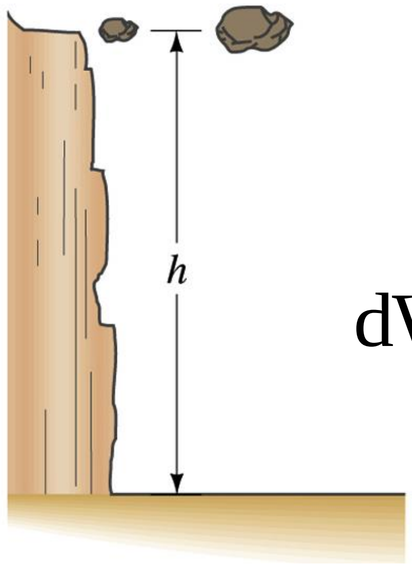
Chapter 25
Electric Potential

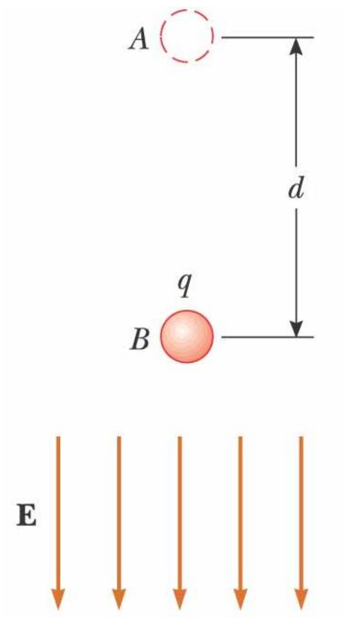
25-1 Potential difference and electric Potential.

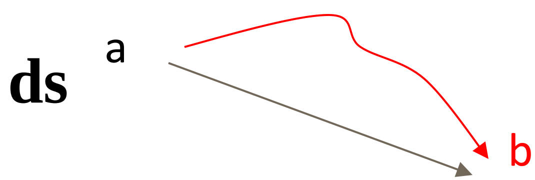
25-2 Potential Difference and electric field.

25-1 Potential difference and electric Potential



$$dW = -dU$$





25-1 Potential difference and electric Potential

A work done by electric field on a charge has infinitesimal displacement $d\vec{s}$, is

$$dW = \vec{F} \cdot d\vec{s}$$

$$= q_0 \vec{E} \cdot d\vec{s}$$

$$dW = -dU$$ → **Work Energy Theorem**

$$-dU = q_0 \vec{E} \cdot d\vec{s}$$

$$\int_a^b dU = -q_0 \int_a^b \vec{E} \cdot d\vec{s}$$

$$\Delta U = U_b - U_a = -q \int_a^b \vec{E} \cdot d\vec{s}$$ → **Change in Potential Energy**

25-1 Potential difference and electric Potential

Now, a physical quantity, electric potential, is defined as potential energy per unit charge $V=U/q_0$.

$$U_b - U_a = -q_0 \int_a^b \vec{E} \cdot d\vec{s}$$

$$\Delta V = V_b - V_a \equiv \frac{U_b - U_a}{q_0} = -\int_a^b \vec{E} \cdot d\vec{s}$$

→ **Potential Difference**

The SI unit of both electric potential and potential difference is joules per coulomb, which is defined as a volt (V):

$$\mathbf{1\ V \equiv 1\ J/C}$$

The difference in potential energy exists only if a test charge is moved between the points.
Electric potential is a scalar characteristic of an electric field, independent of any charges that may be placed in the field.

25-1 Potential difference and electric Potential

Imagine an arbitrary charge q located in an electric field. The work done by an external agent in moving a charge q through an electric field at constant velocity is:

$$W = q\Delta V$$

The SI unit of the work Joule (J):

A unit of energy commonly used in atomic and nuclear physics is the electron volt (eV), which is defined as the energy a charge–field system gains or loses when a charge of magnitude e (an electron or a proton) is moved through a potential difference of 1 V.

Because 1 V = 1 J/C and because the fundamental charge is $1.60 * 10^{-19}$ C, the electron volt is related to the joule as follows:

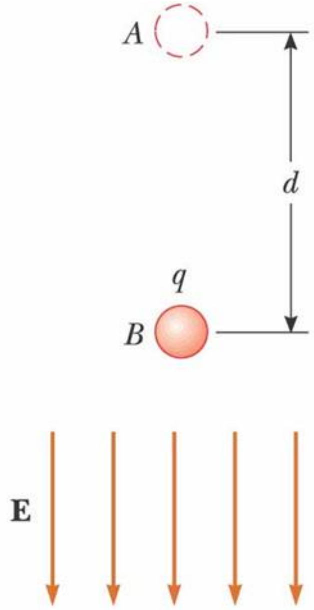
$$1\ \text{eV} = 1.60 \times 10^{-19}\ \text{C}\cdot\text{V} = 1.60 \times 10^{-19}\ \text{J}$$

