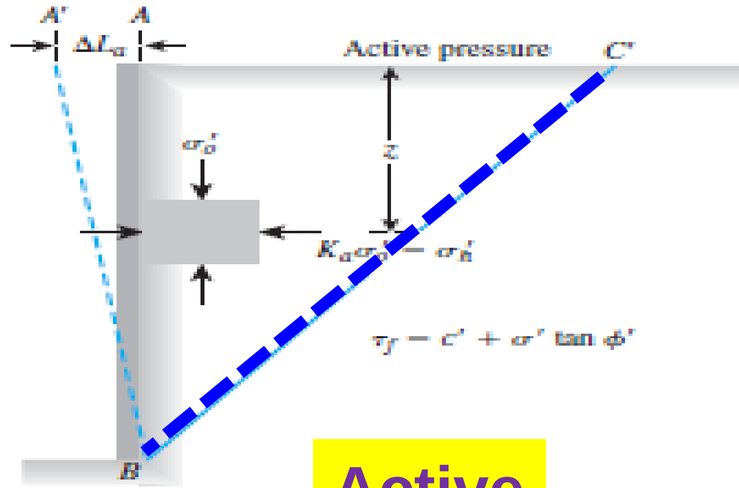
For the wall (retaining wall) in the left side there exist a soil with height less than the soil in the right and as mentioned above the right soil will pushes the wall away, so the wall will be pushed into the left soil (i.e. soil compresses the left soil) this means the soil has a passive effect and the force in this case is called passive force and termed by " P_P".



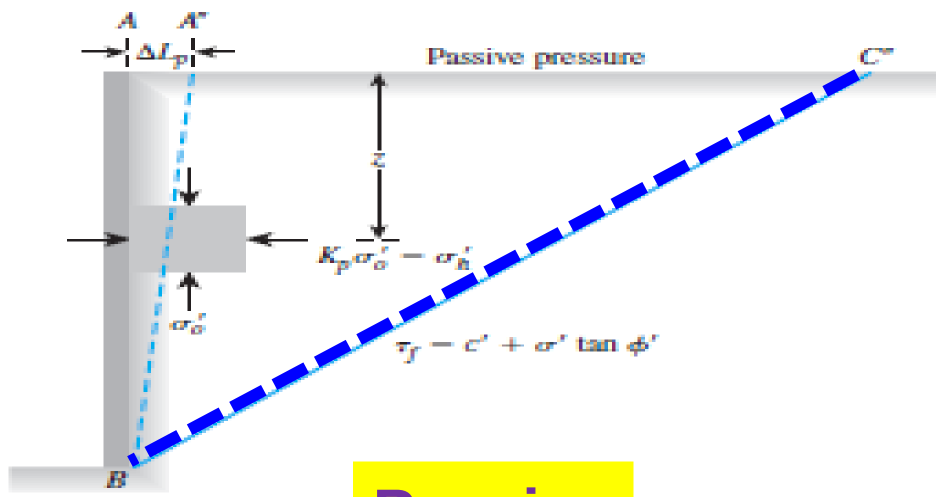
CASES



At-Rest

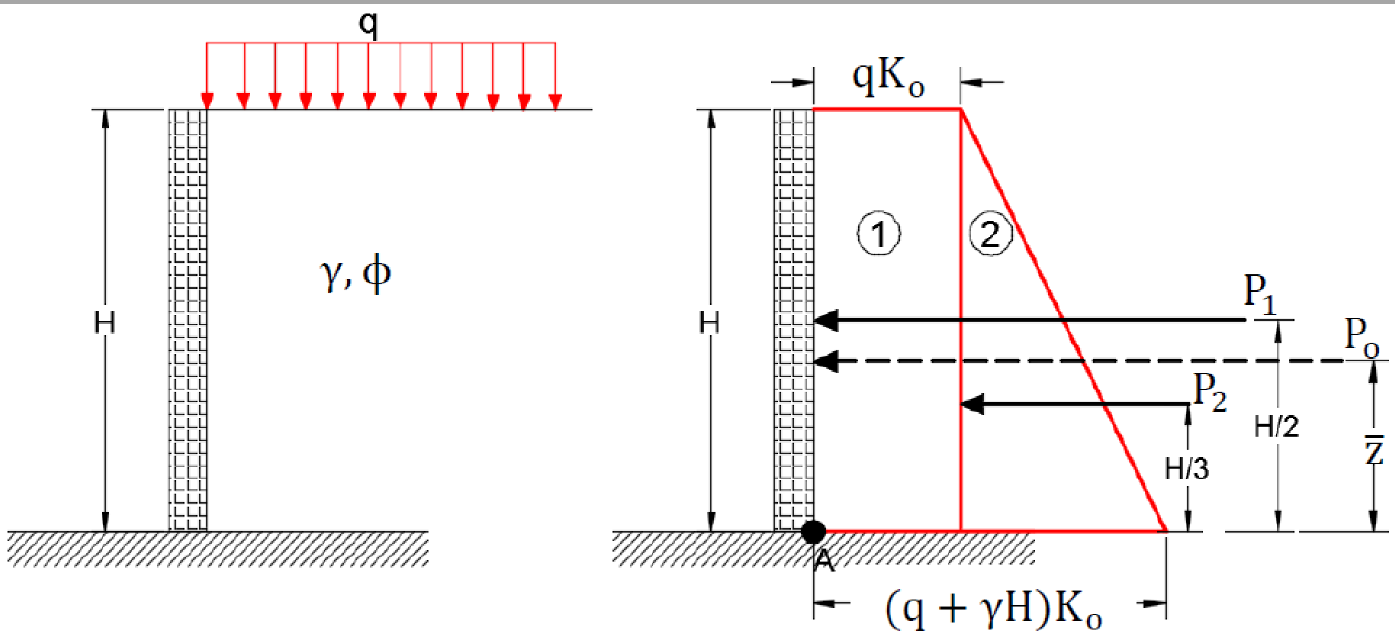


Active



Passive

Lateral Earth Pressure at Rest



$$P_o = P_1 + P_2 = qK_oH + \tfrac{1}{2}\gamma H^2 K_o$$

where

P_1 = area of rectangle 1
P_2 = area of triangle 2

$$\bar{z} = \frac{P_1\left(\frac{H}{2}\right) + P_2\left(\frac{H}{3}\right)}{P_o}$$

Coefficient of Lateral Earth Pressure K_0

Jaky formula

For normally consolidated clays and loose sand.

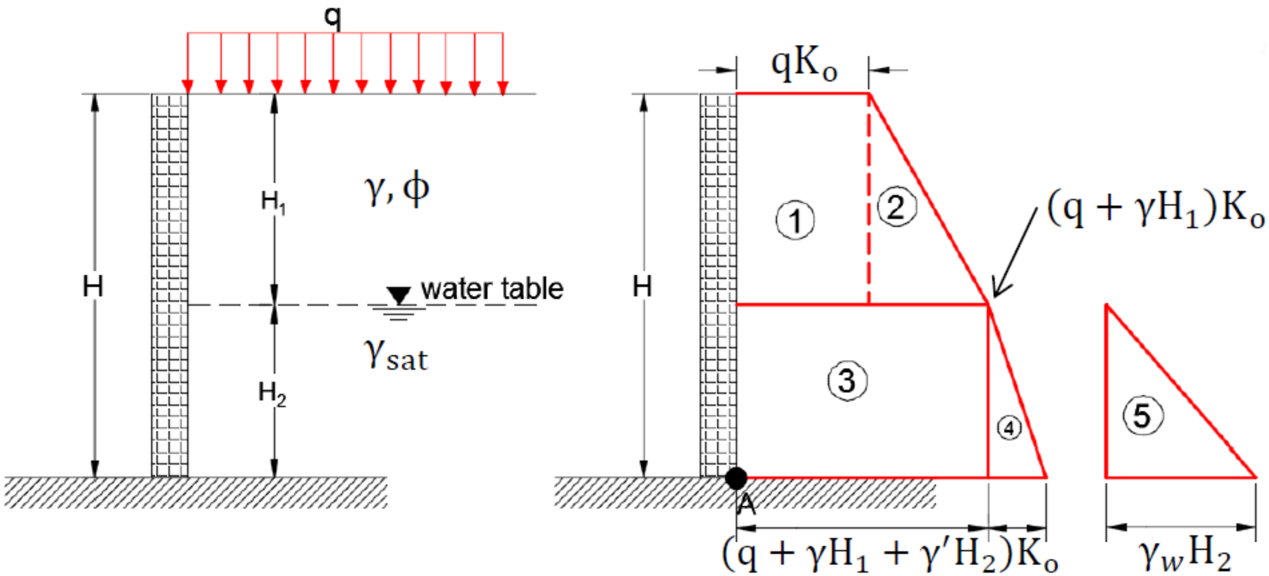
$$K_o \approx 1 - \sin\phi'$$

Mayne and Kulhawy

For Overconsolidated clays

$$K_o = (1 - \sin\phi')\,\mathrm{OCR}^{\sin\phi'}$$

Lateral Earth Pressure at Rest with Water



$$\text{at } z = 0, \qquad \sigma'_h = K_o \sigma'_o = K_o q$$
$$\text{at } z = H_1, \qquad \sigma'_h = K_o \sigma'_o = K_o(q + \gamma H_1)$$

and

$$\text{at } z = H_2, \qquad \sigma'_h = K_o \sigma'_o = K_o(q + \gamma H_1 + \gamma' H_2)$$

$$P_o = A_1 + A_2 + A_3 + A_4 + A_5$$

where A = area of the pressure diagram.

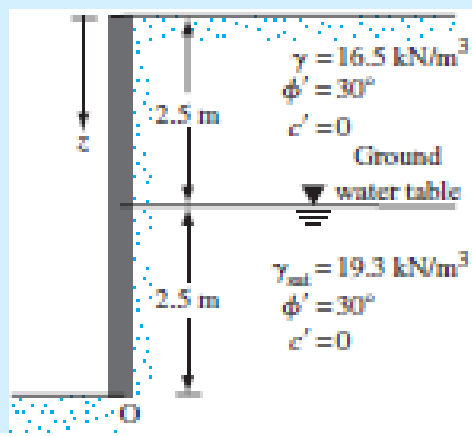
So,

$$P_o = K_o q H_1 + \tfrac{1}{2} K_o \gamma H_1^2 + K_o(q + \gamma H_1)H_2 + \tfrac{1}{2} K_o \gamma' H_2^2 + \tfrac{1}{2} \gamma_w H_2^2$$

EXAMPLE 12.1

Example 12.1

For the retaining wall shown in Figure 12.5a, determine the lateral earth force at rest per unit length of the wall. Also determine the location of the resultant force. Assume OCR = 1.



Solution

$$K_o = 1 - \sin\phi' = 1 - \sin 30° = 0.5$$

At $z = 0$, $\sigma_o' = 0$; $\sigma_h' = 0$

At $z = 2.5$ m, $\sigma_o' = (16.5)(2.5) = 41.25$ kN/m^2;

$$\sigma_h' = K_o\sigma_o' = (0.5)(41.25) = 20.63 \text{ kN/m}^2$$

At $z = 5$ m, $\sigma_o' = (16.5)(2.5) + (19.3 - 9.81)2.5 = 64.98$ kN/m^2;

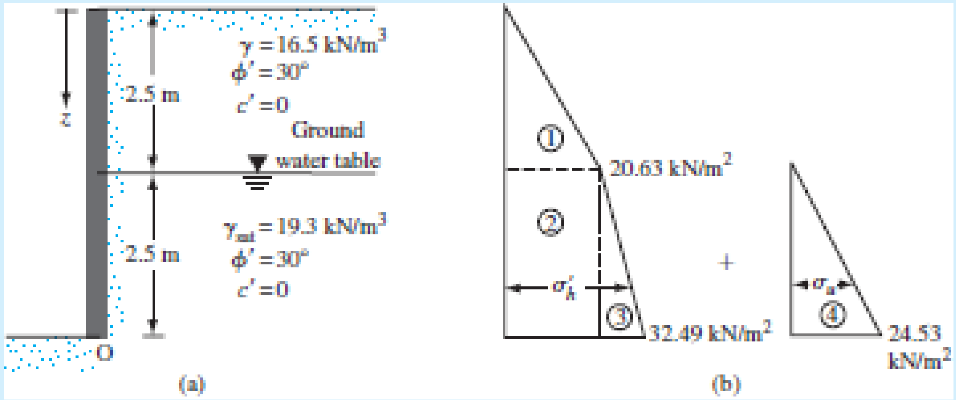
$$\sigma_h' = K_o\sigma_o' = (0.5)(64.98) = 32.49 \text{ kN/m}^2$$

The hydrostatic pressure distribution is as follows:

From $z = 0$ to $z = 2.5$ m, $u = 0$. At $z = 5$ m, $u = \gamma_w(2.5) = (9.81)(2.5) = 24.53$ kN/m^2.

The pressure distribution for the wall is shown in Figure 12.5b.

EXAMPLE 12.1



The total force per unit length of the wall can be determined from the area of the pressure diagram, or

$$P_a = \text{Area 1} + \text{Area 2} + \text{Area 3} + \text{Area 4}$$
$$= \tfrac{1}{2}(2.5)(20.63) + (2.5)(20.63) + \tfrac{1}{2}(2.5)(32.49 - 20.63)$$
$$+ \tfrac{1}{2}(2.5)(24.53) = \mathbf{122.85\ kN/m}$$

The location of the center of pressure measured from the bottom of the wall (point O) =

$$\bar{z} = \frac{(\text{Area 1})\left(2.5 + \frac{2.5}{3}\right) + (\text{Area 2})\left(\frac{2.5}{2}\right) + (\text{Area 3} + \text{Area 4})\left(\frac{2.5}{3}\right)}{P_a}$$

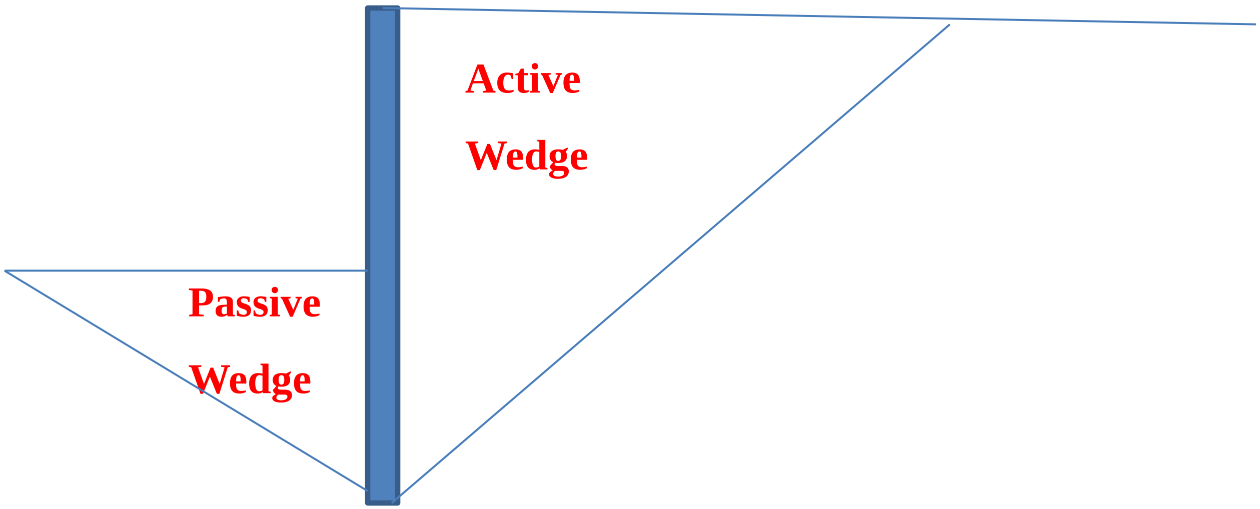
$$= \frac{(25.788)(3.33) + (51.575)(1.25) + (14.825 + 30.663)(0.833)}{122.85}$$

$$= \frac{85.87 + 64.47 + 37.89}{122.85} = \mathbf{1.53\ m}$$

NOTES

- If the lateral strain in the soil is ZERO the corresponding lateral pressure is called the earth pressure at-rest. This is the case before construction.
- In the case of active case the soil is the actuating element and in the case of passive the wall is the actuating element.
- For either the active or passive states to develop, the wall must MOVE. If the wall does not move, an intermediate stress state exists called earth pressure at rest. (i.e. zero lateral strain).
- For greatest economy, retaining structures are designed only sufficiently strong to resist ACTIVE PRESSURE. They therefore must be allowed to move.
- It may at first seem unlikely that a wall ever would be built to PUSH into the soil and mobilize passive earth pressure.

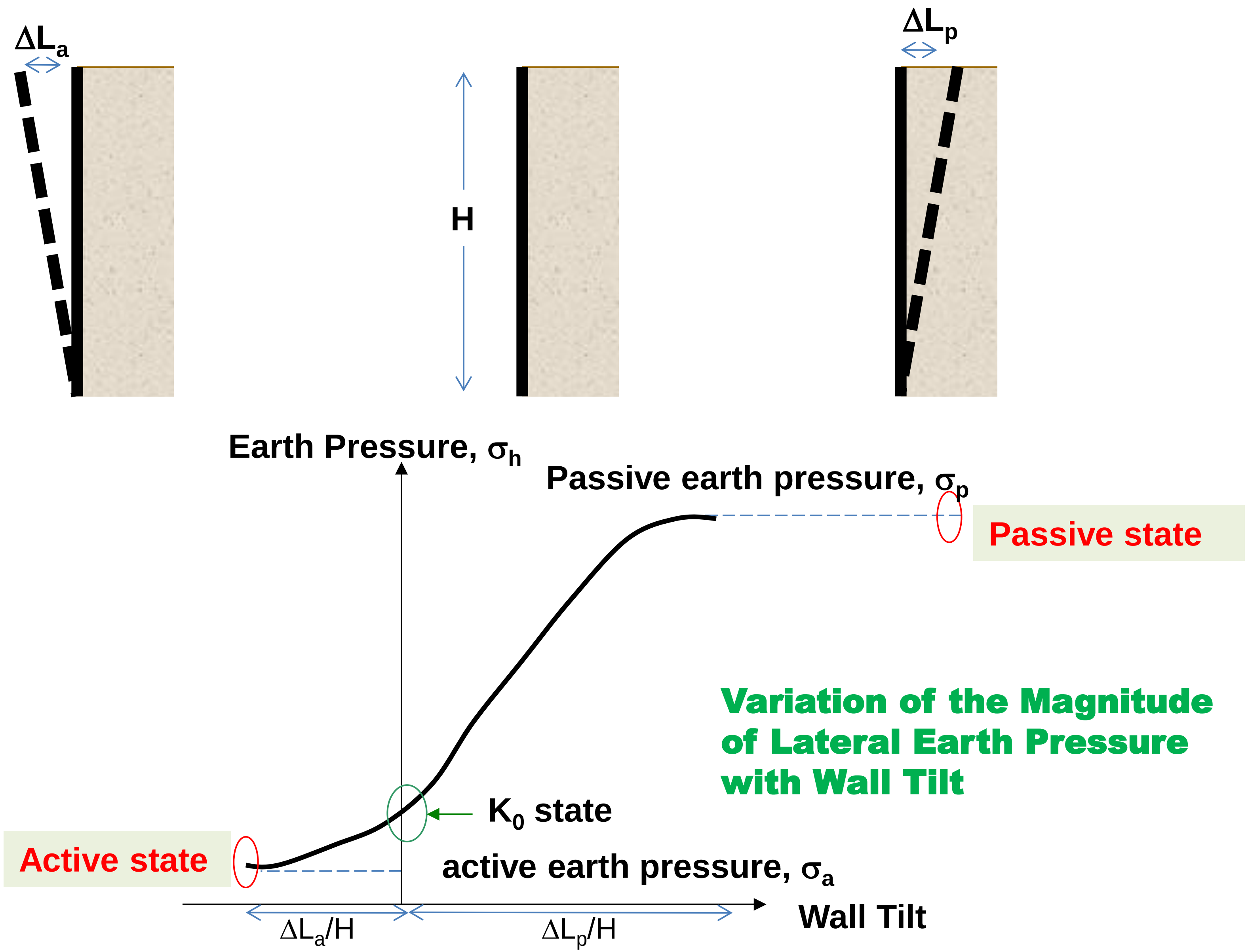
NOTES



- Typically passive earth pressure is developed by anchor plates or blocks, embedded in the soil and where the anchor rod or cable tension pulls the anchor into/against the soil to develop passive resistance. Walls are seldom designed for passive pressure.

