Chapter 26 Capacitance and Dielectrics

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26-1 Definition of Capacitance

The capacitance C of a capacitor is defined as the ratio of the magnitude of the charge on either conductor to the magnitude of the potential difference between the conductors:

$$C \equiv Q/\Delta V$$

The SI units of coulombs per volt. The SI unit of capacitance is the farad (F)

$$1\ \text{F} = 1\ \text{C/V}$$

The farad is an extremely large unit, typically you will see

microfarads (mF=10^{-6}F),

nanofarads (nF=10^{-9}F), and

picofarads (pF=10^{-12}F)

- Capacitance will always be a positive quantity
- The capacitance of a given capacitor is constant
- The capacitance is a measure of the capacitor's ability to store charge

26.2 Calculating Capacitance

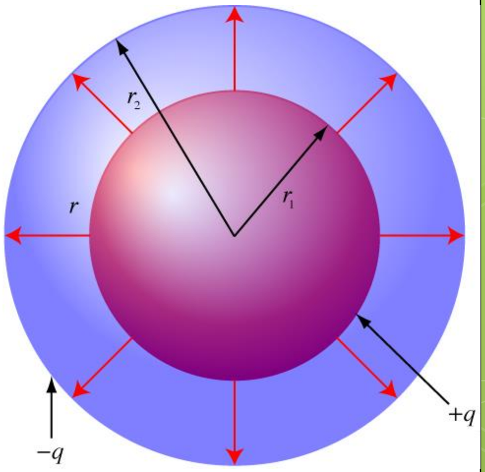
Let's assume that there is inner sphere ($r_1 = R$) has charge +q concentric with outer sphere that has charge –q (concentric spherical capacitor).

We obtain the capacitance of a single conducting sphere by taking our result for a spherical capacitor and moving the outer spherical conductor infinitely far away ($r_2 \rightarrow \infty$) i.e, $V = 0$ for the infinitely large shell.

$$C = \frac{Q}{\Delta V}$$ we know that $$\Delta V = k_e Q\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$C = \frac{Q}{k_e Q\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}$$ substitute for $$r_1 = R, r_2 \rightarrow \infty$$

$$C = \frac{1}{k_e\left(\frac{1}{R}\right)} = \frac{R}{k_e} = 4\pi \in_0 R$$



Note, this is independent of the charge and the potential difference.

26.2 Calculating Capacitance

Parallel - Plate Capacitors

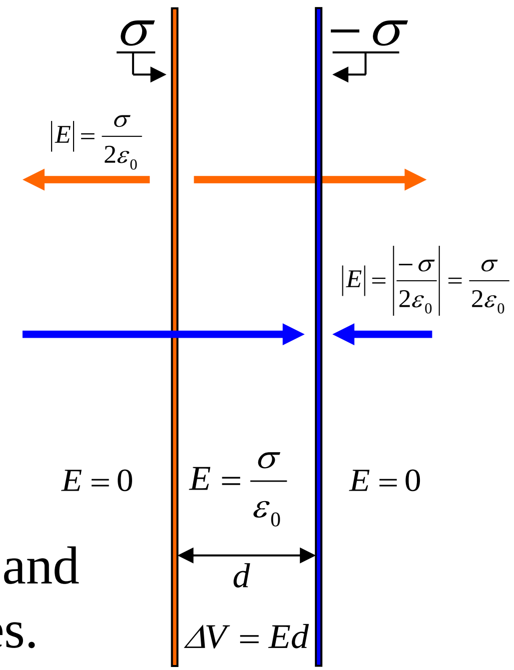
The charge density on the plates is σ = Q/A, where A is the area of each plate, which are equal. Q is, the charge on each plate, equal with opposite signs. The electric field is uniform between the plates and zero elsewhere.

$$C=\frac{Q}{\Delta V}\rightarrow Q=C\Delta V$$

$$\Delta V=Ed=\frac{\sigma}{\varepsilon_o}d=\frac{Qd}{A\varepsilon_o}$$

$$\Delta V=(\frac{d}{A\varepsilon_o})Q\rightarrow \quad Q=(\frac{A\varepsilon_o}{d})\Delta V$$

$$C=\frac{A\varepsilon_o}{d}$$



The capacitance is proportional to the area of its plates and inversely proportional to the distance between the plates.

26.2 Calculating Capacitance

Parallel - Plate Capacitors

The electric field lines for a parallel-plate capacitor reveals that the field is uniform in the central region between the plates and nonuniform at the edges of the plates. Such end effects can be neglected if the plate separation is small compared with the length of the plates.