25.2 Potential Differences in a Uniform Electric Field

The two eq.s below hold in all electric fields, whether uniform or varying, but they can be simplified for a uniform field.

$$U_b - U_a = -q_0 \int_a^b \vec{E} \cdot d\vec{s} \qquad \Delta V = V_b - V_a \equiv \frac{U_b - U_a}{q_0} = -\int_a^b \vec{E} \cdot d\vec{s}$$

Consider a uniform electric field, the potential difference between two points A and B separated by a distance |**s**| = d, where **s** is parallel to the field lines.



$$V_B - V_A = \Delta V = -\int_A^B \mathbf{E} \cdot d\mathbf{s} = -\int_A^B (E \cos 0°)\, ds = -\int_A^B E\, ds$$

$$\Delta V = -E \int_A^B ds = -Ed$$

The negative sign indicates that $V_B < V_A$. Electric field lines always point in the direction of decreasing electric potential.

$$\Delta U = q_0\, \Delta V = -\, q_0 E d$$

25.2 Potential Differences in a Uniform Electric Field

$$\Delta U = q_0\, \Delta V = -\, q_0 E d$$

- A system consisting of a positive charge and an electric field loses electric potential energy when the charge moves in the direction of the field.
- As the charged particle gains kinetic energy, the charge–field system loses an equal amount of potential energy.

$$\Delta U + \Delta K = 0$$

- A system consisting of a negative charge and an electric field gains electric potential energy when the charge moves in the direction of the field.

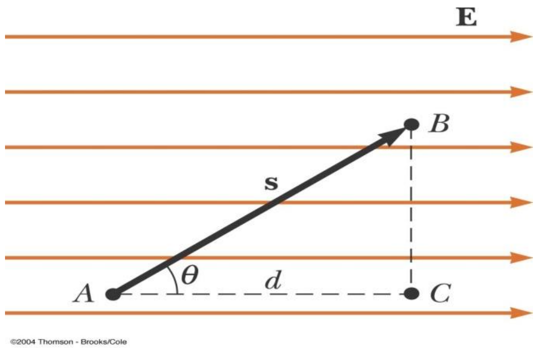
25.2 Potential Differences in a Uniform Electric Field

More general case of a charged particle that moves between A and B in a uniform electric field such that the vector **s** is not parallel to the field lines.

$$\Delta V = V_B - V_A = -\int_A^B \vec{E}\,.\vec{ds} = -\vec{E}\,.\int_A^B \vec{ds} = -\vec{E}\,.\vec{d} = E\, s\cos\theta$$

The change in potential energy of the charge–field system is:

$$\Delta U = q_0\,\Delta V = -\,q_0\,\mathbf{E.d}$$



The name equipotential surface is given to any surface consisting of a continuous distribution of points having the same electric potential.

25.2 Potential Differences in a Uniform Electric Field

Example 25.1 The Electric Field Between Two Parallel Plates of Opposite Charge

A battery produces a specified potential difference ΔV between conductors attached to the battery terminals. A 12-V battery is connected between two parallel plates, as shown in Figure 25.5. The separation between the plates is $d = 0.30$ cm, and we assume the electric field between the plates to be uniform. (This assumption is reasonable if the plate separation is small relative to the plate dimensions and if we do not consider locations near the plate edges.) Find the magnitude of the electric field between the plates.

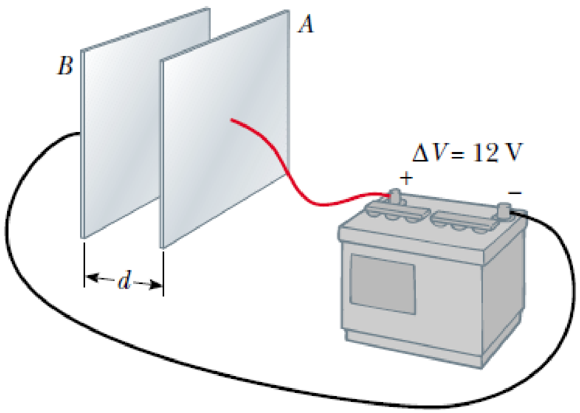


Figure 25.5 (Example 25.1) A 12-V battery connected to two parallel plates. The electric field between the plates has a magnitude given by the potential difference ΔV divided by the plate separation d.

Solution The electric field is directed from the positive plate (A) to the negative one (B), and the positive plate is at a higher electric potential than the negative plate is. The potential difference between the plates must equal the potential difference between the battery terminals. We can understand this by noting that all points on a conductor in equilibrium are at the same electric potential[1]; no potential difference exists between a terminal and any portion of the plate to which it is connected. Therefore, the magnitude of the electric field between the plates is, from Equation 25.6,

$$E = \frac{|V_B - V_A|}{d} = \frac{12\text{ V}}{0.30 \times 10^{-2}\text{ m}} = \boxed{4.0 \times 10^3 \text{ V/m}}$$

The configuration of plates in Figure 25.5 is called a *parallel-plate capacitor*, and is examined in greater detail in Chapter 26.