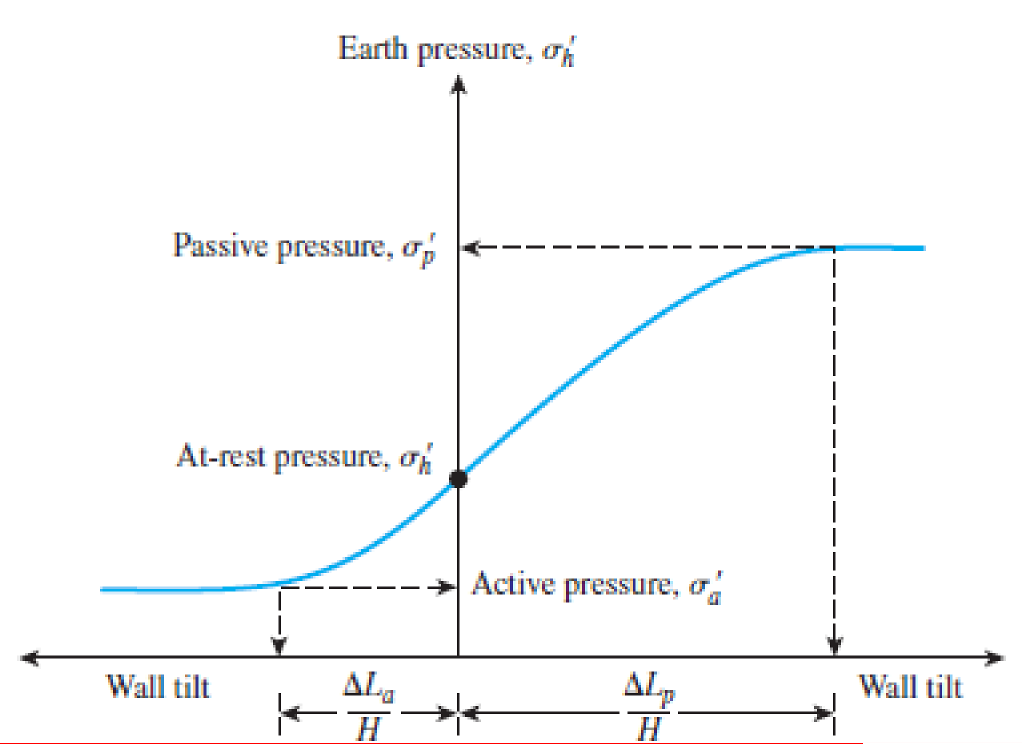
- In most retaining walls of limited height, movement may occur by simple translation or, more frequently, by rotation about the bottom.

NOTES



Variation of the Magnitude of Lateral Earth Pressure with Wall Tilt

NOTES



Active or passive condition will only be reached if the wall is allowed to yield sufficiently. The amount of wall necessary depends on:-

- **Soil type (sand vs. clay)**
- **Soil density (Loose vs. dense)**
- **Pressure (Active vs. passive)**

Typical Values of $\Delta L_a/H$ and $\Delta L_p/H$

Soil type	$\Delta L_a/H$	$\Delta L_p/H$
Loose sand	0.001–0.002	0.01
Dense sand	0.0005–0.001	0.005
Soft clay	0.02	0.04
Stiff clay	0.01	0.02

Lateral Earth Pressure Theories

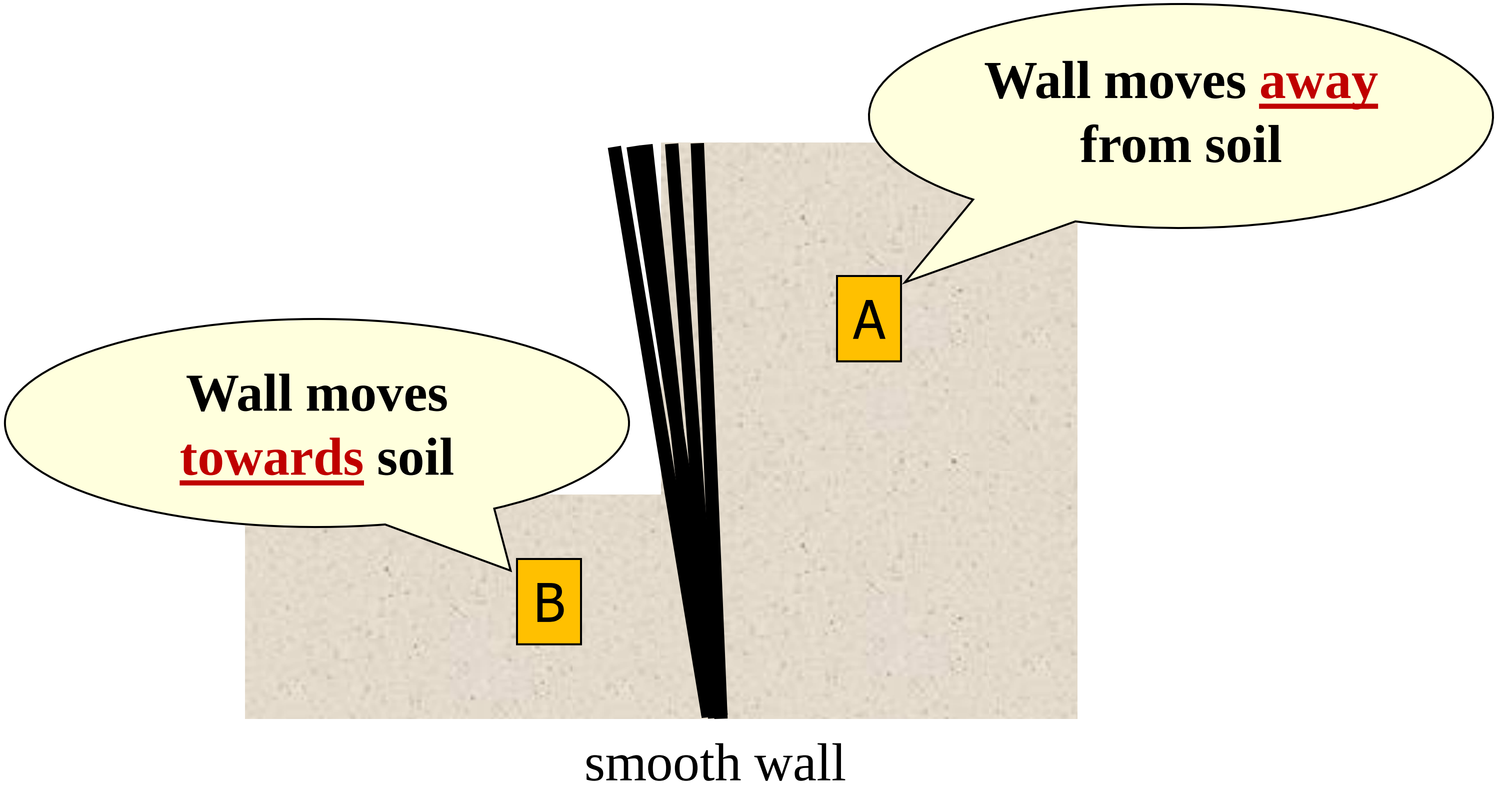
- Since late 17th century many theories of earth of earth pressure have been proposed by various investigators. Of the theories the following two are the most popular and used for computation of active and passive earth pressures:.

  1. Rankine's Theory (No wall friction)
  2. Coulomb's Theory (With wall friction)

- Those are usually called the classical lateral earth pressure theories.

- In both theories it is required that the soil mass, or at least certain parts of the mass, is in a state of PLASTIC EQUILIBRIUM. The soil mass is on verge of failure. Failure here is defined to be the state of stress which satisfies the Mohr-Coulomb criterion.

Active vs. Passive Earth Pressures



Let's look at the soil elements A and B during the wall movement.

- **In most retaining walls of limited height, movement may occur by simple translation or, more frequently, by rotation about the bottom.**

Rankine's Earth Pressure Theory

- Rankine (1857) investigated the stress condition in a soil at a state of PLASTIC EQUILIBRIUM.
- Developed based on semi infinite "loose granular" soil mass for which the soil movement is uniform.
- Used stress states of soil mass to determine lateral pressures on a frictionless wall

Assumptions:

- Vertical wall
- Smooth retaining wall
- Horizontal ground surface
- Homogeneous soil

Active Earth Pressure

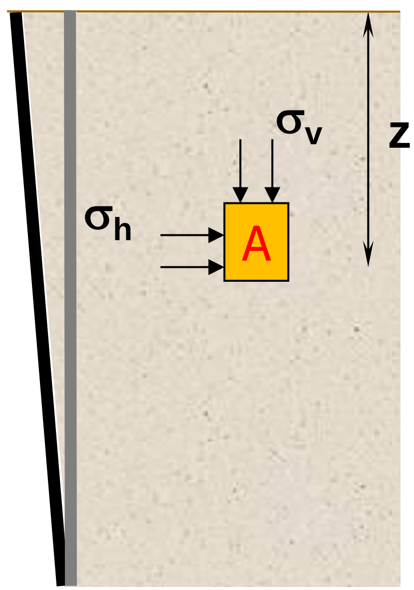
Active earth pressure

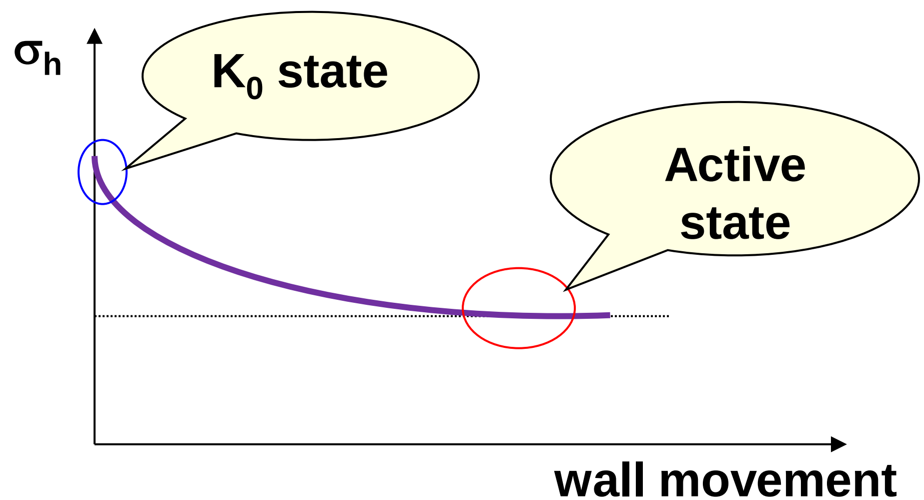
- $\sigma_v = \gamma z$
- Initially, there is no lateral movement.

  $\therefore \sigma_h = K_0 \sigma_v = K_0 \gamma z$
- As the wall moves away from the soil,
- σ_v remains the same; and
- σ_h decreases till failure occurs.

Active state

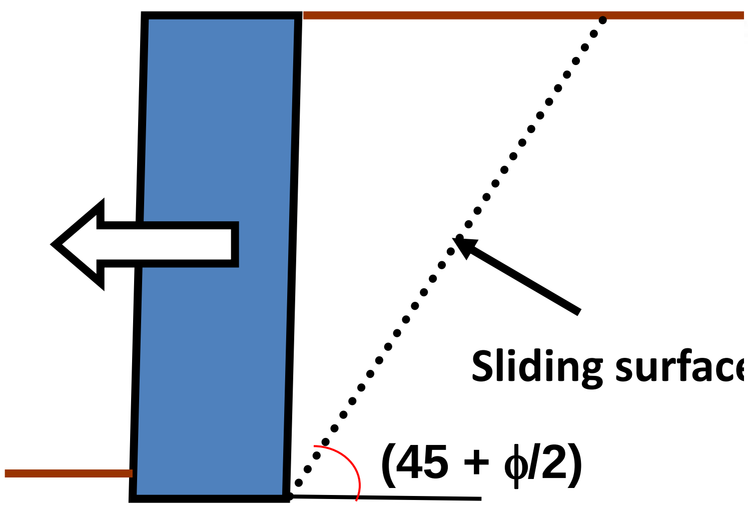
$\sigma_h \rightarrow \sigma_a$

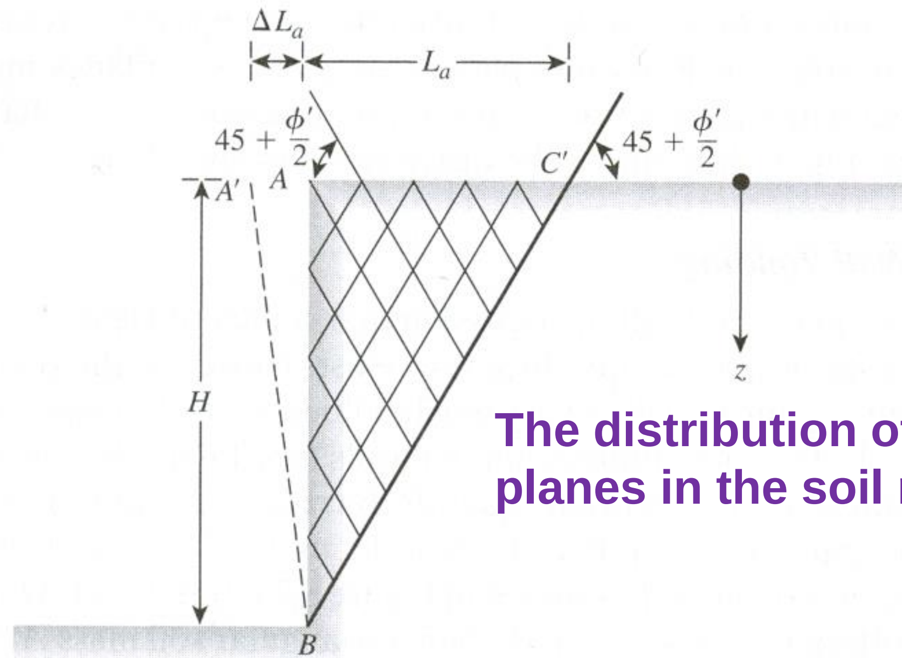




Orientation of Failure Planes

- From Mohr Circle the failure planes in the soil make **± (45 + ϕ/2)**-degree angles with the direction of the major principal plane—that is, the horizontal.
- These are called potential ***slip planes.***

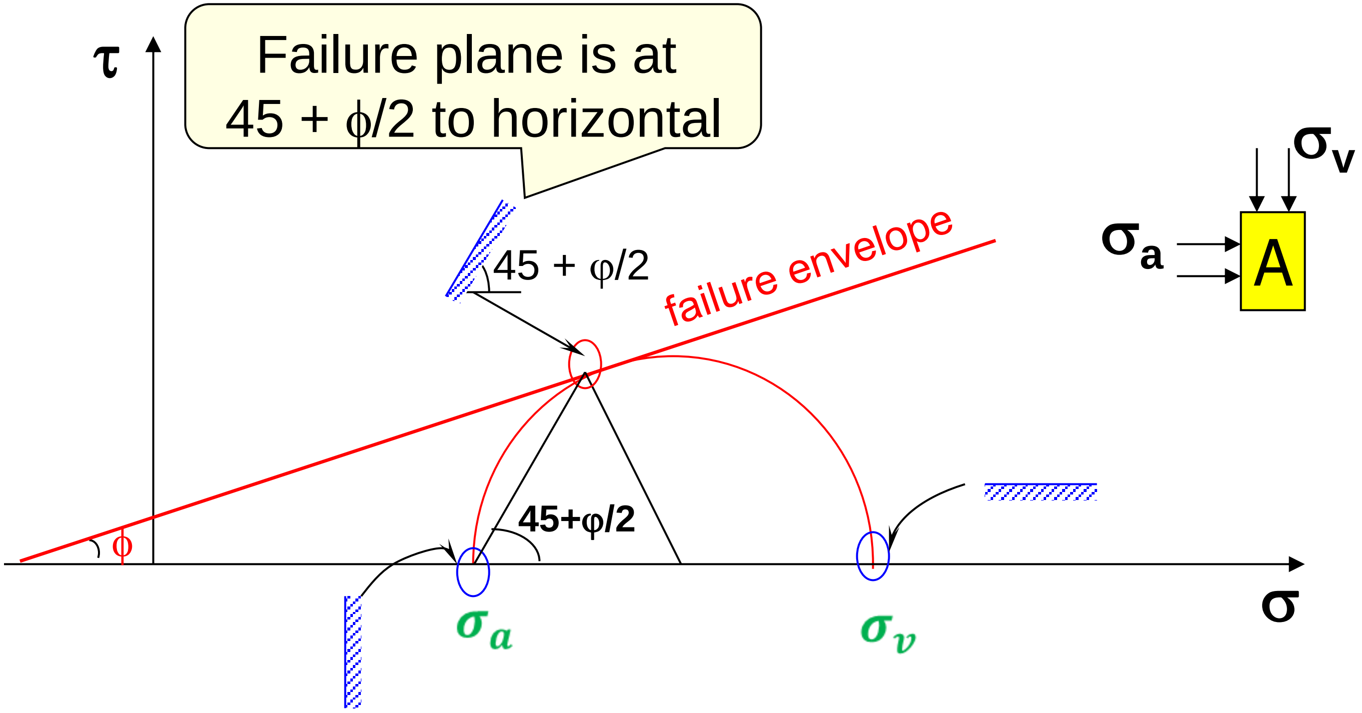




The distribution of slip planes in the soil mass.

- **Because the slip planes make angles of (45 + ϕ/2) degrees with the major principal plane, the soil mass in the state of plastic equilibrium is bounded by the plane *BC'*. The soil inside the zone *ABC'* undergoes the same unit deformation in the horizontal direction everywhere, which is equal to $\Delta L_a/L_a$.**

Active Earth Pressure



Active Earth Pressure

Retaining wall with a vertical back and a horizontal backfill

Mohr–Coulomb failure envelope defined by the equation

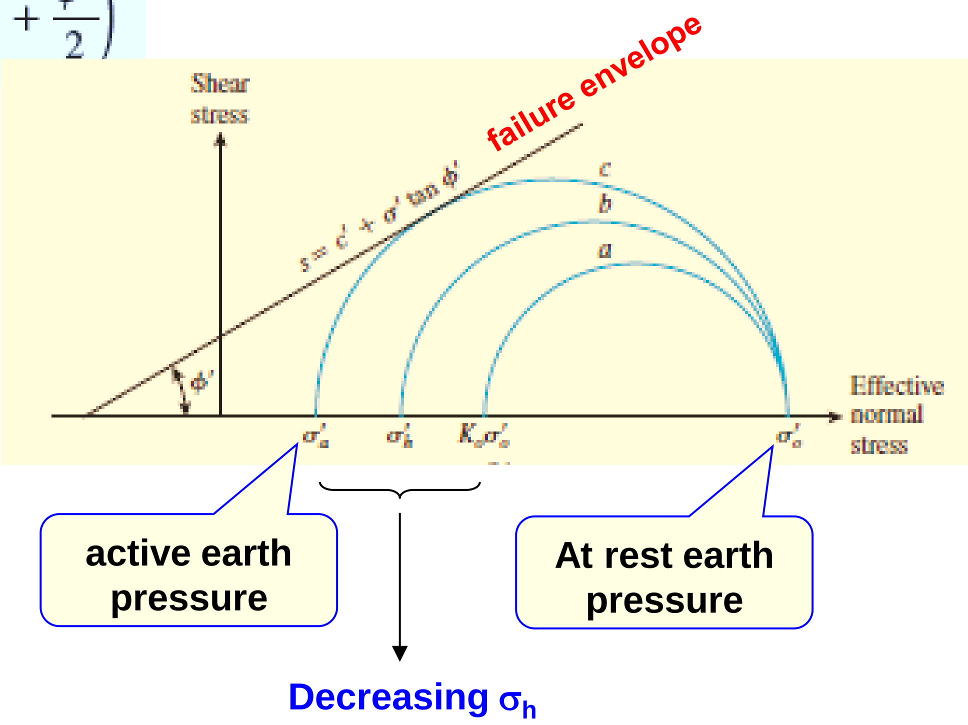
$$s = c' + \sigma' \tan\phi'$$

The principal stresses for a Mohr's circle that touches the Mohr–Coulomb failure envelope:

$$\sigma_1' = \sigma_3' \tan^2\left(45 + \frac{\phi'}{2}\right) + 2c' \tan\left(45 + \frac{\phi'}{2}\right)$$

$$\sigma_o' = \sigma_a' \tan^2\left(45 + \frac{\phi'}{2}\right) + 2c' \tan\left(45 + \frac{\phi'}{2}\right)$$