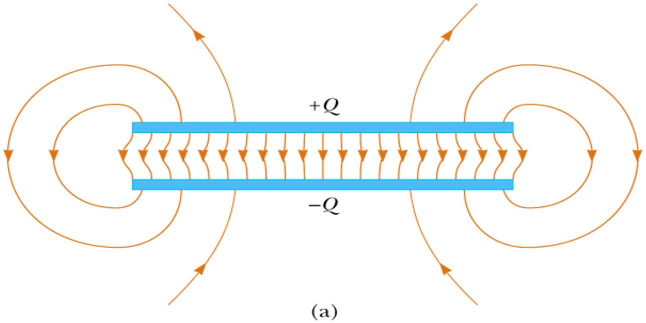
Example 26.1 Parallel-Plate Capacitor

A parallel-plate capacitor with air between the plates has an area $A = 2.00 \times 10^{-4}\ \text{m}^2$ and a plate separation $d = 1.00$ mm. Find its capacitance.

Solution From Equation 26.3, we find that

$$C = \frac{\epsilon_0 A}{d} = \frac{(8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(2.00 \times 10^{-4}\ \text{m}^2)}{1.00 \times 10^{-3}\ \text{m}}$$

$$= 1.77 \times 10^{-12}\ \text{F} = \boxed{1.77\ \text{pF}}$$



(a)

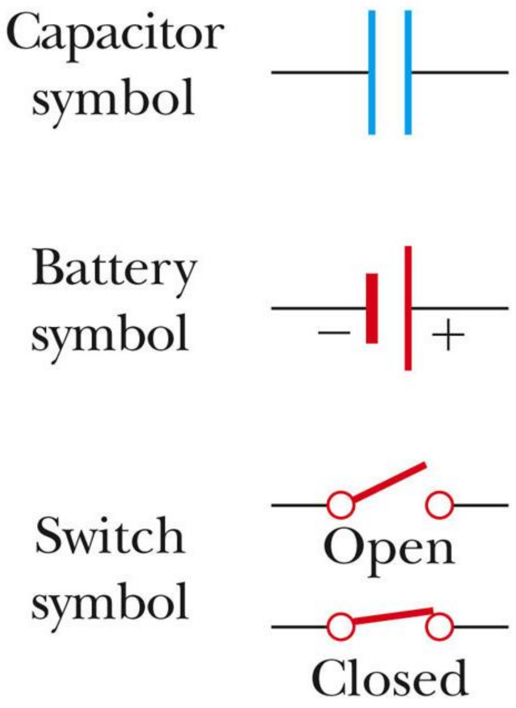
26.3 Combinations of Capacitors

A circuit diagram is a simplified representation of an actual circuit

Circuit symbols are used to represent the various elements

Lines are used to represent wires

The battery's positive terminal is indicated by the longer line



26.3 Parallel Combinations of Capacitors

The individual potential differences across capacitors connected in parallel are all the same and are equal to the potential difference applied across the combination.

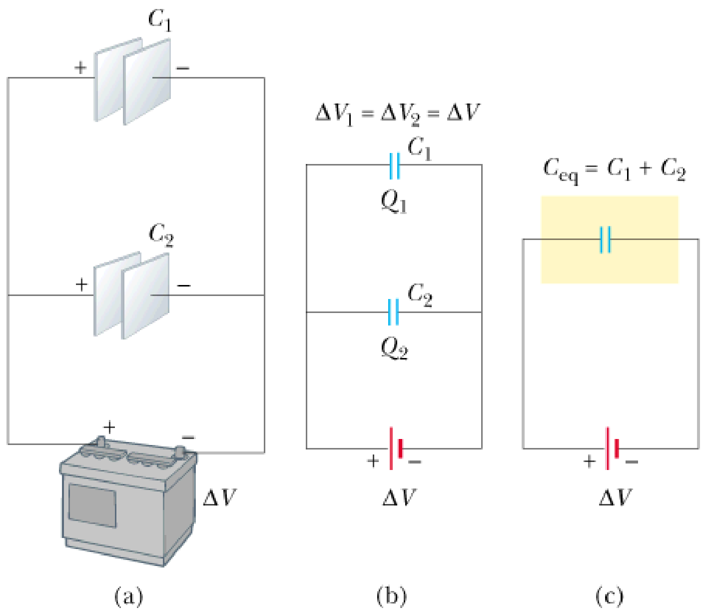
The total charge on capacitors connected in parallel is the sum of the charges on the individual capacitors:

$Q = Q_1 + Q_2$

$Q_1 = C_1 \Delta V, \qquad Q_2 = C_2 \Delta V$

$C_{eq} \Delta V = C_1 \Delta V + C_2 \Delta V$

$C_{eq} = C_1 + C_2$



26.3 Series Combinations of Capacitors

The charges on capacitors connected in series are the same.

the total potential difference across any number of capacitors connected in series is the sum of the potential differences across the individual capacitors.