25.2 Potential Differences in a Uniform Electric Field

Example 25.2 Motion of a Proton in a Uniform Electric Field

Interactive

A proton is released from rest in a uniform electric field that has a magnitude of 8.0×10^4 V/m (Fig. 25.6). The proton undergoes a displacement of 0.50 m in the direction of **E**.

(A) Find the change in electric potential between points A and B.

Solution Because the positively charged proton moves in the direction of the field, we expect it to move to a position of lower electric potential. From Equation 25.6, we have

$$\Delta V = -Ed = -(8.0 \times 10^4 \text{ V/m})(0.50 \text{ m}) = \boxed{-4.0 \times 10^4 \text{ V}}$$

(B) Find the change in potential energy of the proton–field system for this displacement.

Solution Using Equation 25.3,

$$\begin{aligned}\Delta U &= q_0 \, \Delta V = e \, \Delta V \\ &= (1.6 \times 10^{-19} \text{ C})(-4.0 \times 10^4 \text{ V}) \\ &= \boxed{-6.4 \times 10^{-15} \text{ J}}\end{aligned}$$

The negative sign means the potential energy of the system decreases as the proton moves in the direction of the electric field. As the proton accelerates in the direction of the field, it gains kinetic energy and at the same time the system loses electric potential energy.

(C) Find the speed of the proton after completing the 0.50 m displacement in the electric field.

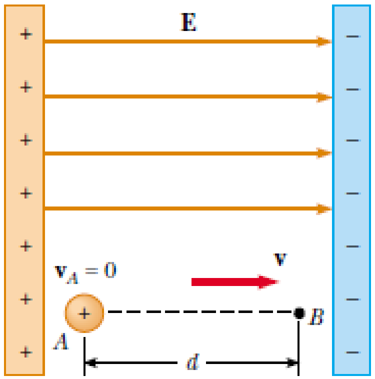


Figure 25.6 (Example 25.2) A proton accelerates from A to B in the direction of the electric field.

Solution The charge–field system is isolated, so the mechanical energy of the system is conserved:

$$\Delta K + \Delta U = 0$$

$$(\tfrac{1}{2}mv^2 - 0) + e \, \Delta V = 0$$

$$\begin{aligned}v &= \sqrt{\frac{-(2e\,\Delta V)}{m}} \\ &= \sqrt{\frac{-2(1.6 \times 10^{-19} \text{ C})(-4.0 \times 10^4 \text{ V})}{1.67 \times 10^{-27} \text{ kg}}} \\ &= \boxed{2.8 \times 10^6 \text{ m/s}}\end{aligned}$$

What If? **What if the situation is exactly the same as that shown in Figure 25.6, but no proton is present? Could both parts (A) and (B) of this example still be answered?**

25.2 Potential Differences in a Uniform Electric Field

Answer Part (A) of the example would remain exactly the same because the potential difference between points A and B is established by the source charges in the parallel plates. The potential difference does not depend on the presence of the proton, which plays the role of a test charge. Part (B) of the example would be meaningless if the proton is not present. A change in potential energy is related to a change in the charge–field system. In the absence of the proton, the system of the electric field alone does not change.

Examples:

If a 9 V battery has a charge of 46 C how much chemical energy does the battery have?

E = V Q = 9 V x 46C = 414 Joules

A pair of oppositely charged, parallel plates are separated by 5.33 mm. A potential difference of 600 V exists between the plates. (a) What is the magnitude of the electric field strength between the plates? (b) What is the magnitude of the force on an electron between the plates?

$$d = 0.00533m, \quad \Delta V = 600V, \quad E = ?, \quad q_{e^-} = 1.6x10^{-19}C$$

$$\Delta V = Ed$$

$$600 = E(0.0053)$$

$$E = 113{,}207.55 N/C$$

$$E = \frac{F_e}{q} = \frac{F_e}{1.6x10^{-19}C}$$

$$F_e = 1.81 \times 10^{-14}\,\mathrm{N}$$

Examples:

Calculate the speed of a proton that is accelerated from rest through a potential difference of 120 V

$$q_p = 1.6x10^{-19} C$$

$$m_p = 1.67x10^{-27} kg$$

$$V = 120V$$

$$v = ?$$

NOTE:
K+U=E
ΔU+ ΔK=0→ ΔK=- ΔU
W=- ΔU
→W= ΔK

$$W = q\Delta V \Rightarrow \Delta V = \frac{W}{q} = \frac{\Delta K}{q} = \frac{\frac{1}{2}mv^2}{q}$$

$$v = \sqrt{\frac{2q\Delta V}{m}} = \sqrt{\frac{2(1.6x10^{-19})(120)}{1.67x10^{-27}}} = 1.52\times10^5 \text{ m/s}$$