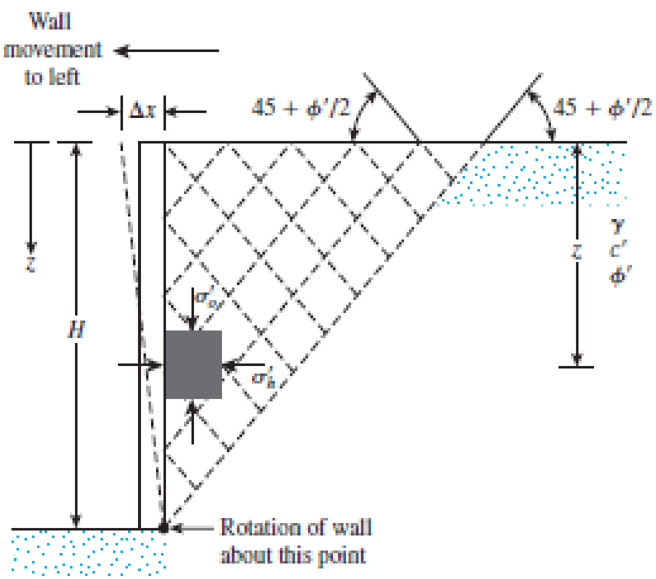
$$\sigma_a' = \frac{\sigma_o'}{\tan^2\left(45 + \frac{\phi'}{2}\right)} - \frac{2c'}{\tan\left(45 + \frac{\phi'}{2}\right)}$$

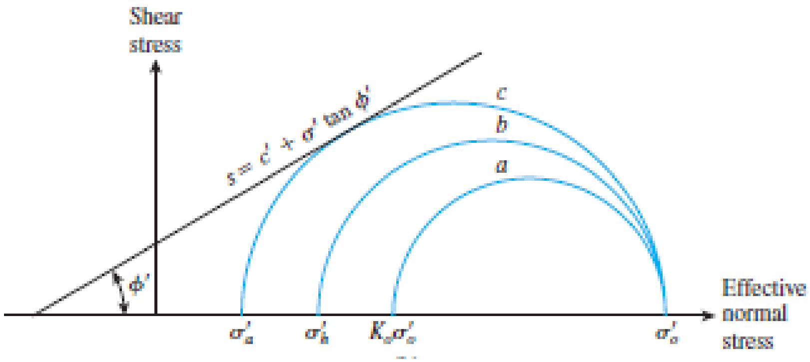


$$\sigma_a' = \sigma_o' \tan^2\left(45 - \frac{\phi'}{2}\right) - 2c' \tan\left(45 - \frac{\phi'}{2}\right)$$
$$= \sigma_o' K_a - 2c'\sqrt{K_a}$$

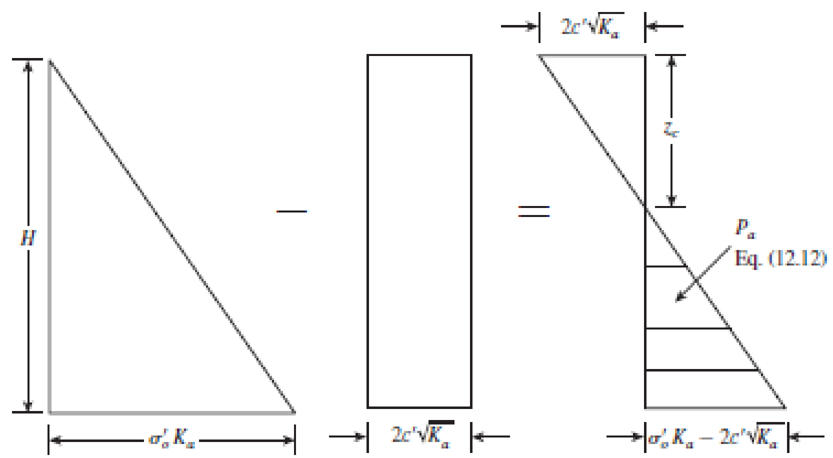
$K_a = \tan^2(45 - \phi'/2) =$ Rankine active-pressure coefficient.

Active Earth Pressure





$$\sigma'_a = \sigma'_o \tan^2\left(45 - \frac{\phi'}{2}\right) - 2c' \tan\left(45 - \frac{\phi'}{2}\right)$$
$$= \sigma'_o K_a - 2c'\sqrt{K_a}$$



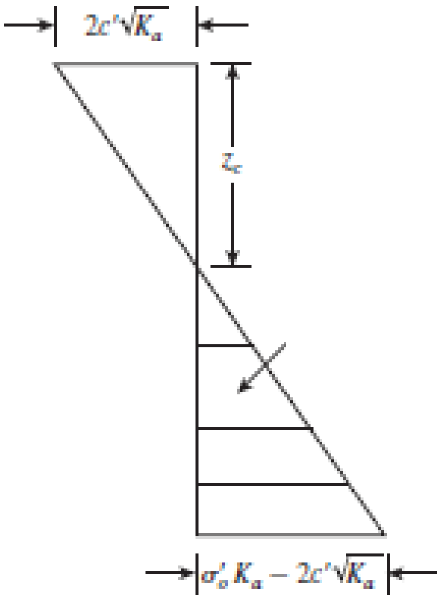
Active Earth Pressure

Tensile stress in the soil will cause a crack along the soil–wall interface

The ***depth of tensile crack***

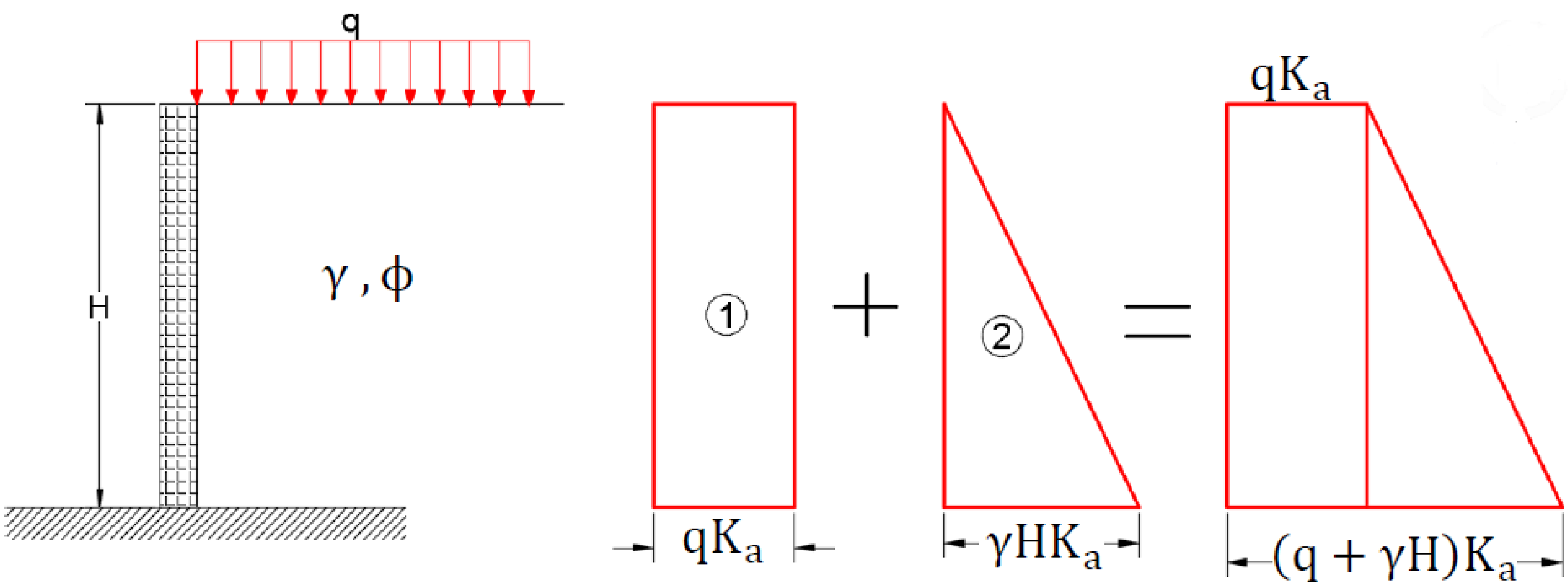
$$\gamma z_c K_a - 2c'\sqrt{K_a} = 0$$

$$z_c = \frac{2c'}{\gamma\sqrt{K_a}}$$



Active Earth Pressure

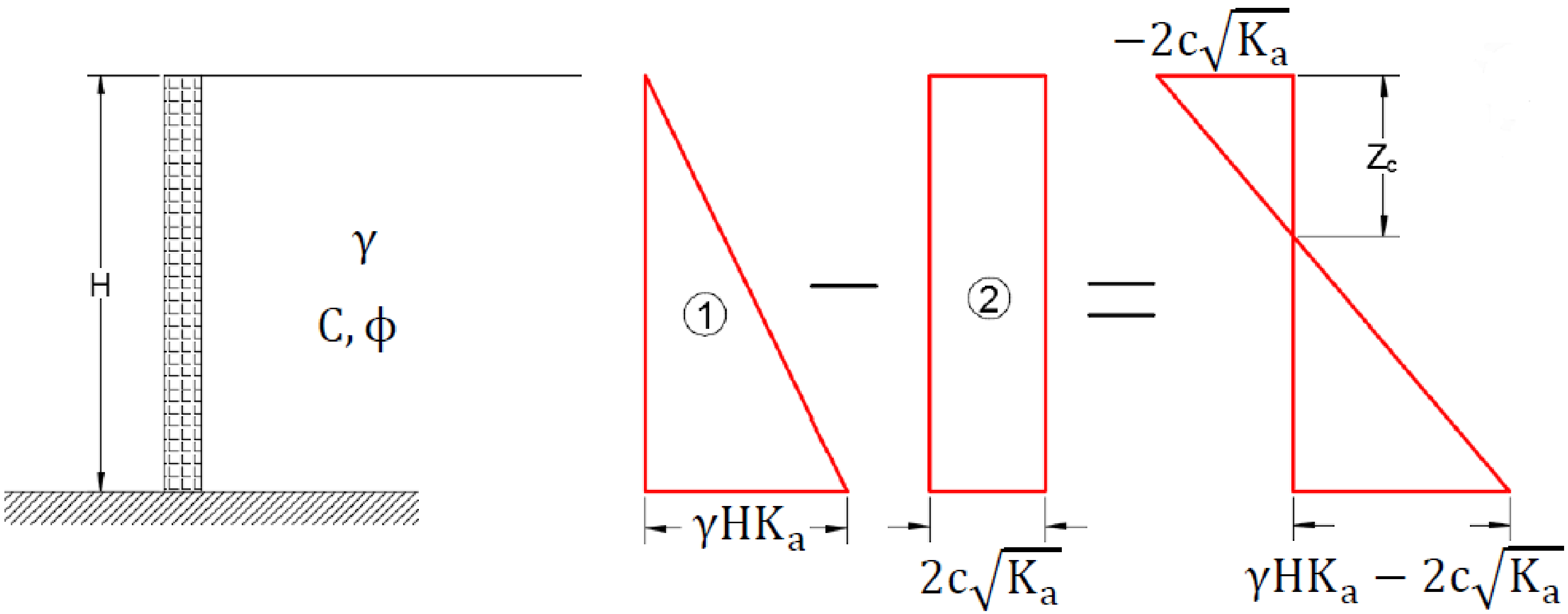
In case of granular soil (pure sand):



$$P_a = P_1 + P_2$$

Active Earth Pressure

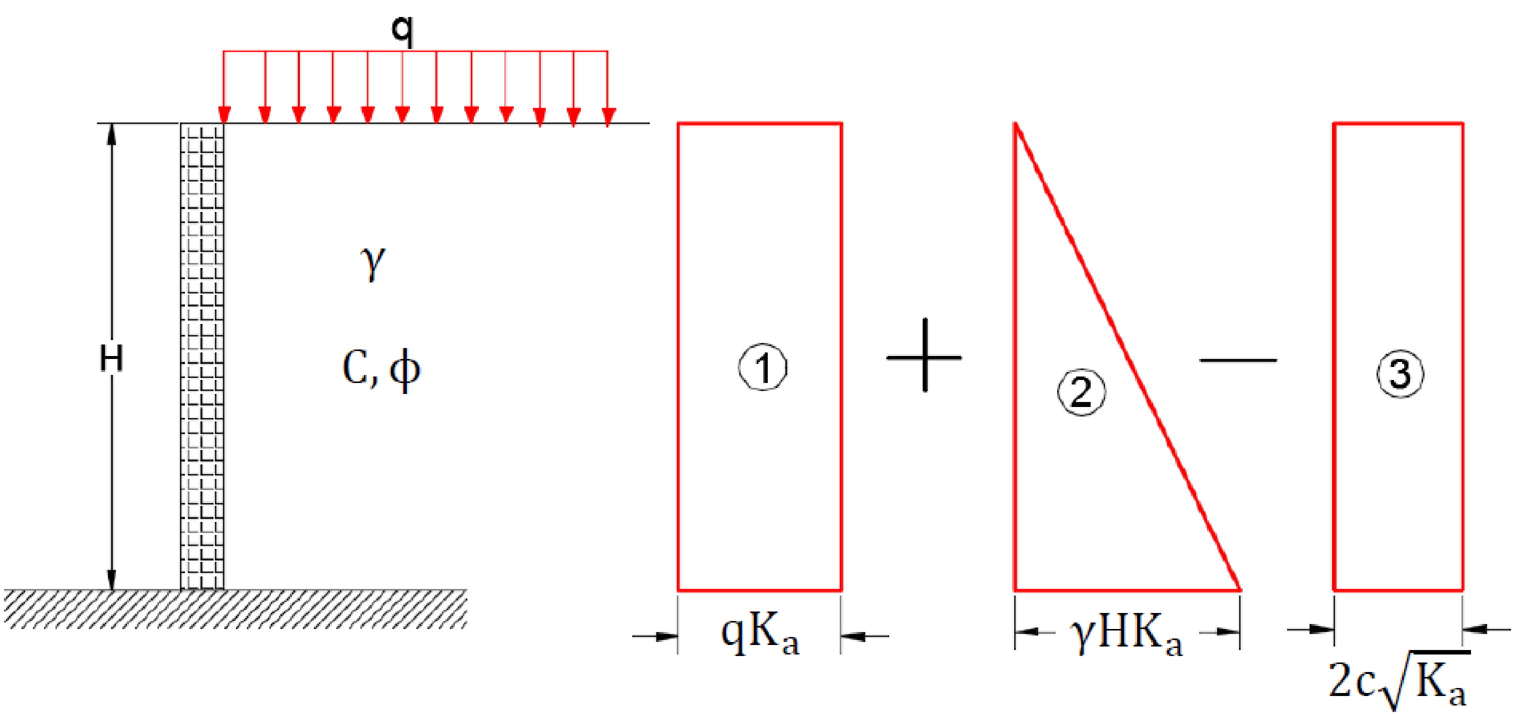
If the soil is $C - \phi$ soil:



Active Earth Pressure

If the soil is C − ϕ soil:

If there exist surcharge:



$$\sigma_{h,a} = (q + \gamma H)K_a - 2c\sqrt{K_a}$$

EXAMPLE 12.2

Example 12.2

A 6-m-high retaining wall is to support a soil with unit weight $\gamma = 17.4\ \text{kN/m}^3$, soil friction angle $\phi' = 26°$, and cohesion $c' = 14.36\ \text{kN/m}^2$. Determine the Rankine active force per unit length of the wall both before and after the tensile crack occurs, and determine the line of action of the resultant in both cases.

Solution

Before tensile crack occurs

For $\phi' = 26°$,

$$K_a = \tan^2\left(45 - \frac{\phi'}{2}\right) = \tan^2(45 - 13) = 0.39$$

$$\sqrt{K_a} = 0.625$$

$$\sigma'_a = \gamma H K_a - 2c'\sqrt{K_a}$$

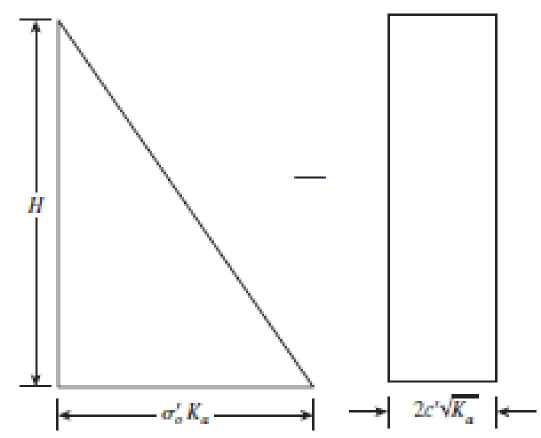
From Figure 12.6c, at $z = 0$,

$$\sigma'_a = -2c'\sqrt{K_a} = -2(14.36)(0.625) = -17.95\ \text{kN/m}^2$$

and at $z = 6$ m,

$$\sigma'_a = (17.4)(6)(0.39) - 2(14.36)(0.625)$$
$$= 40.72 - 17.95 = 22.77\ \text{kN/m}^2$$

$$P_a = \tfrac{1}{2}\gamma H^2 K_a - 2c'H\sqrt{K_a}$$
$$= \tfrac{1}{2}(6)(40.72) - (6)(17.95) = 122.16 - 107.7 = \mathbf{14.46\ kN/m}$$



EXAMPLE 12.2

The line of action of the resultant can be determined by taking the moment of the area of the pressure diagrams about the bottom of the wall, or

$$P_a\bar{z} = (122.16)\left(\frac{6}{3}\right) - (107.7)\left(\frac{6}{2}\right)$$

Thus,

$$\bar{z} = \frac{244.32 - 323.1}{14.46} = \mathbf{-5.45\ m.}$$

After tensile crack occurs

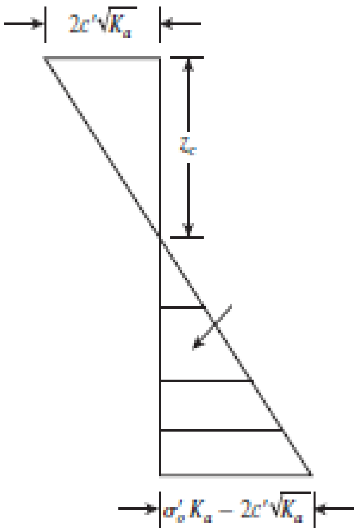
$$z_c = \frac{2c'}{\gamma\sqrt{K_a}} = \frac{2(14.36)}{(17.4)(0.625)} = 2.64\ \text{m}$$

Using Eq. (12.11) gives

$$P_a = \tfrac{1}{2}(H - z_c)(\gamma H K_a - 2c'\sqrt{K_a}) = \tfrac{1}{2}(6 - 2.64)(22.77) = \mathbf{38.25\ kN/m}$$

Figure 12.6c indicates that the force $P_a = 38.25$ kN/m is the area of the hatched triangle. Hence, the line of action of the resultant will be located at a height $\bar{z} = (H - z_c)/3$ above the bottom of the wall, or

$$\bar{z} = \frac{6 - 2.64}{3} = \mathbf{1.12\ m}$$ ■

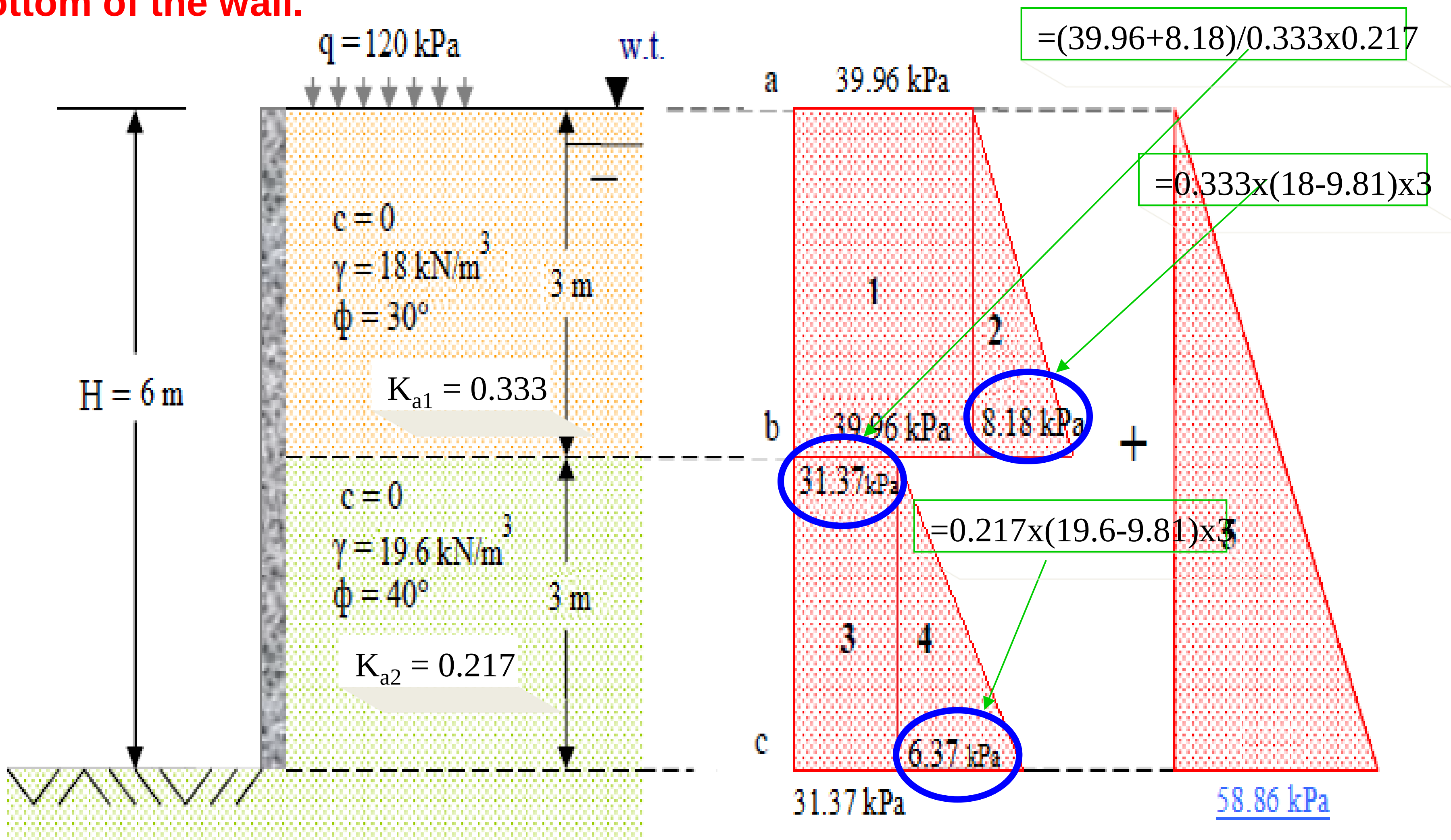


EXAMPLE 12.3

READ EXAMPLE 12.3

EXAMPLE

Draw the pressure diagram on the wall in an active pressure condition, and find the total resultant F on the wall and its location with respect to the bottom of the wall.