$\Delta V = \Delta V_1 + \Delta V_2$

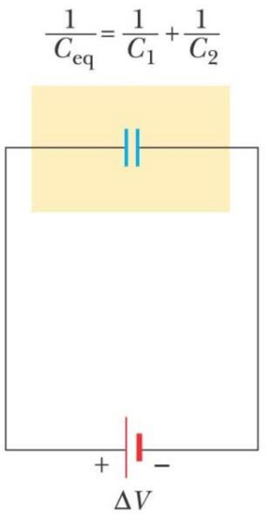
$= Q/C_{eq}$

$Q = Q_1 = Q_2$

$\Delta V_1 = Q/C_1$ & $\Delta V_2 = Q/C_2$

$Q/C_{eq} = Q/C_1 + Q/C_2$

$1/C_{eq} = 1/C_1 + 1/C_2$



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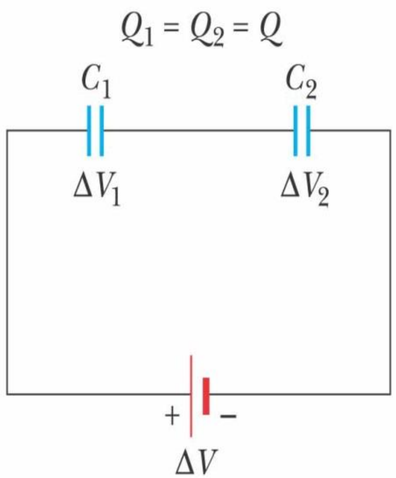

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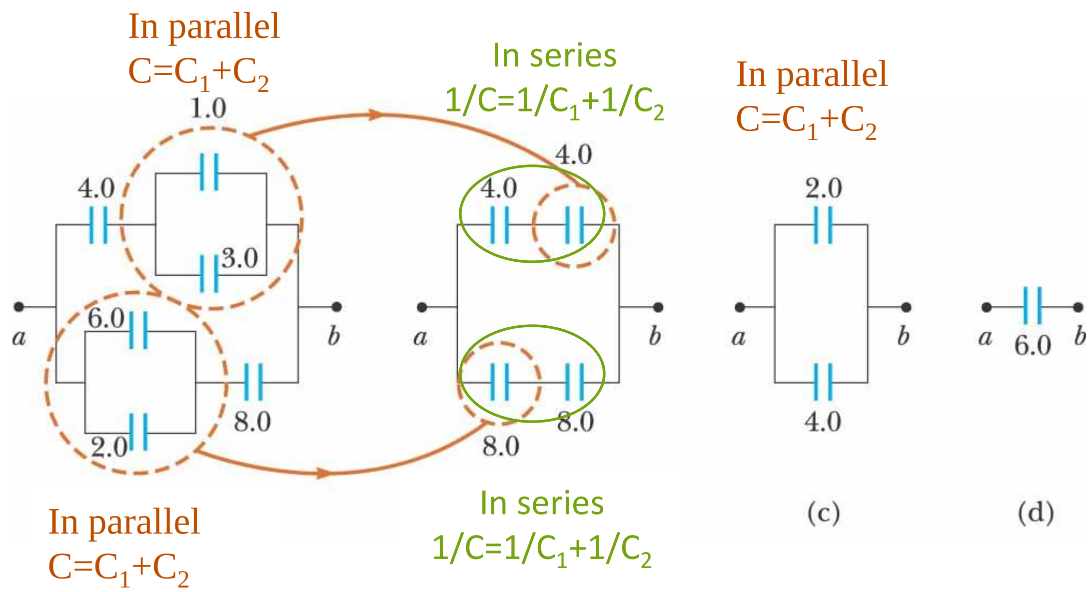
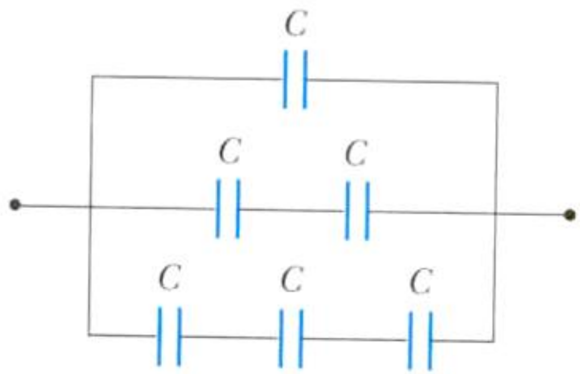
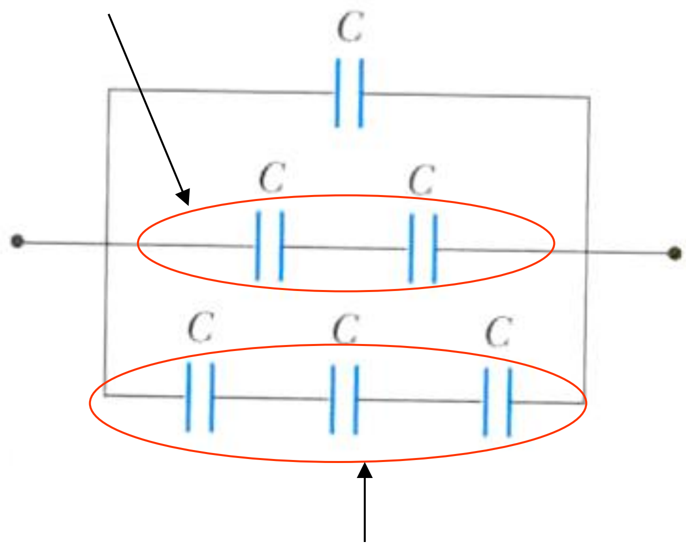
26.3 Combinations of Capacitors