SOLUTION

Step 1

$K_{a1} = \tan^2 (45° - 30°/2) = 0.333$

$K_{a2} = \tan^2 (45° - 40°/2) = 0.217$

Step 2

The stress on the wall at point *a* is:

$p_a = q\,K_{a1} = (120)\,(0.333) = \underline{39.96\ \text{kPa}}$

The stress at *b* (within the top stratum) is:

$p_{b-} = (q + \gamma' h)\,K_{a1}$

$= [120 + (18 - 9.81)\,(3)]\,[0.333] = \underline{48.14\ \text{kPa}}$

The stress at *b* (within bottom stratum) is:

$p_{b+} = (q + \gamma' h)\,K_{a2}$

$= [120 + (18 - 9.81)\,(3)]\ [0.217] = \underline{31.37\ \text{kPa}}$

The stress at point *c* is:

$p_{c'} = [q + (\gamma' h)_1 + (\gamma' h)_2]\,K_{a2}$

$= [120 + (18 - 9.81)\,(3) + (19.6 - 9.81)\,(3)]\,[0.217] = \underline{37.75\ \text{kPa}}$

The pressure of the water upon the wall is:

$p_w = \gamma_w h = (9.81)\,(6) = \underline{58.86\ \text{kPa}}$

SOLUTION

Step 3

The forces from each area:

$F_1 = (3)\,(39.96) = 119.88$ kN/m

$F_2 = ½\,(3)\,(8.18) = 12.27$ kN/m

$F_3 = (3)\,(31.37) = 94.11$ kN/m

$F_4 = ½\,(3)\,(6.37) = 9.555$ kN/m

$F_5 = ½\,(58.86)\,(6) = 176.58$ kN/m

$\mathbf{F_{total}} = \mathbf{412.395}$ kN/m

Step 4

The location of forces ŷ is at:

ŷ **412.395** =119.88 (4.5) + 12.27 (4) + 94.11 (1.5) + 9.555 (1) + 176.58 (2)

=539.46+ 49.08 + 141.165 + 9.555 + 353.16 = 1092.42 kN

ŷ = 2.65 m from bottom of wall

General Case: Inclined Wall with Inclined Backfill

Granular backfill (c' =0)

α = inclination of backfill with horizontal

θ = inclination of wall with vertical

β = inclination of P_a with the normal to the wall

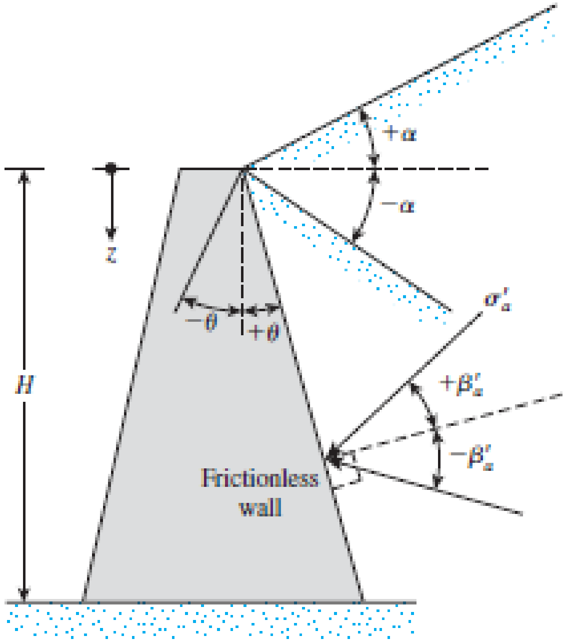
$$\sigma'_a = \frac{\gamma z \cos\alpha \sqrt{1 + \sin^2\phi' - 2\sin\phi' \cos\psi_a}}{\cos\alpha + \sqrt{\sin^2\phi' - \sin^2\alpha}}$$

$$\psi_a = \sin^{-1}\left(\frac{\sin\alpha}{\sin\phi'}\right) - \alpha + 2\theta.$$

$$\beta'_a = \tan^{-1}\left(\frac{\sin\phi' \sin\psi_a}{1 - \sin\phi' \cos\psi_a}\right)$$

$$P_a = \frac{1}{2}\gamma H^2 K_a$$

$$K_{a(R)} = \frac{\cos(\alpha - \theta)\sqrt{1 + \sin^2\phi' - 2\sin\phi' \cos\psi_a}}{\cos^2\theta\left(\cos\alpha + \sqrt{\sin^2\phi' - \sin^2\alpha}\right)}$$



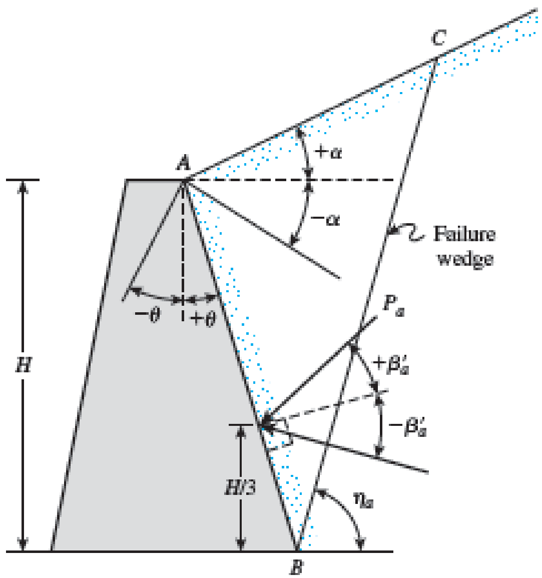
General Case : Inclined Wall with Inclined Backfill

Granular backfill (c' =0)

$$P_a = \frac{1}{2}\gamma H^2 K_a$$

$$K_{a(R)} = \frac{\cos(\alpha - \theta)\sqrt{1 + \sin^2\phi' - 2\sin\phi'\cos\psi_a}}{\cos^2\theta\left(\cos\alpha + \sqrt{\sin^2\phi' - \sin^2\alpha}\right)}$$

where $\psi_a = \sin^{-1}\left(\frac{\sin\alpha}{\sin\phi'}\right) - \alpha + 2\theta.$ $\quad \beta'_a = \tan^{-1}\left(\frac{\sin\phi'\sin\psi_a}{1 - \sin\phi'\cos\psi_a}\right)$



$$\eta_a = \frac{\pi}{4} + \frac{\phi'}{2} + \frac{\alpha}{2} - \frac{1}{2}\sin^{-1}\left(\frac{\sin\alpha}{\sin\phi'}\right)$$

Table 12.1 gives the variation of K_a for various values of α, θ, and ϕ'

Table 12.2 gives the variation of β_a' for various values of α, θ, and ϕ

Table 12.1 Variation of $K_{a(R)}$ [Eq. (12.17)]

α (deg)	θ (deg)	$K_{a(R)}$, ϕ' (deg) 28	30	32	34	36	38	40
	0	0.361	0.333	0.307	0.283	0.260	0.238	0.217
	2	0.363	0.335	0.309	0.285	0.262	0.240	0.220
	4	0.368	0.341	0.315	0.291	0.269	0.248	0.228
0	6	0.376	0.350	0.325	0.302	0.280	0.260	0.242
	8	0.387	0.362	0.338	0.316	0.295	0.276	0.259
	10	0.402	0.377	0.354	0.333	0.314	0.296	0.280
	15	0.450	0.428	0.408	0.390	0.373	0.358	0.345

Table 12.2 Variation of β'_a [Eq. (12.15)]

α (deg)	θ (deg)	β'_a, ϕ' (deg) 28	30	32	34	36	38	40
	0	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	2	3.525	3.981	4.484	5.041	5.661	6.351	7.124
	4	6.962	7.848	8.821	9.893	11.075	12.381	13.827
0	6	10.231	11.501	12.884	14.394	16.040	17.837	19.797
	8	13.270	14.861	16.579	18.432	20.428	22.575	24.876
	10	16.031	17.878	19.850	21.951	24.184	26.547	29.039
	15	21.582	23.794	26.091	28.464	30.905	33.402	35.940

Vertical Wall with Inclined Backfill

Granular backfill (c' =0)

$$P_a = \frac{1}{2}\gamma H^2 K_a$$

$$K_a = \cos\alpha \frac{\cos\alpha - \sqrt{\cos^2\alpha - \cos^2\phi'}}{\cos\alpha + \sqrt{\cos^2\alpha - \cos^2\phi'}}$$

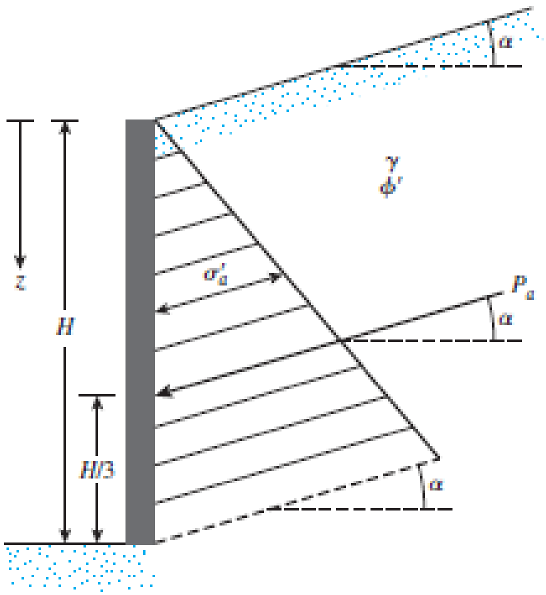


Table 12.3 Values of K_a [Eq. (12.19)]

α (deg)	ϕ' (deg) →												
↓	28	29	30	31	32	33	34	35	36	37	38	39	40
0	0.3610	0.3470	0.3333	0.3201	0.3073	0.2948	0.2827	0.2710	0.2596	0.2486	0.2379	0.2275	0.2174
1	0.3612	0.3471	0.3335	0.3202	0.3074	0.2949	0.2828	0.2711	0.2597	0.2487	0.2380	0.2276	0.2175
2	0.3618	0.3476	0.3339	0.3207	0.3078	0.2953	0.2832	0.2714	0.2600	0.2489	0.2382	0.2278	0.2177
3	0.3627	0.3485	0.3347	0.3214	0.3084	0.2959	0.2837	0.2719	0.2605	0.2494	0.2386	0.2282	0.2181
4	0.3639	0.3496	0.3358	0.3224	0.3094	0.2967	0.2845	0.2726	0.2611	0.2500	0.2392	0.2287	0.2186
5	0.3656	0.3512	0.3372	0.3237	0.3105	0.2978	0.2855	0.2736	0.2620	0.2508	0.2399	0.2294	0.2192

Vertical Wall with Inclined Backfill

(c'– ϕ') backfill

$$\sigma'_a = \gamma z K_a = \gamma z K'_a \cos\alpha$$

$$Z_c = \frac{2c}{\gamma}\sqrt{\frac{1+\sin\phi}{1-\sin\phi}}$$

$$P_a = \frac{1}{2} \times (\gamma H K'_a \cos\alpha) \times (H - Z_c)$$

$$K'_a = \frac{1}{\cos^2\phi'}\left\{\begin{array}{l} 2\cos^2\alpha + 2\left(\dfrac{c'}{\gamma z}\right)\cos\phi'\sin\phi' \\ -\sqrt{\left[4\cos^2\alpha(\cos^2\alpha - \cos^2\phi') + 4\left(\dfrac{c'}{\gamma z}\right)^2\cos^2\phi' + 8\left(\dfrac{c'}{\gamma z}\right)\cos^2\alpha\sin\phi'\cos\phi'\right]}\end{array}\right\} - 1$$

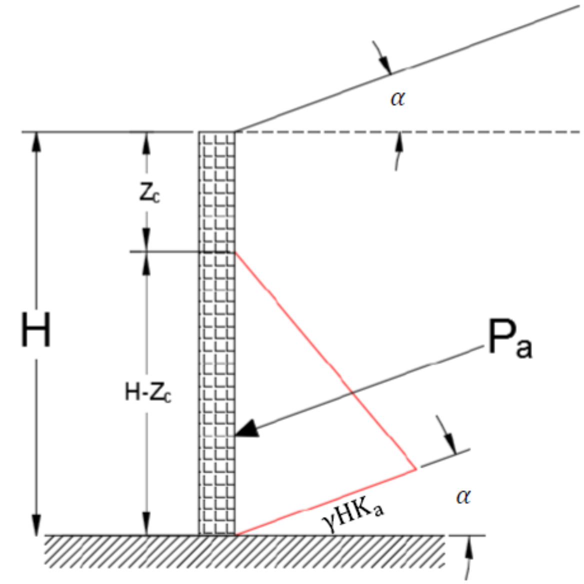


Table 12.4 Values of K'_a

		$\frac{c'}{\gamma z}$			
ϕ' (deg)	**α (deg)**	**0.025**	**0.05**	**0.1**	**0.5**
15	0	0.550	0.512	0.435	−0.179
	5	0.566	0.525	0.445	−0.184
	10	0.621	0.571	0.477	−0.186
	15	0.776	0.683	0.546	−0.196
20	0	0.455	0.420	0.350	−0.210
	5	0.465	0.429	0.357	−0.212
	10	0.497	0.456	0.377	−0.218
	15	0.567	0.514	0.417	−0.229
25	0	0.374	0.342	0.278	−0.231
	5	0.381	0.348	0.283	−0.233
	10	0.402	0.366	0.296	−0.239
	15	0.443	0.401	0.321	−0.250
30	0	0.305	0.276	0.218	−0.244
	5	0.309	0.280	0.221	−0.246
	10	0.323	0.292	0.230	−0.252
	15	0.350	0.315	0.246	−0.263

EXAMPLE 12.4

READ EXAMPLE 12.4

EXAMPLE 12.5

Example 12.5

For the retaining wall shown in Figure 12.10, $H = 7.5$ m, $\gamma = 18$ kN/m^3, $\phi' = 20°$, $c' = 13.5$ kN/m^2, and $\alpha = 10°$. Calculate the Rankine active force, P_a, per unit length of the wall and the location of the resultant force after the occurrence of the tensile crack.

Solution

From Eq. (12.24),

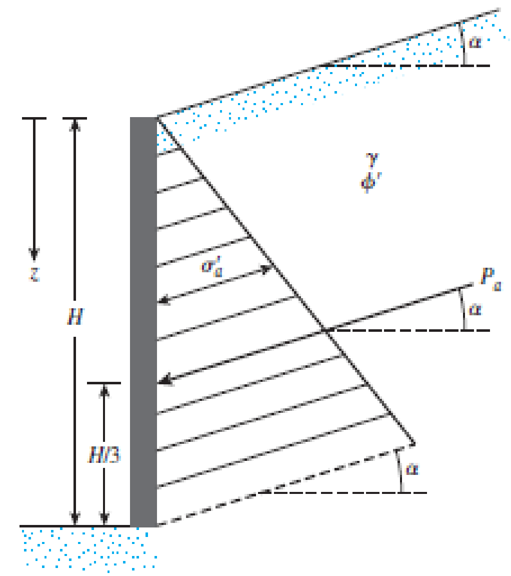
$$z_r = \frac{2c'}{\gamma}\sqrt{\frac{1+\sin\phi'}{1-\sin\phi'}} = \frac{(2)(13.5)}{18}\sqrt{\frac{1+\sin 20}{1-\sin 20}} = 2.14 \text{ m}$$

At $z = 7.5$ m,

$$\frac{c'}{\gamma z} = \frac{13.5}{(18)(7.5)} = 0.1$$

From Table 12.4, for $\phi' = 20°$, $c'/\gamma z = 0.1$, and $\alpha = 10°$, the value of K_a' is 0.377, so at $z = 7.5$ m,

$$\sigma_a' = \gamma z K_a' \cos\alpha = (18)(7.5)(0.377)(\cos 10) = 50.1 \text{ kN/m}^2$$



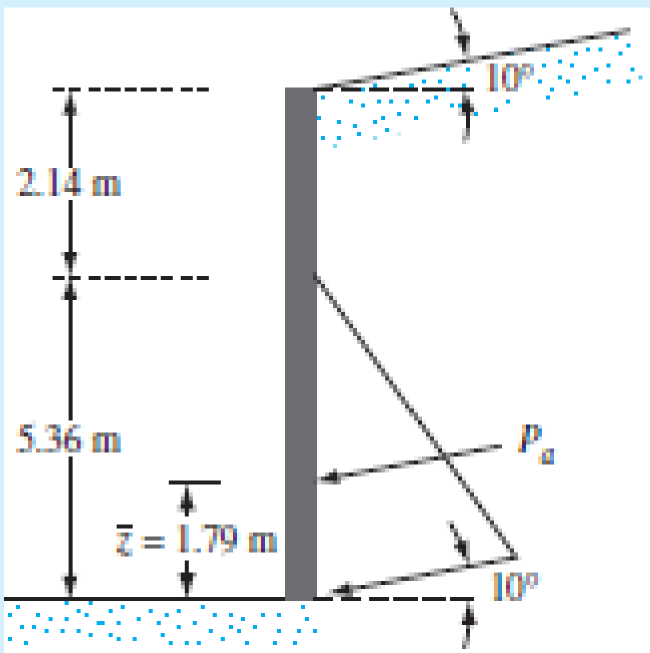
EXAMPLE 12.5

After the occurrence of the tensile crack, the pressure distribution on the wall will be as shown in Figure 12.11, so

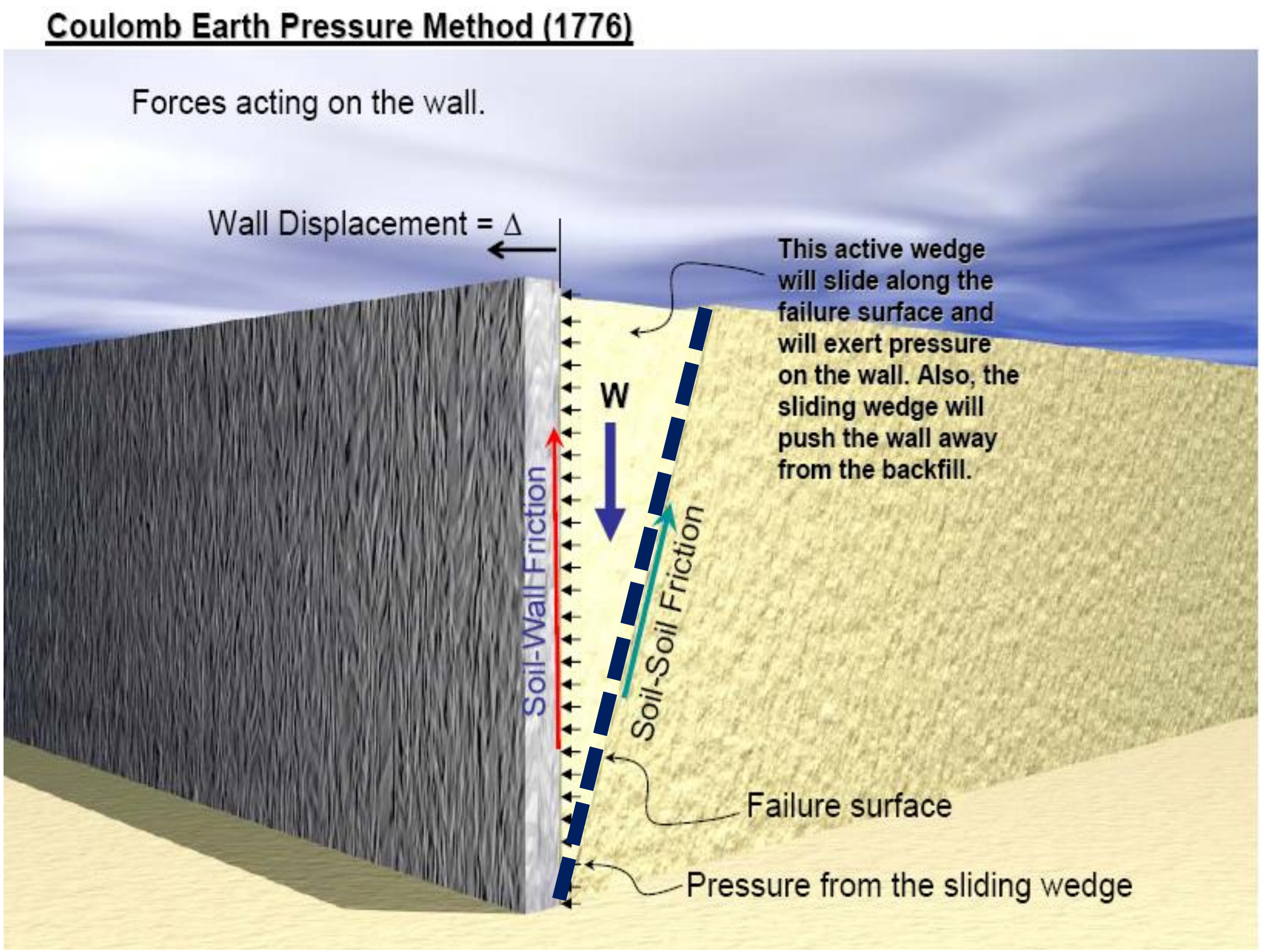
$$P_a = \left(\frac{1}{2}\right)(50.1)(7.5 - 2.14) = \mathbf{134.3\ kN/m}$$

and

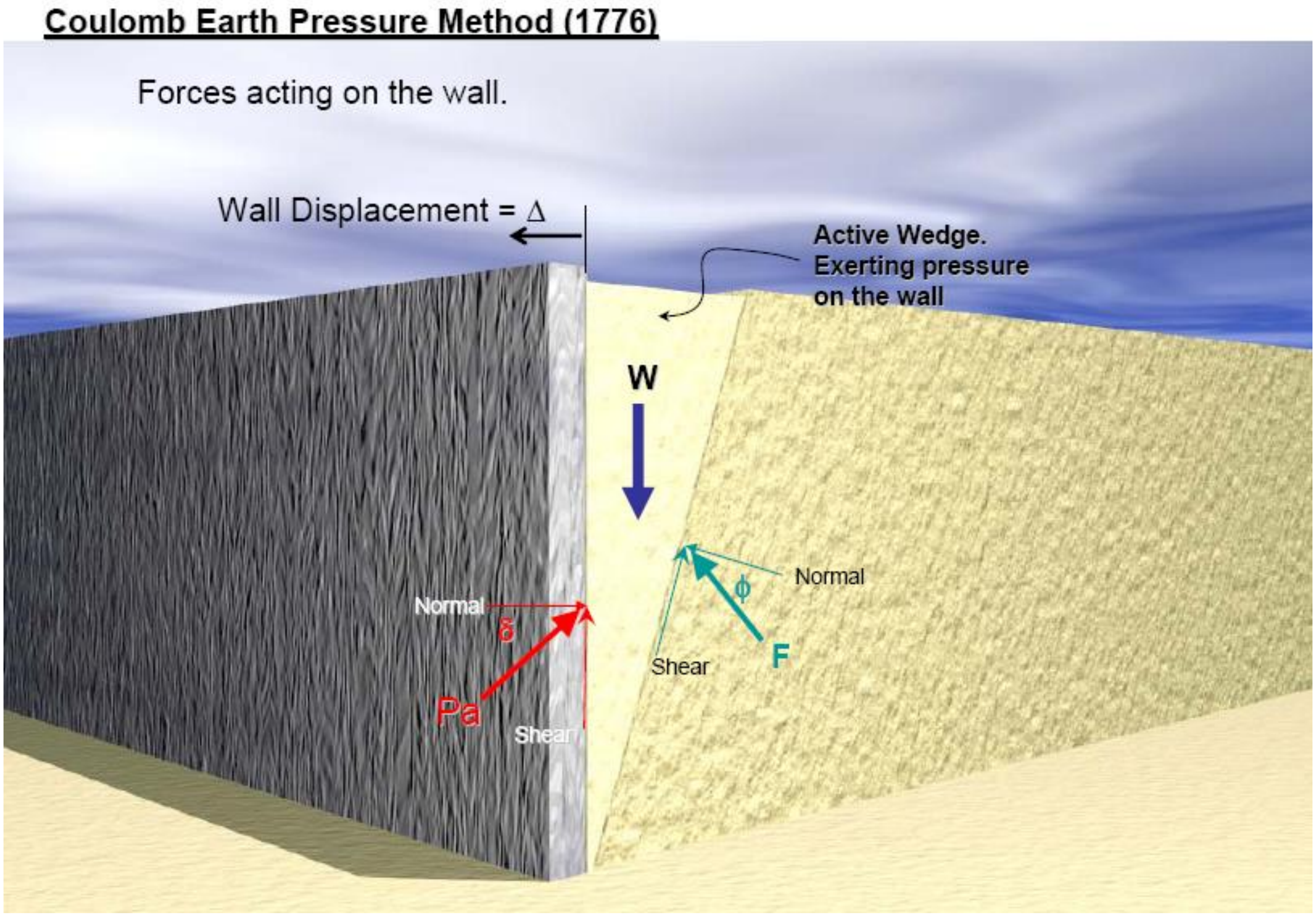
$$\bar{z} = \frac{7.5 - 2.14}{3} = \mathbf{1.79\ m}$$



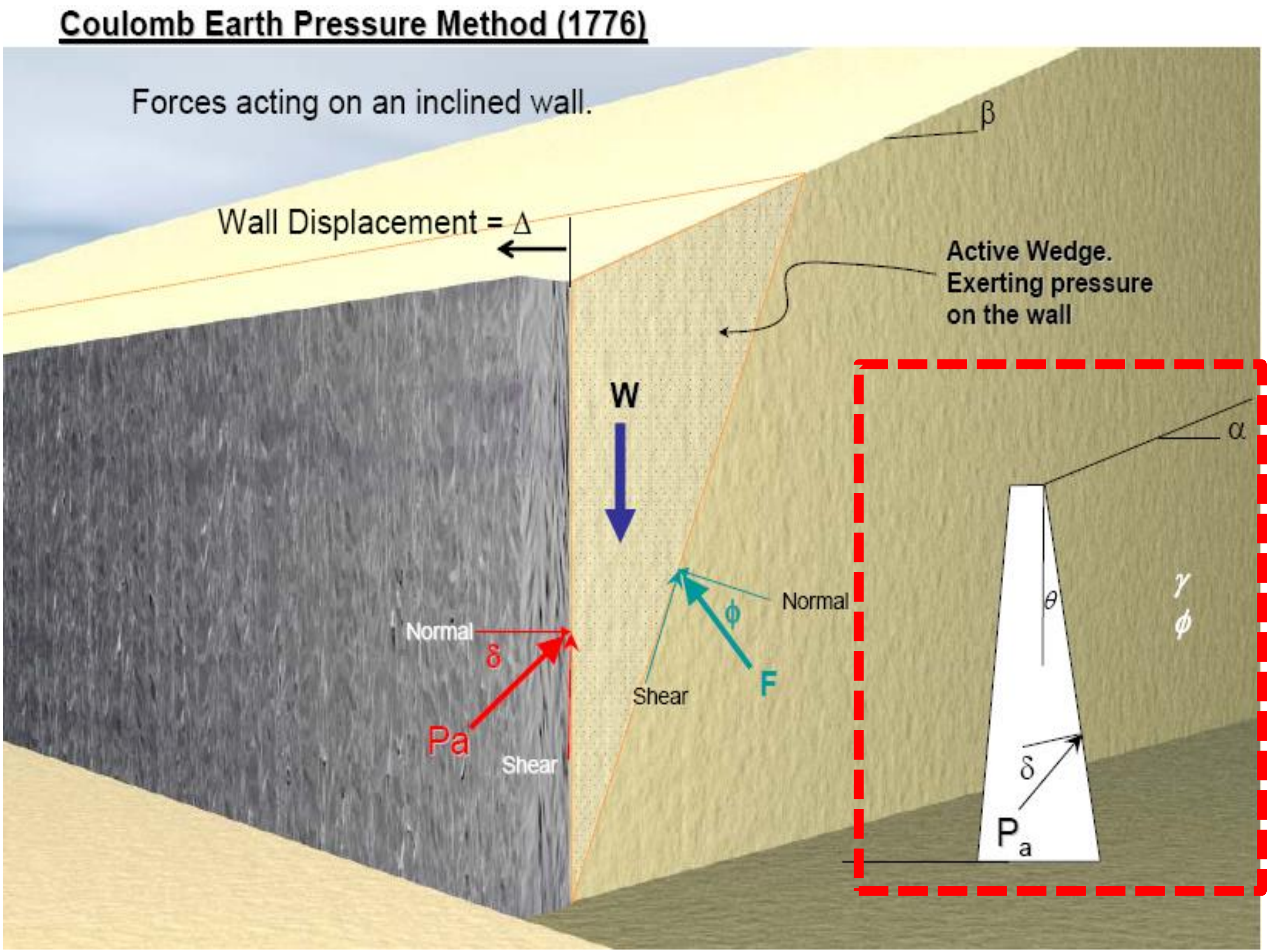
COULOMB'S EARTH PRESSURE



COULOMB'S EARTH PRESSURE



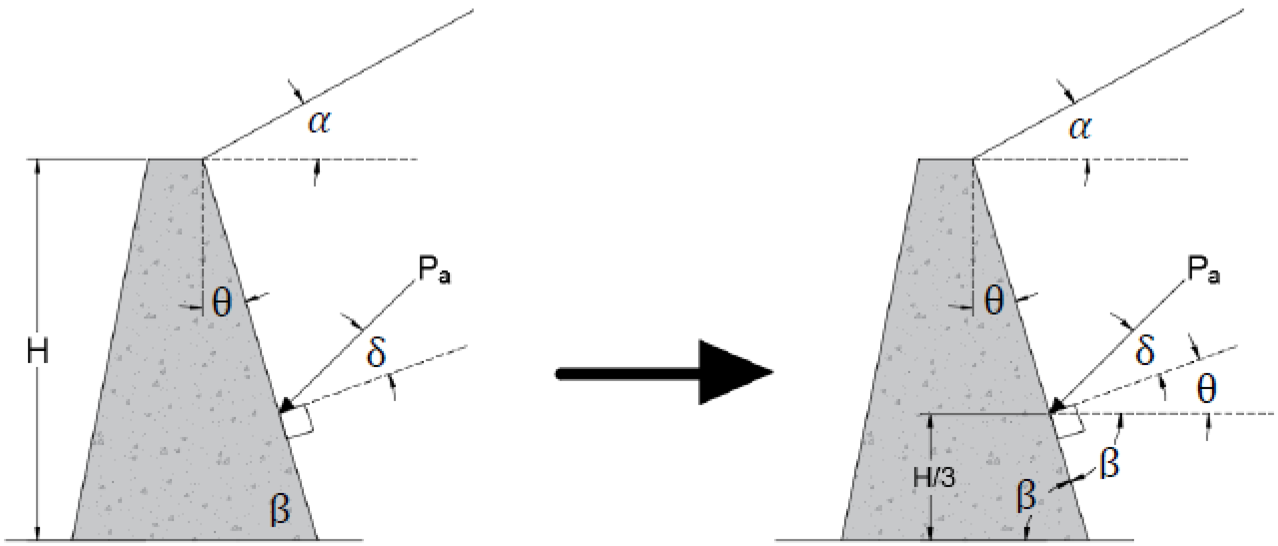
COULOMB'S EARTH PRESSURE



Coulomb's Active Earth Pressure

Granular backfill (c' =0)

General case (inclined wall and inclined backfill):



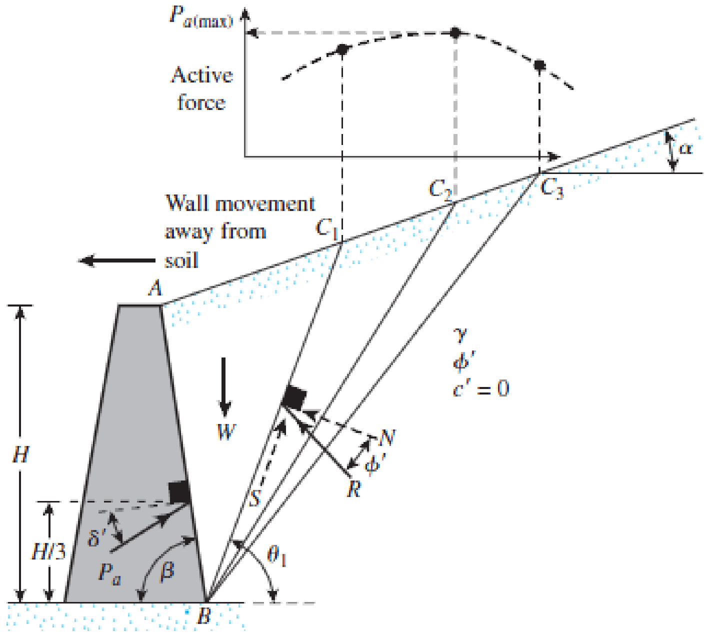
α = inclination of backfill with horizontal

θ = inclination of wall with vertical

β = inclination of wall with the horizontal

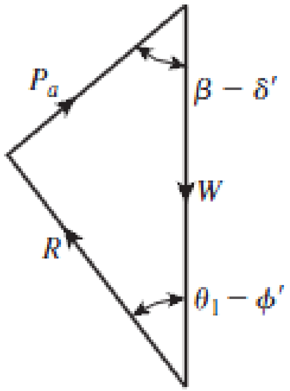
δ = friction angle between soil and wall

Coulomb's Active Earth Pressure



The forces acting on this wedge (per unit length at right angles to the cross section shown) are as follows:

1. The weight of the wedge, W.
2. The resultant, R, of the normal and resisting shear forces along the surface, BC_1. The force R will be inclined at an angle ϕ' to the normal drawn to BC_1.
3. The active force per unit length of the wall, P_a, which will be inclined at an angle δ' to the normal drawn to the back face of the wall.



Coulomb's Active Earth Pressure

$$P_a = \frac{1}{2} K_a \gamma H^2$$

K_a = Coulomb's active earth pressure coefficient

$$= \frac{\sin^2(\beta + \phi')}{\sin^2\beta \sin(\beta - \delta')\left[1 + \sqrt{\frac{\sin(\phi' + \delta')\sin(\phi' - \alpha)}{\sin(\beta - \delta')\sin(\alpha + \beta)}}\right]^2}$$

When α = 0°, β =90°, δ' = 0°, Coulomb's active earth pressure coefficient becomes equal to (1-sin ϕ')/(1+ sin ϕ'), which is the same as Rankine's active earth pressure coefficient.

Coulomb's Active Earth Pressure

K_a = Coulomb's active earth pressure coefficient

$$= \frac{\sin^2(\beta + \phi')}{\sin^2\beta \sin(\beta - \delta')\left[1 + \sqrt{\frac{\sin(\phi' + \delta')\sin(\phi' - \alpha)}{\sin(\beta - \delta')\sin(\alpha + \beta)}}\right]^2}$$

The values of the active earth pressure coefficient, K_a for a vertical retaining wall **, β =90º** with horizontal backfill **α = 0º** are given in Table 12.5

Table 12.5 Values of K_a [Eq. (12.26)] for $\beta = 90°$ and $\alpha = 0°$

	δ′ (deg)					
φ′ (deg)	**0**	**5**	**10**	**15**	**20**	**25**
28	0.3610	0.3448	0.3330	0.3251	0.3203	0.3186
30	0.3333	0.3189	0.3085	0.3014	0.2973	0.2956
32	0.3073	0.2945	0.2853	0.2791	0.2755	0.2745
34	0.2827	0.2714	0.2633	0.2579	0.2549	0.2542
36	0.2596	0.2497	0.2426	0.2379	0.2354	0.2350
38	0.2379	0.2292	0.2230	0.2190	0.2169	0.2167
40	0.2174	0.2098	0.2045	0.2011	0.1994	0.1995
42	0.1982	0.1916	0.1870	0.1841	0.1828	0.1831

Coulomb's Active Earth Pressure

K_a = Coulomb's active earth pressure coefficient

$$= \frac{\sin^2(\beta + \phi')}{\sin^2\beta \sin(\beta - \delta')\left[1 + \sqrt{\dfrac{\sin(\phi' + \delta')\sin(\phi' - \alpha)}{\sin(\beta - \delta')\sin(\alpha + \beta)}}\right]^2}$$

The wall friction angle δ' can be determined in the laboratory by means of direct shear test. It is assumed to be between **ϕ'/2** and **2ϕ'/3**

Table 12.6 Values of K_a [from Eq. (12.26)] for $\delta' = \frac{2}{3}\phi'$

		β (deg)					
α (deg)	ϕ' (deg)	90	85	80	75	70	65
0	28	0.3213	0.3588	0.4007	0.4481	0.5026	0.5662
	29	0.3091	0.3467	0.3886	0.4362	0.4908	0.5547
	30	0.2973	0.3349	0.3769	0.4245	0.4794	0.5435
	31	0.2860	0.3235	0.3655	0.4133	0.4682	0.5326
	32	0.2750	0.3125	0.3545	0.4023	0.4574	0.5220
	33	0.2645	0.3019	0.3439	0.3917	0.4469	0.5117
	34	0.2543	0.2916	0.3335	0.3813	0.4367	0.5017
	35	0.2444	0.2816	0.3235	0.3713	0.4267	0.4919

Table 12.7 Values of K_a [from Eq. (12.26)] for $\delta' = \phi'/2$

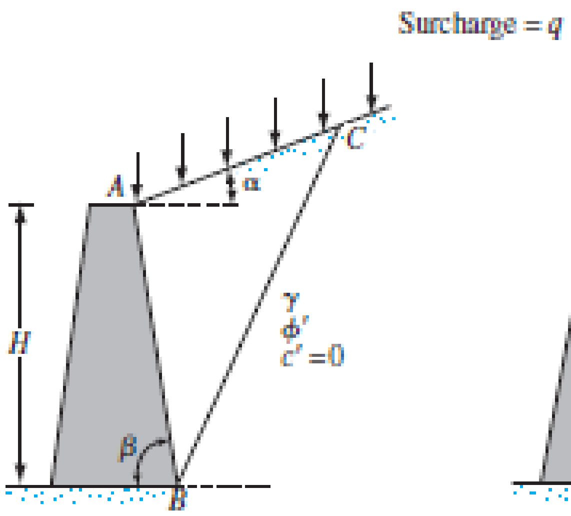
		β (deg)					
α (deg)	ϕ' (deg)	90	85	80	75	70	65
0	28	0.3264	0.3629	0.4034	0.4490	0.5011	0.5616
	29	0.3137	0.3502	0.3907	0.4363	0.4886	0.5492
	30	0.3014	0.3379	0.3784	0.4241	0.4764	0.5371
	31	0.2896	0.3260	0.3665	0.4121	0.4645	0.5253
	32	0.2782	0.3145	0.3549	0.4005	0.4529	0.5137
	33	0.2671	0.3033	0.3436	0.3892	0.4415	0.5025
	34	0.2564	0.2925	0.3327	0.3782	0.4305	0.4915
	35	0.2461	0.2820	0.3221	0.3675	0.4197	0.4807

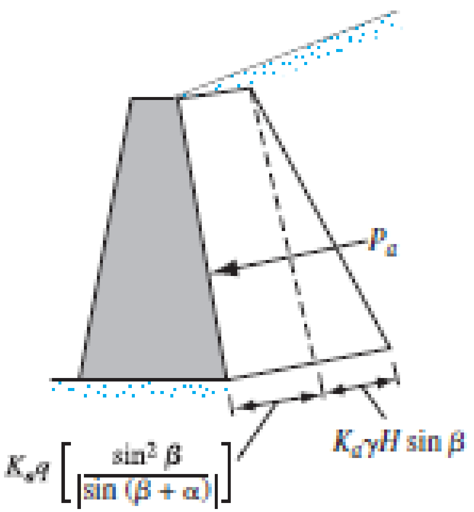
Coulomb's Active Earth Pressure

If a uniform surcharge of intensity *q* is located above the backfill

$$P_a = \tfrac{1}{2} K_a \gamma_{eq} H^2$$

$$\gamma_{eq} = \gamma + \left[\frac{\sin\beta}{\sin(\beta + \alpha)}\right]\left(\frac{2q}{H}\right)$$





EXAMPLE 12.6

Example 12.6

Consider the retaining wall shown in Figure 12.12a. Given: $H = 5$ m; unit weight of soil = 17.6 kN/m^3; angle of friction of soil = 35°; wall friction-angle, $\delta' = \frac{2}{3}\phi'$, soil cohesion, $c' = 0$; $\alpha = 0$, and $\beta = 90°$. Calculate the Coulomb's active force per unit length of the wall.

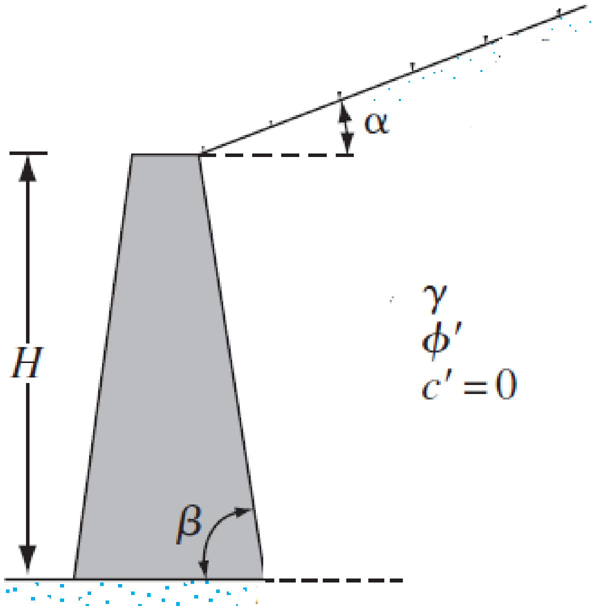
Solution

From Eq. (12.25)

$$P_a = \frac{1}{2}\gamma H^2 K_a$$

From Table 12.6, for $\alpha = 0°$, $\beta = 90°$, $\phi' = 35°$, and $\delta' = \frac{2}{3}\phi' = 23.33°$, $K_a = 0.2444$. Hence,

$$P_a = \frac{1}{2}(17.6)(5)^2(0.2444) = \mathbf{53.77\ kN/m}$$



EXAMPLE 12.7

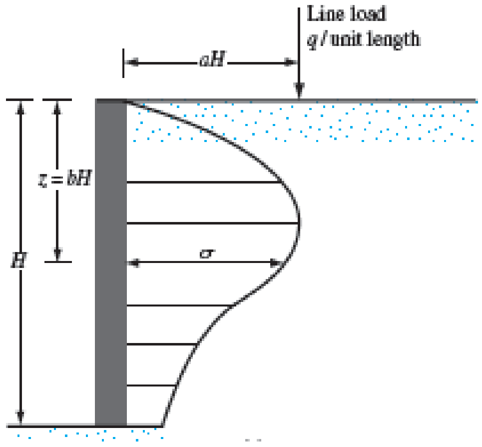
READ EXAMPLE 12.7

Lateral Earth Pressure Due to Surcharge

Line load of intensity *q*/unit length

$$\sigma = \frac{2q}{\pi H} \frac{a^2 b}{(a^2 + b^2)^2}$$

σ = horizontal stress at depth $z = bH$



Because soil is not a perfectly elastic medium. The modified forms of the equation above are :

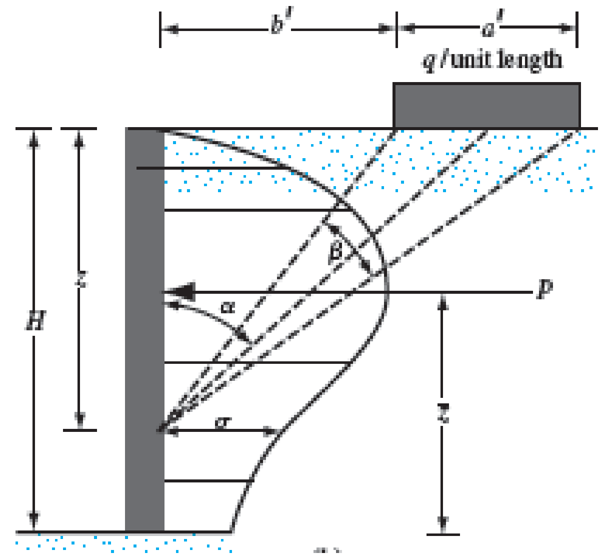
$$\sigma = \frac{4q}{\pi H} \frac{a^2 b}{(a^2 + b^2)} \qquad \text{for } a > 0.4$$

$$\sigma = \frac{q}{H} \frac{0.203b}{(0.16 + b^2)^2} \qquad \text{for } a \le 0.4$$

Lateral Earth Pressure Due to Surcharge

Strip load of intensity *q*/unit area

$$\sigma = \frac{2q}{\pi}(\beta - \sin\beta\cos 2\alpha)$$



The total force per unit length (P) due to the *strip loading only*

$$P = \frac{q}{90}[H(\theta_2 - \theta_1)]$$

$$\bar{z} = H - \left[\frac{H^2(\theta_2 - \theta_1) + (R - Q) - 57.3a'H}{2H(\theta_2 - \theta_1)}\right]$$

$$\theta_1 = \tan^{-1}\left(\frac{b'}{H}\right)(\text{deg})$$

$$\theta_2 = \tan^{-1}\left(\frac{a' + b'}{H}\right)(\text{deg})$$

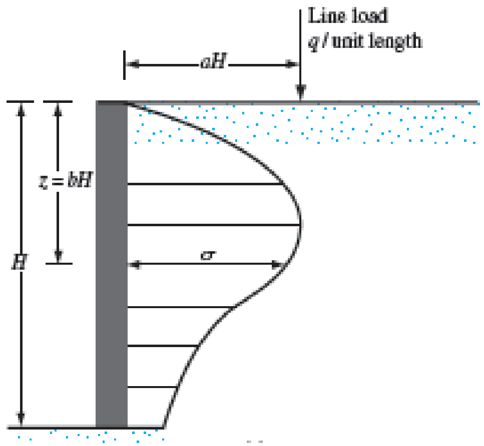
$$R = (a' + b')^2(90 - \theta_2)$$

$$Q = b'^2(90 - \theta_1)$$

EXAMPLE 12.8

EXAMPLE 12.8

Refer to Figure 12.14a which shows a line load surcharge. Given: $H = 6$ m, $a = 0.25$, and $q = 3$ kN/m. Calculate the variation of the lateral stress σ on the retaining structure at $z = 1, 2, 3, 4, 5,$ and 6 m.



Solution

For $a = 0.25$, which is less than 0.4, we will use Eq. (12.31). Now the following table can be prepared.

z (m)	H (m)	$b = z/H$	a	σ (kN/m^2)
1	6	0.167	0.25	**0.48**
2	6	0.333	0.25	**0.46**
3	6	0.5	0.25	**0.302**
4	6	0.667	0.25	**0.185**
5	6	0.833	0.25	**0.116**
6	6	1	0.25	**0.073**

$$\sigma = \frac{q}{H}\frac{0.203b}{(0.16 + b^2)^2} \quad \text{for } a \le 0.4$$

EXAMPLE 12.9

Example 12.9

Refer to Figure 12.14b. Here, $a' = 2$ m, $b' = 1$ m, $q = 40$ kN/m^2, and $H = 6$ m. Determine the total force on the wall (kN/m) caused by the strip loading only.

Solution

From Eqs. (12.35) and (12.36),

$$\theta_1 = \tan^{-1}\left(\frac{1}{6}\right) = 9.46°$$

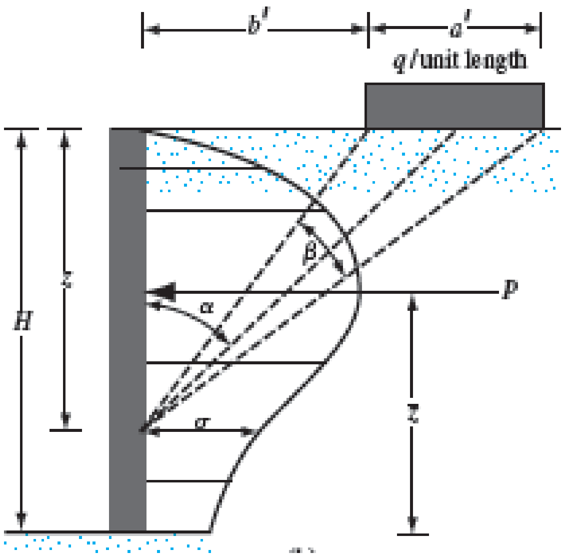
$$\theta_2 = \tan^{-1}\left(\frac{2+1}{6}\right) = 26.57°$$

$$\theta_1 = \tan^{-1}\left(\frac{b'}{H}\right)(\text{deg})$$

$$\theta_2 = \tan^{-1}\left(\frac{a'+b'}{H}\right)(\text{deg})$$

From Eq. (12.34)

$$P = \frac{q}{90}[H(\theta_2 - \theta_1)] = \frac{40}{90}[6(26.57 - 9.46)] = \mathbf{45.63\ kN/m}$$



EXAMPLE 12.10

Example 12.10

Refer to Example 12.9. Determine the location of the resultant $\bar{z}$.

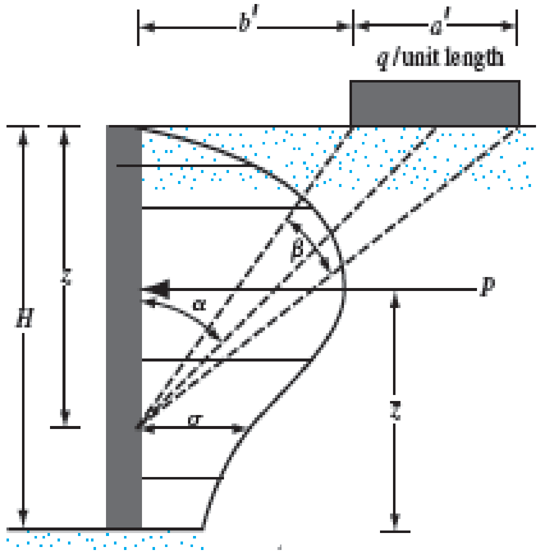
Solution

From Eqs. (12.38) and (12.39),

$$R = (a' + b')^2(90 - \theta_2) = (2 + 1)^2(90 - 26.57) = 570.87$$
$$Q = b'^2(90 - \theta_1) = (1)^2(90 - 9.46) = 80.54$$

From Eq. (12.37),

$$\bar{z} = H - \left[\frac{H^2(\theta_2 - \theta_1) + (R - Q) - 57.3a'H}{2H(\theta_2 - \theta_1)}\right]$$
$$= 6 - \left[\frac{(6)^2(26.57 - 9.46) + (570.87 - 80.54) - (57.3)(2)(6)}{(2)(6)(26.57 - 9.46)}\right] = \mathbf{3.96\ m}$$

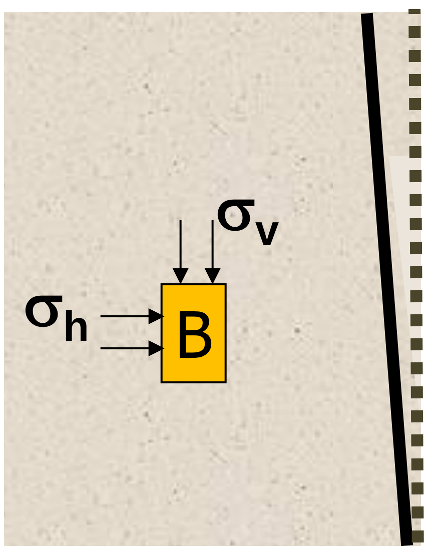


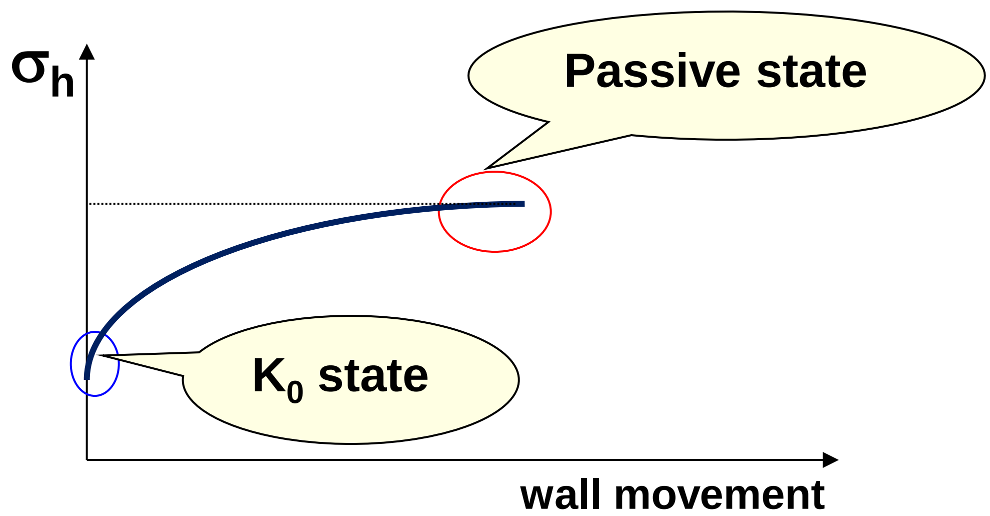
Passive Earth Pressure

Passive earth pressure

- Initially, soil is in K_0 state.
- As the wall moves towards (pushed into) the soil mass,
- σ_v remains the same, and
- σ_h **increases** till failure occurs. $\sigma_h \rightarrow \sigma_p$

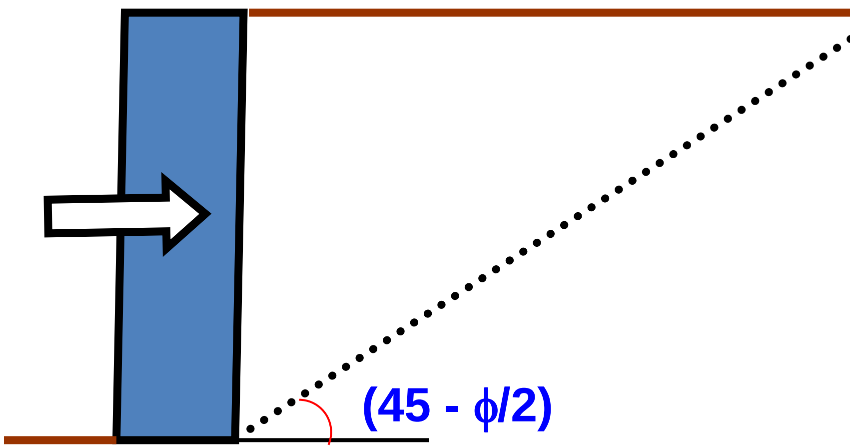
Passive state

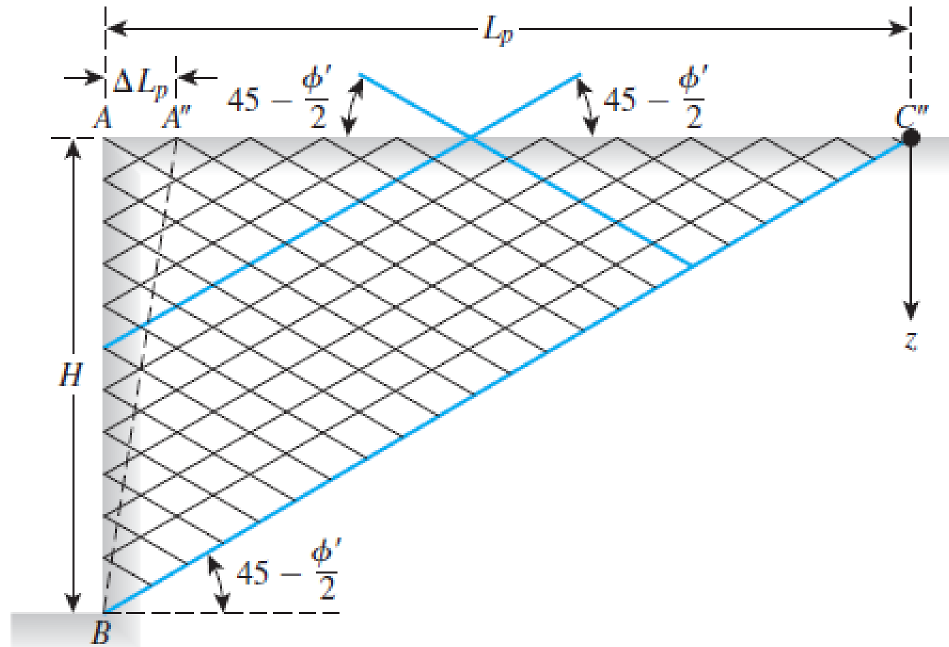




Orientation of Failure Planes

- From Mohr Circle the failure planes in the soil make ± (45 - ϕ/2)-degree angles with the direction of the major principal plane—that is, the horizontal.
- These are called potential ***slip planes***.

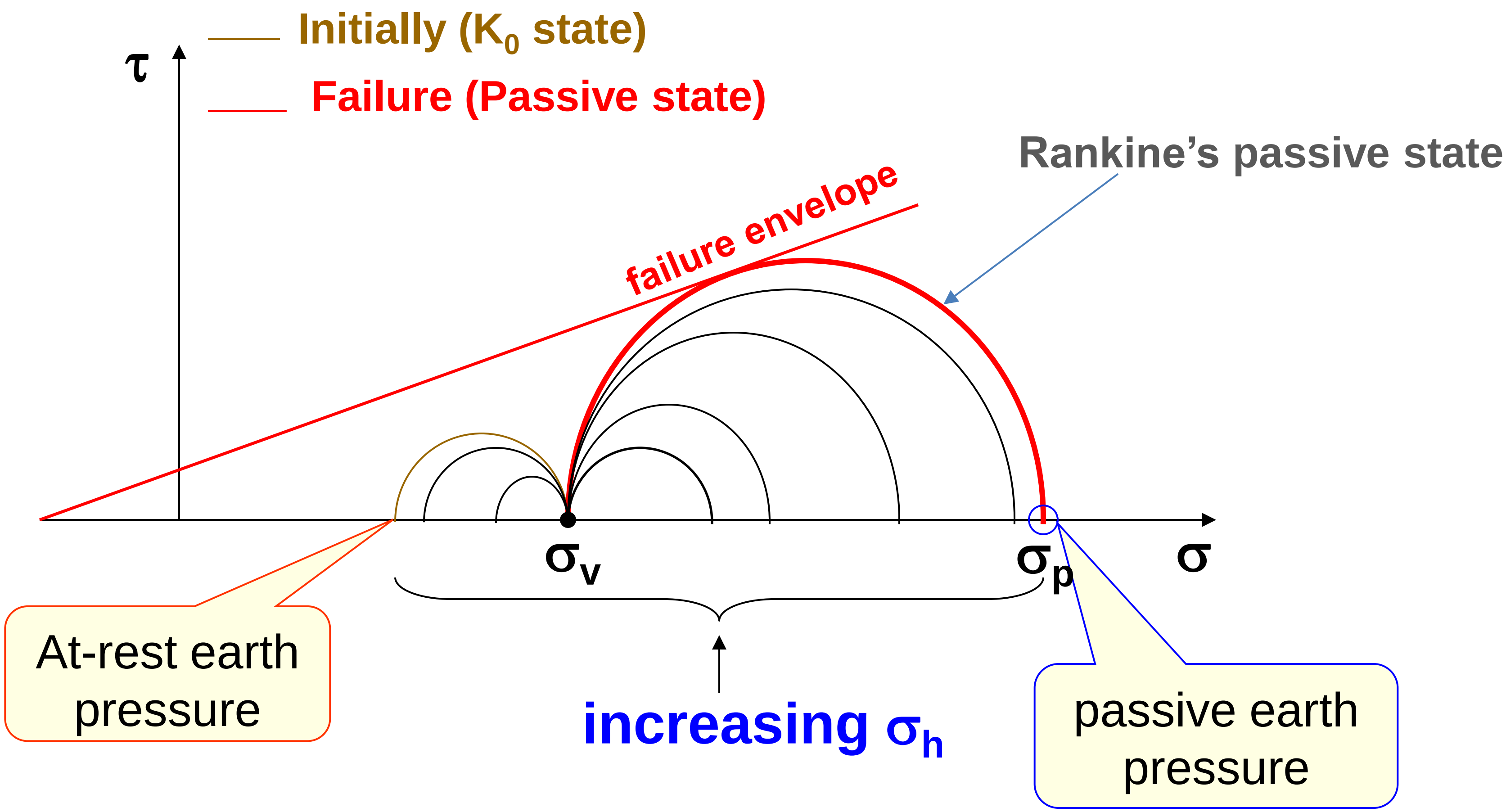




- Because the slip planes make angles of (45 - ϕ/2) degrees with the major principal plane, the soil mass in the state of plastic equilibrium is bounded by the plane ***BC***′. The soil inside the zone ***ABC***″ <u>undergoes the same unit deformation in the horizontal direction everywhere, which is equal to</u> $\Delta L_p / L'_p$.

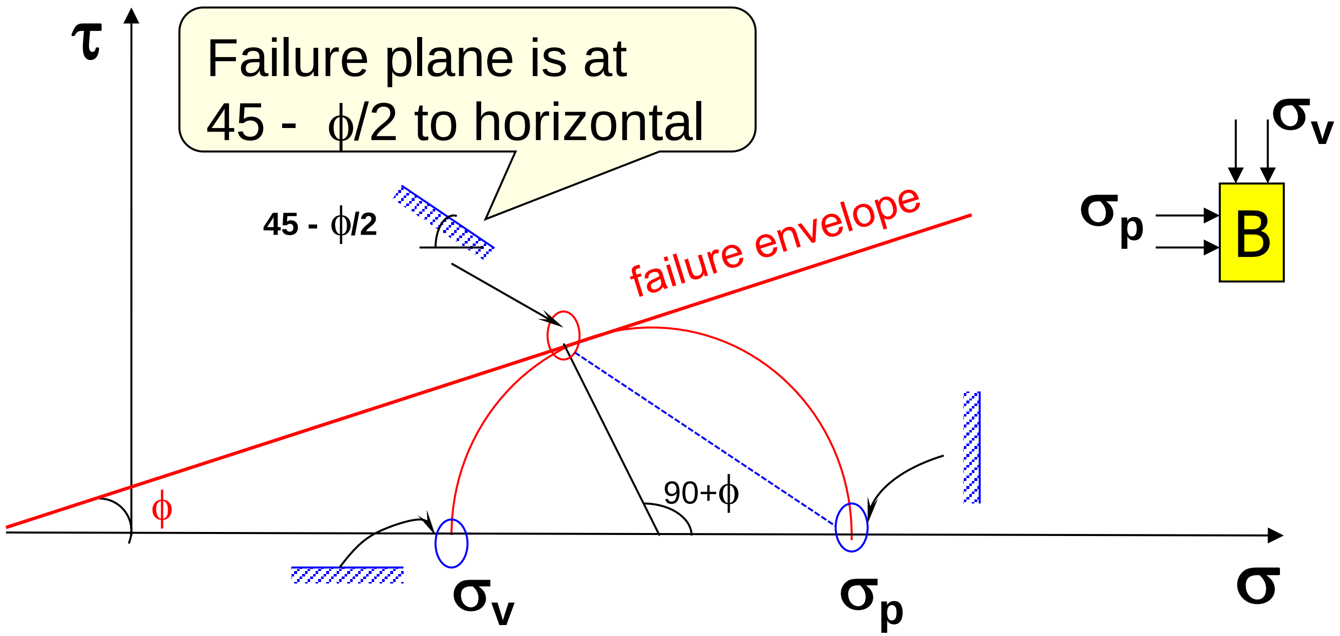
Passive Earth Pressure

- **As the wall moves towards the soil,**

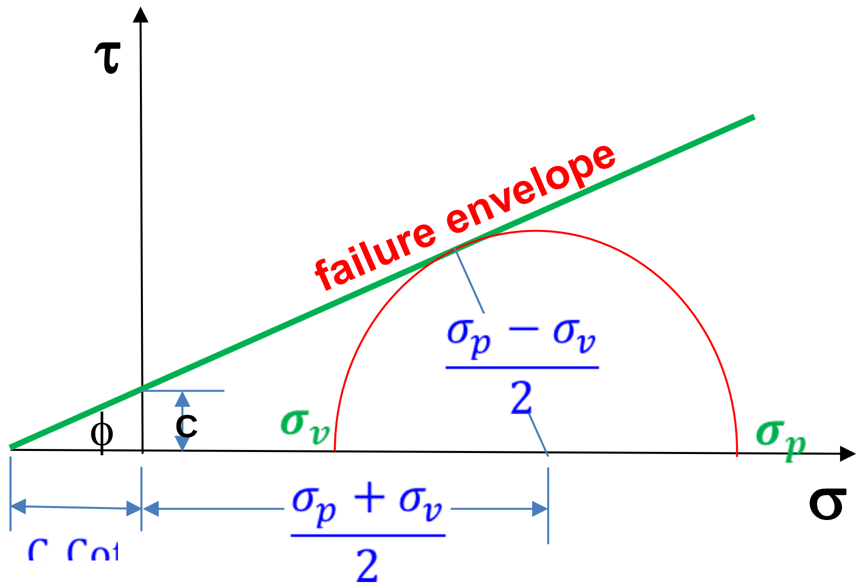


Passive Earth Pressure

Failure plane



Passive Earth Pressure



$$Sin\,\emptyset = \frac{\frac{\sigma_p - \sigma_v}{2}}{C.\,Cot\,\emptyset + \frac{\sigma_p + \sigma_v}{2}}$$

$$\sigma'_p = \sigma'_o \tan^2\left(45 + \frac{\phi'}{2}\right) + 2c' \tan\left(45 + \frac{\phi'}{2}\right)$$

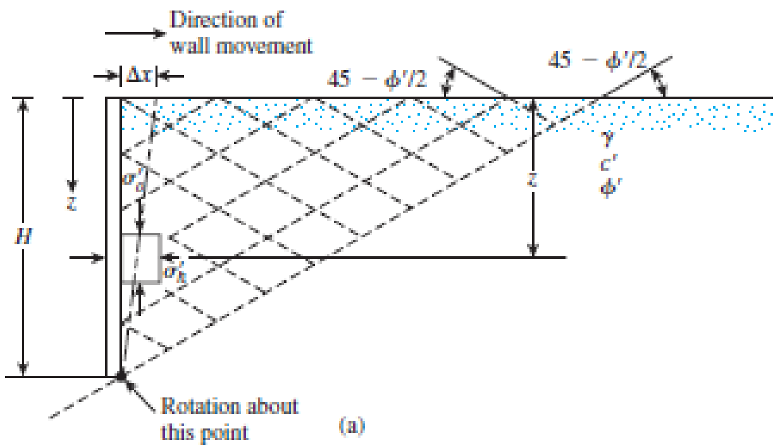
K_p = Rankine passive earth pressure coefficient

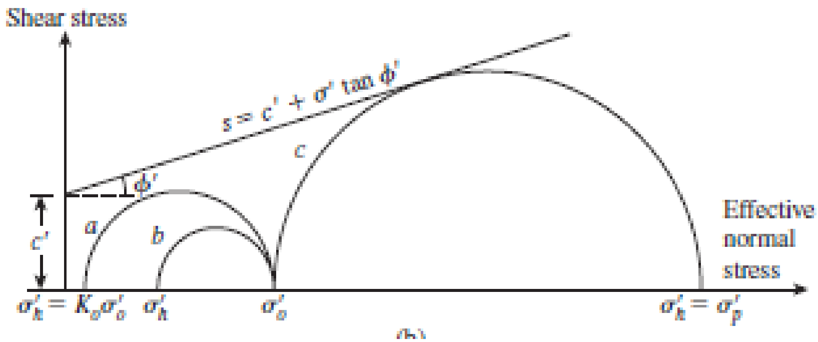
$$= \tan^2\left(45 + \frac{\phi'}{2}\right)$$

$$\sigma'_p = \sigma'_o K_p + 2c'\sqrt{K_p}$$

$$P_p = \frac{1}{2}\gamma H^2 K_p + 2c' H\sqrt{K_p}$$

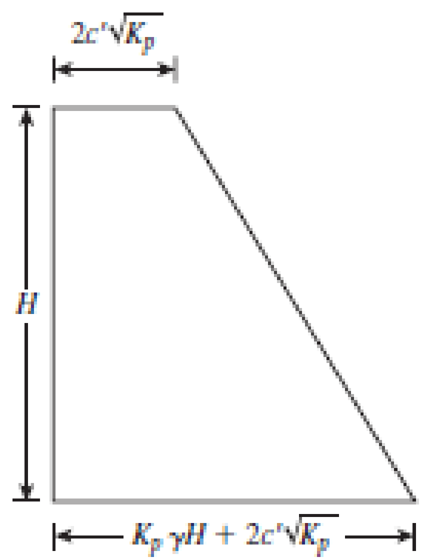
Passive Earth Pressure





$$\sigma'_p = \sigma'_o K_p + 2c'\sqrt{K_p}$$

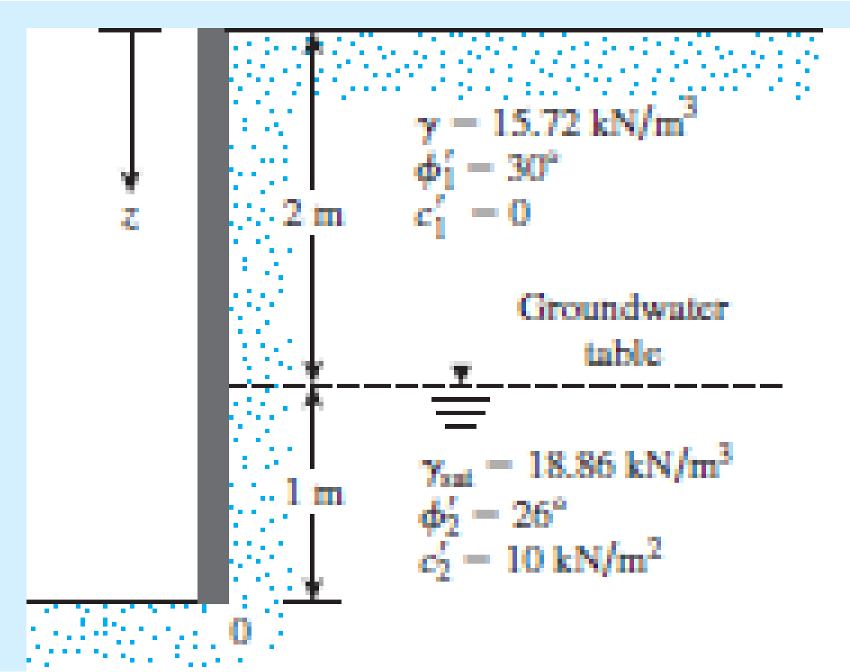
$$P_p = \frac{1}{2}\gamma H^2 K_p + 2c' H\sqrt{K_p}$$



EXAMPLE 12.13

Example 12.13

A 3-m-high wall is shown in Figure 12.20a. Determine the Rankine passive force per unit length of the wall.



EXAMPLE 12.13

Solution

For the top layer

$$K_{p(1)} = \tan^2\left(45 + \frac{\phi_1'}{2}\right) = \tan^2(45 + 15) = 3$$

From the bottom soil layer

$$K_{p(2)} = \tan^2\left(45 + \frac{\phi_2'}{2}\right) = \tan^2(45 + 13) = 2.56$$

$$\sigma_p' = \sigma_o' K_p + 2c'\sqrt{K_p}$$

where

σ_o' = effective vertical stress

at $z = 0$, $\sigma_o' = 0$, $c_1' = 0$, $\sigma_p' = 0$

at $z = 2$ m, $\sigma_o' = (15.72)(2) = 31.44$ kN/m^2, $c_1' = 0$

So, for the top soil layer

$$\sigma_p' = 31.44 K_{p(1)} + 2(0)\sqrt{K_{p(1)}} = 31.44(3) = 94.32 \text{ kN/m}^2$$

At this depth, that is $z = 2$ m, for the bottom soil layer

$$\sigma_p' = \sigma_o' K_{p(2)} + 2c_2'\sqrt{K_{p(2)}} = 31.44(2.56) + 2(10)\sqrt{2.56}$$
$$= 80.49 + 32 = 112.49 \text{ kN/m}^2$$

Again, at $z = 3$ m,

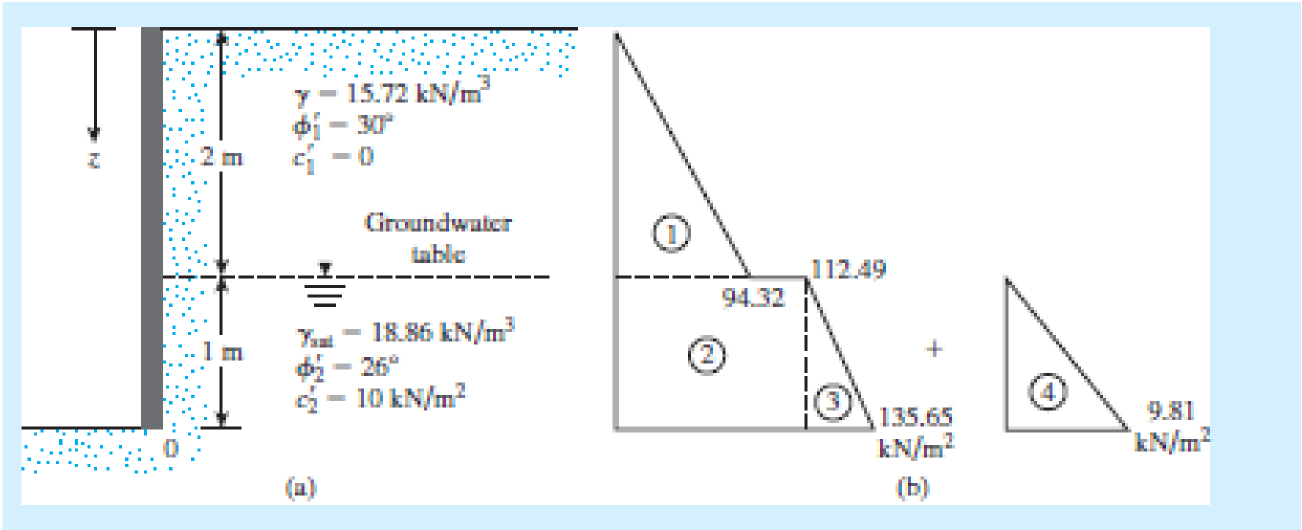
$$\sigma_o' = (15.72)(2) + (\gamma_{sat} - \gamma_w)(1)$$
$$= 31.44 + (18.86 - 9.81)(1) = 40.49 \text{ kN/m}^2$$

Hence,

$$\sigma_p' = \sigma_o' K_{p(2)} + 2c_2'\sqrt{K_{p(2)}} = 40.49(2.56) + (2)(10)(1.6)$$
$$= \mathbf{135.65 \text{ kN/m}^2}$$

For $z = 0$ to 2 m, $u = 0$; $z = 3$ m, $u = (1)(\gamma_w) = 9.81$ kN/m^2.

EXAMPLE 12.13



Area no.	Area	
1	$\left(\frac{1}{2}\right)(2)(94.32)$	$= 94.32$
2	$(112.49)(1)$	$= 112.49$
3	$\left(\frac{1}{2}\right)(1)(135.65 - 112.49)$	$= 11.58$
4	$\left(\frac{1}{2}\right)(9.81)(1)$	$= 4.905$
		$P_p \approx 223.3$ kN/m

Rankine Passive Earth Pressure

Vertical Wall and Inclined Backfill

Granular backfill c' = 0

$$\sigma'_p = \gamma z K_p$$

$$P_p = \frac{1}{2}\gamma H^2 K_p$$

$$K_p = \cos\alpha \frac{\cos\alpha + \sqrt{\cos^2\alpha - \cos^2\phi'}}{\cos\alpha - \sqrt{\cos^2\alpha - \cos^2\phi'}}$$

Table 12.9 Passive Earth Pressure Coefficient K_p [from Eq. (12.63)]

	ϕ' (deg)→						
↓α (deg)	28	30	32	34	36	38	40
0	2.770	3.000	3.255	3.537	3.852	4.204	4.599
5	2.715	2.943	3.196	3.476	3.788	4.136	4.527
10	2.551	2.775	3.022	3.295	3.598	3.937	4.316
15	2.284	2.502	2.740	3.003	3.293	3.615	3.977
20	1.918	2.132	2.362	2.612	2.886	3.189	3.526
25	1.434	1.664	1.894	2.135	2.394	2.676	2.987

Rankine Passive Earth Pressure

Vertical Wall and Inclined Backfill

$c'-\phi'$ Soil

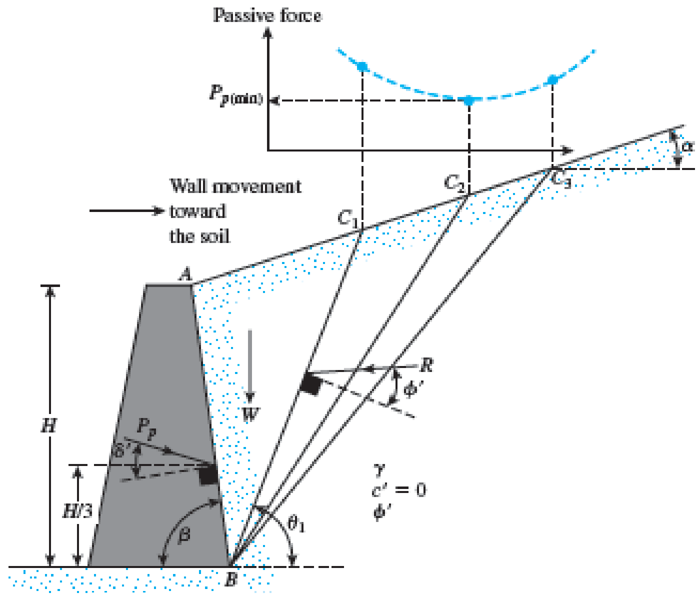
$$\sigma'_p = \gamma z K_p = \gamma z K'_p \cos\alpha$$

$$K'_p = \frac{1}{\cos^2\phi'}\left\{\begin{array}{c} 2\cos^2\alpha + 2\left(\frac{c'}{\gamma z}\right)\cos\phi'\sin\phi' \\ + \sqrt{4\cos^2\alpha(\cos^2\alpha - \cos^2\phi') + 4\left(\frac{c'}{\gamma z}\right)^2\cos^2\phi' + 8\left(\frac{c'}{\gamma z}\right)\cos^2\alpha\sin\phi'\cos\phi'} \end{array}\right\} - 1$$

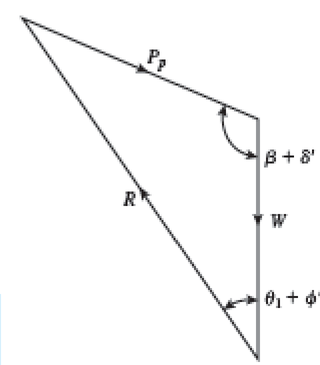
Table 12.10 Values of K'_p

		$c'/\gamma z$			
ϕ' (deg)	α (deg)	0.025	0.050	0.100	0.500
15	0	1.764	1.829	1.959	3.002
	5	1.716	1.783	1.917	2.971
	10	1.564	1.641	1.788	2.880
	15	1.251	1.370	1.561	2.732
20	0	2.111	2.182	2.325	3.468
	5	2.067	2.140	2.285	3.435
	10	1.932	2.010	2.162	3.339
	15	1.696	1.786	1.956	3.183
25	0	2.542	2.621	2.778	4.034
	5	2.499	2.578	2.737	3.999
	10	2.368	2.450	2.614	3.895
	15	2.147	2.236	2.409	3.726
30	0	3.087	3.173	3.346	4.732
	5	3.042	3.129	3.303	4.674
	10	2.907	2.996	3.174	4.579
	15	2.684	2.777	2.961	4.394

Coulomb's Passive Earth Pressure



1. The weight of the wedge, W
2. The resultant, R, of the normal and shear forces on the plane BC_1, and
3. The passive force, P_p



$$P_p = \frac{1}{2}\gamma H^2 K_p$$

K_p = Coulomb's passive pressure coefficient

$$= \frac{\sin^2(\beta - \phi')}{\sin^2\beta \sin(\beta + \delta')\left[1 - \sqrt{\dfrac{\sin(\phi' + \delta')\sin(\phi' + \alpha)}{\sin(\beta + \delta')\sin(\beta + \alpha)}}\right]^2}$$

Coulomb's Passive Earth Pressure

K_p = Coulomb's passive pressure coefficient

$$= \frac{\sin^2(\beta - \phi')}{\sin^2\beta \sin(\beta + \delta')\left[1 - \sqrt{\frac{\sin(\phi' + \delta')\sin(\phi' + \alpha)}{\sin(\beta + \delta')\sin(\beta + \alpha)}}\right]^2}$$

Table 12.11 Values of K_p [from Eq. (12.67)] for $\beta = 90°$ and $\alpha = 0°$

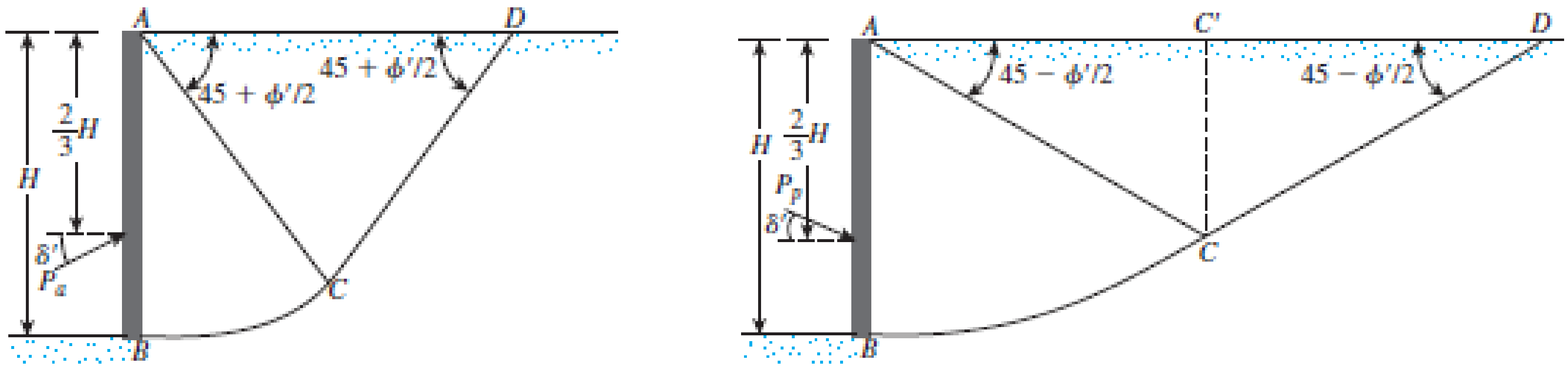
	δ' (deg)				
ϕ' (deg)	0	5	10	15	20
15	1.698	1.900	2.130	2.405	2.735
20	2.040	2.313	2.636	3.030	3.525
25	2.464	2.830	3.286	3.855	4.597
30	3.000	3.506	4.143	4.977	6.105
35	3.690	4.390	5.310	6.854	8.324
40	4.600	5.590	6.946	8.870	11.772

Comments on the Failure Surface Assumption for Coulomb's Pressure Calculations

The fundamental assumption in Coulomb's pressure calculation methods for active and passive pressure is the acceptance of *plane failure surface.*

However, for walls with friction, this assumption does not hold in practice. The nature of *actual* failure surface in the soil mass for active and passive pressure is shown in Figure below, respectively (for a vertical wall with a horizontal backfill).

Note that the failure surface *BC* is curved and that the failure surface *CD* is a plane.



Comments on the Failure Surface Assumption for Coulomb's Pressure Calculations

Although the actual failure surface in soil for the case of active pressure is somewhat different from that assumed in the calculation of the Coulomb pressure, the results are not greatly different.

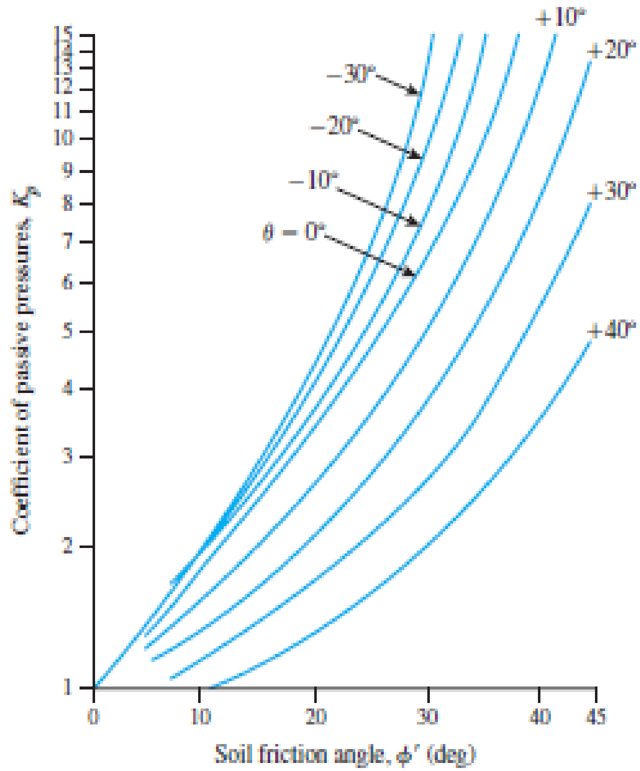
However, in the case of passive pressure, as the value of δ' increases, Coulomb's method of calculation gives increasingly erroneous values of P_p .

This factor of error could lead to an unsafe condition because the values of P_p would become higher than the soil resistance.

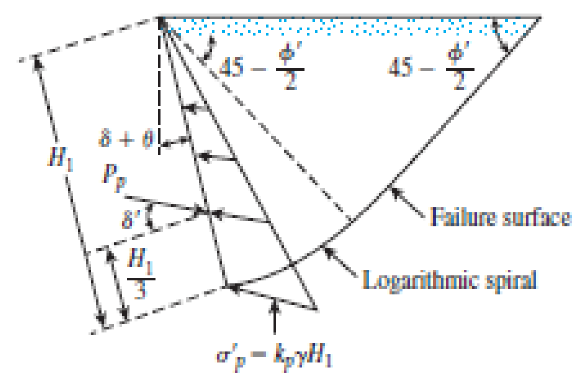
Several studies have been conducted to determine the passive force Pp , assuming that the curved portion *BC* in is an arc of a circle, an ellipse, or a logarithmic spiral (e.g., Caquot and Kerisel, 1948; Terzaghi and Peck, 1967; Shields and Tolunay,1973; Zhu and Qian, 2000).

Caquot and Kerisel Solution for Passive Earth Pressure (Granular Backfill)

$$P_p = \frac{1}{2}\gamma H_1^2 K_p$$



for $\delta'/\phi' = 1.$

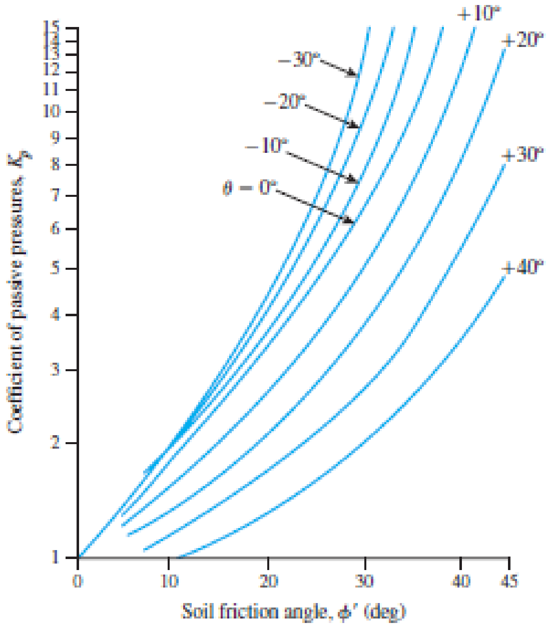


Caquot and Kerisel Solution for Passive Earth Pressure (Granular Backfill)

If $\delta'/\phi' \neq 1$,

1. Assume δ' and ϕ'.
2. Calculate δ'/ϕ'.
3. Using the ratio of δ'/ϕ' (step 2), determine the reduction factor, R', from Table 12.12.
4. Determine K_p from Figure 12.23 for $\delta'/\phi' = 1$
5. Calculate K_p for the required δ'/ϕ' as

$$K_p = (R')[K_{p(\delta'/\phi'=1)}]$$



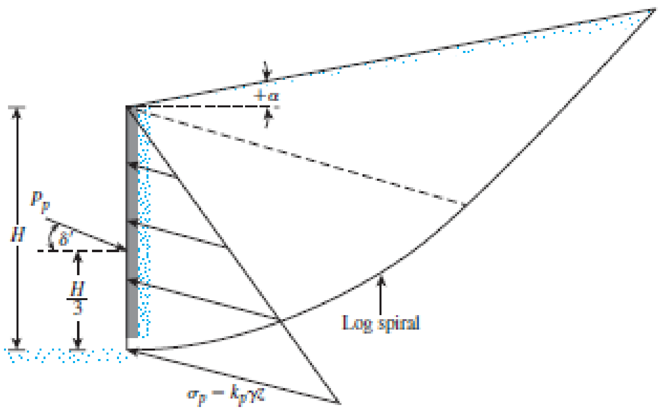
for $\delta'/\phi' = 1$.

Table 12.12 Caquot and Kerisel's Reduction Factor, R', for Passive Pressure Calculation

	δ'/ϕ'							
ϕ'	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0.0
10	0.978	0.962	0.946	0.929	0.912	0.898	0.881	0.864
15	0.961	0.934	0.907	0.881	0.854	0.830	0.803	0.775
20	0.939	0.901	0.862	0.824	0.787	0.752	0.716	0.678
25	0.912	0.860	0.808	0.759	0.711	0.666	0.620	0.574
30	0.878	0.811	0.746	0.686	0.627	0.574	0.520	0.467
35	0.836	0.752	0.674	0.603	0.536	0.475	0.417	0.362
40	0.783	0.682	0.592	0.512	0.439	0.375	0.316	0.262
45	0.718	0.600	0.500	0.414	0.339	0.276	0.221	0.174

Caquot and Kerisel Solution for Passive Earth Pressure (Granular Backfill)

$$P_p = \frac{1}{2}\gamma H^2 K_p$$



Step 1. Determine α/ϕ' (note the sign of α).

Step 2. Knowing ϕ' and α/ϕ', use Figure 12.24a to determine K_p for $\delta'/\phi' = 1$.

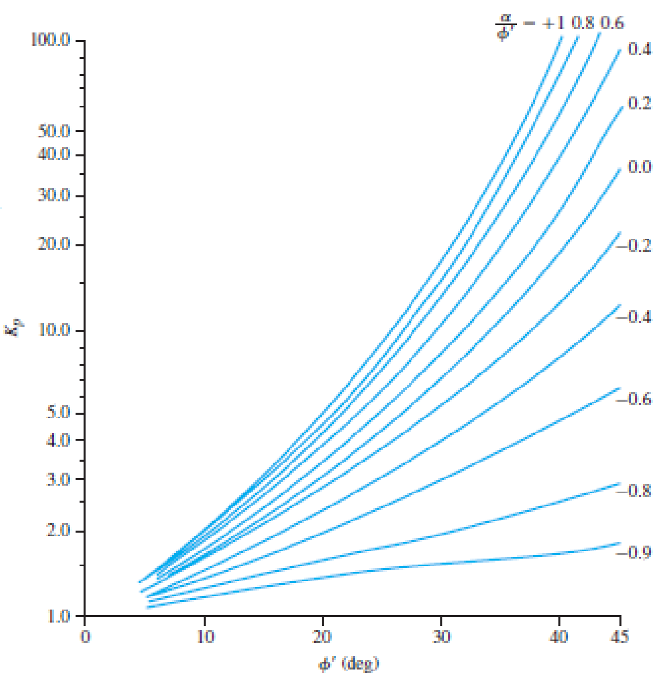
Step 3. Calculate δ'/ϕ'.

Step 4. Go to Table 12.12 to determine the reduction factor, R'.

Step 5. $K_p = (R')\,[K_{p(\delta'/\phi'=1)}]$.

Table 12.12 Caquot and Kerisel's Reduction Factor, R', for Passive Pressure Calculation

	δ'/ϕ'							
ϕ'	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0.0
10	0.978	0.962	0.946	0.929	0.912	0.898	0.881	0.864
15	0.961	0.934	0.907	0.881	0.854	0.830	0.803	0.775
20	0.939	0.901	0.862	0.824	0.787	0.752	0.716	0.678
25	0.912	0.860	0.808	0.759	0.711	0.666	0.620	0.574
30	0.878	0.811	0.746	0.686	0.627	0.574	0.520	0.467
35	0.836	0.752	0.674	0.603	0.536	0.475	0.417	0.362
40	0.783	0.682	0.592	0.512	0.439	0.375	0.316	0.262
45	0.718	0.600	0.500	0.414	0.339	0.276	0.221	0.174



EXAMPLE 12.14

Example 12.14

Consider a 3-m-high (H) retaining wall a vertical back ($\theta = 0°$) and a horizontal granular backfill Given: $\gamma = 15.7$ kN/m^3, $\delta' = 15°$, and $\phi' = 30°$. Estimate the passive force, P_p, by using

a. Coulomb's theory
b. Caquot and Kerisel's theory

Solution

Part a

From Eq. (12.66)

$$P_p = \frac{1}{2} K_p \gamma H^2$$

From Table 12.11, for $\phi' = 30°$ and $\delta' = 15°$, the value of K_p is 4.977. Thus,

$$P_p = \left(\frac{1}{2}\right)(4.977)(15.7)(3)^2 = \mathbf{351.6\ kN/m}$$

Part b

From Eq. (12.68), with $\theta = 0, H_1 = H$,

$$P_p = \frac{1}{2} \gamma H^2 K_p$$

From Figure 12.23a, for $\phi' = 30°$ and $\delta'/\phi' = 1$, the value of $K_{p(\delta'/\phi'=1)}$ is about 5.9. Also, from Table 12.12, with $\phi' = 30°$ and $\delta'/\phi' = 0.5$, the value of R' is 0.746.

Hence,

$$P_p = \frac{1}{2} \gamma H^2 K_p = \frac{1}{2}(15.7)(3)^2(0.746 \times 5.9) \approx \mathbf{311\ kN/m}$$ ■

EXAM

The soil conditions adjacent to a sheet pile wall are given in the Figure below. A surcharge pressure of 50 kN/m² being carried on the surface behind the wall.
For soil 1, a sand above the water table, c' = 0 kN/m² and ϕ' = 38° and γ = 18 KN/m³.
For soil 2, a saturated clay, c' = 10 kN/m² and ϕ' = 28° and γ_{sat} = 20 KN/m³.

- **Calculate K_a and K_p for each of soils (1) and (2).**
- **Complete the given table for the Rankine active pressure at 6 and 9 m depth behind the wall shown in the Figure.**
- **Complete the given table for the Rankine passive pressure at 1.5 and 4.5 m depth in front of the wall shown in the Figure.**

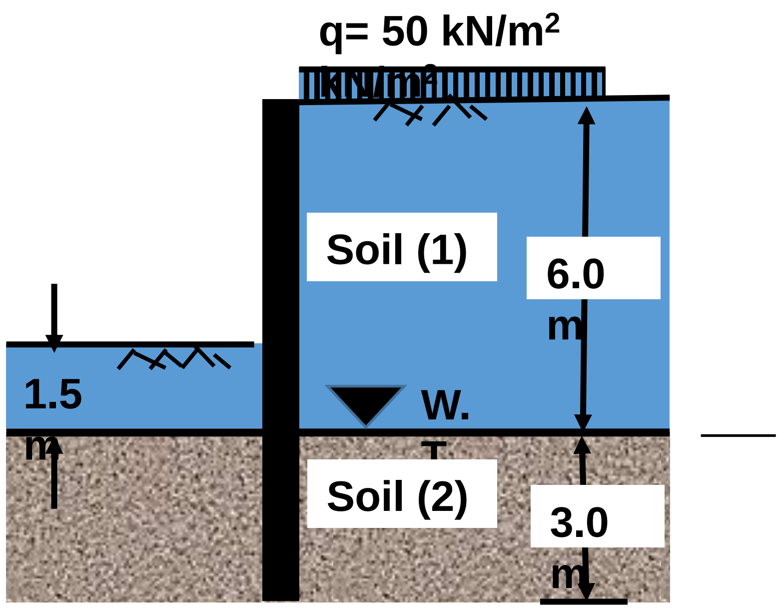


Table . Active and passive earth pressures on sheet pile wall

Depth (meter)	Soil		
		Active Pressure (kN/m^2)	
0	1		
6	1		
6	2		
9	2		
		Passive Pressure (kN/m^2)	
0	1		
1.5	1		
1.5	2		
4.5	2		

SOLUTION

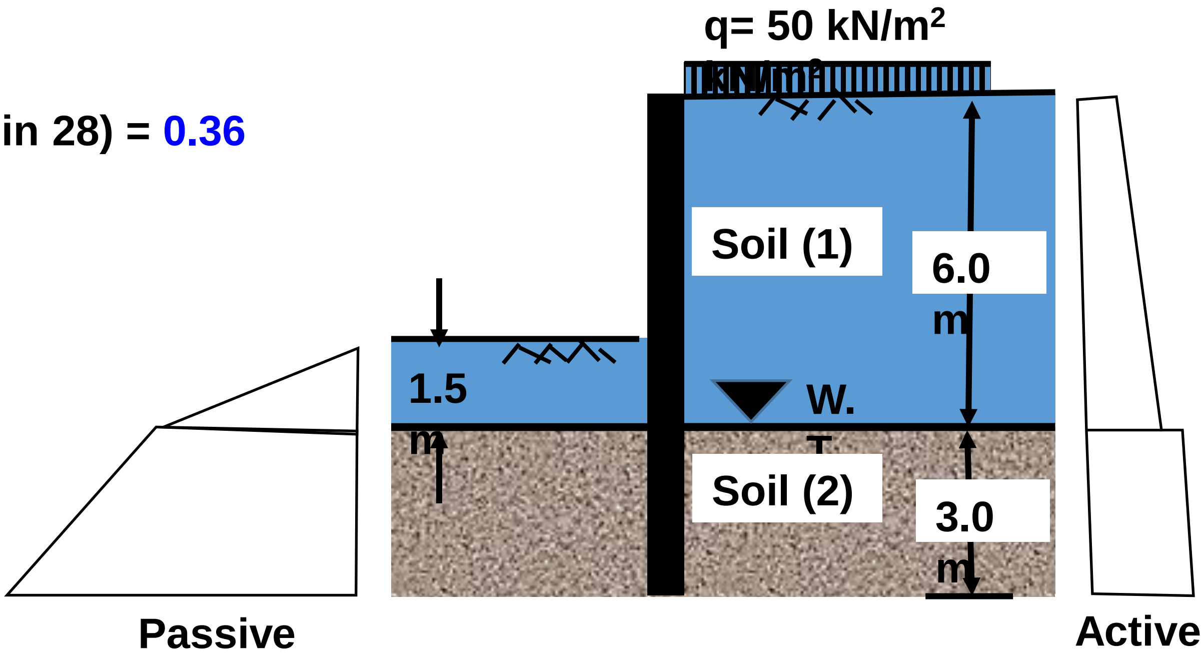
SOLUTION

Soil 1: K_a = (1-sin 38)/(1+sin 38) = **0.24**

K_p = 1/K_a = **4.17**

Soil 2: K_a = (1-sin 28)/1+sin 28) = **0.36**

K_p = 1/K_a = **2.78**



SOLUTION

Depth (meter)	Soil		
		Active Pressure (kN/m^2)	
0	1	0.24 x 50	= 12
6	1	0.24 x (50 + 18 x 6)	= 37.9
6	2	0.36 x (50 + 18 x 6) - 2 x$\sqrt{0.36}$ x 10	= 44.9
9	2	0.36 x (50 + 18 x 6 + 10.2 x 3) - 2 x$\sqrt{0.36}$ x 10 + 9.81 x 3	= 85.33
		Passive Pressure (kN/m^2)	
0	1		= 0
1.5	1	4.17 x 18 x 1.5	= 112.6
1.5	2	2.78 x 18 x 1.5 + 2 x $\sqrt{2.78}$ x 10	= 108.4
4.5	2	2.78 x (18 x 1.5 + 10.2 x 3)+ 2 x $\sqrt{2.78}$ x 10 + 9.81 x 3	= 222.93

RECOMMENDED PROCEDURE

1. Calculate the appropriate k for each soil
2. Calculate σ_V at a specified depth
3. Add q if any
4. Multiply the sum of (σ_V + q) by the appropriate k (for upper and lower soil) and subtract (or add for passive) cohesion part if exists.
5. Calculate water pressure
6. Divide each trapezoidal area into a rectangle and a triangle
7. Calculate areas and that give the lateral forces
8. Locate point of application for each force
9. Find the resultant force
10. Take moments about the base of the wall and find location of the resultant