- The 1.0-μF and 3.0-μF capacitors are in parallel as are the 6.0-μF and 2.0-μF capacitors
- These parallel combinations are in series with the capacitors next to them
- The series combinations are in parallel and the final equivalent capacitance can be found



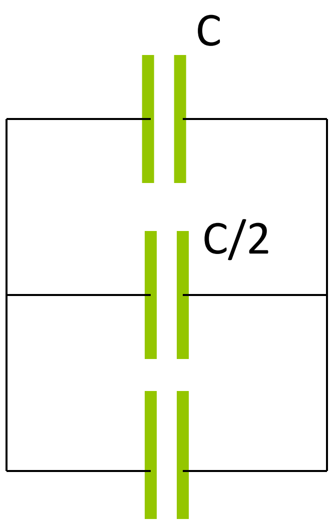
In series use $1/C_{eq}=1/C+1/C$



In series use $1/C_{eq}=1/C+1/C+1/C$

In parallel

$C_{eq}=C+C/2+C/3$



26.4 Energy Stored in a Charged Capacitor

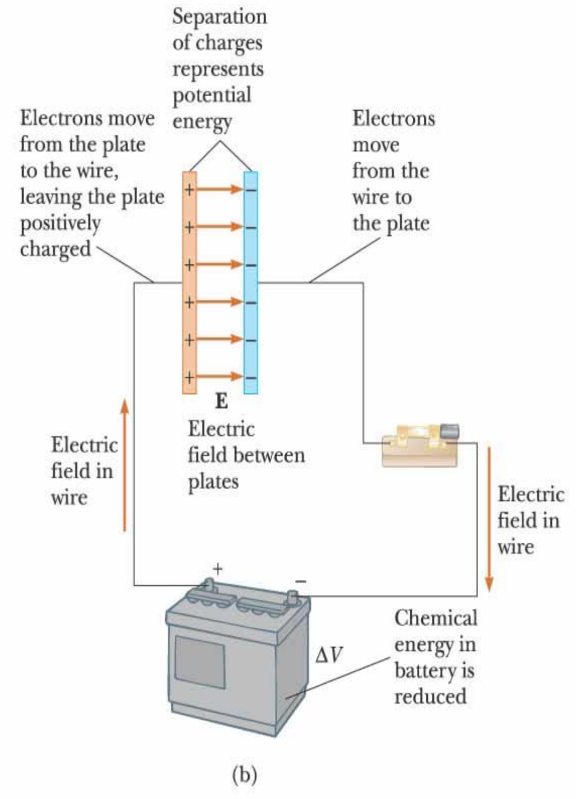
- Consider the circuit to be a system.

- Before the switch is closed, the energy is stored as chemical energy in the battery.

- When the switch is closed, the energy is transformed from chemical to electric potential energy.

- The electric potential energy is related to the separation of the positive and negative charges on the plates.

- A capacitor can be described as a device that stores energy as well as charge.



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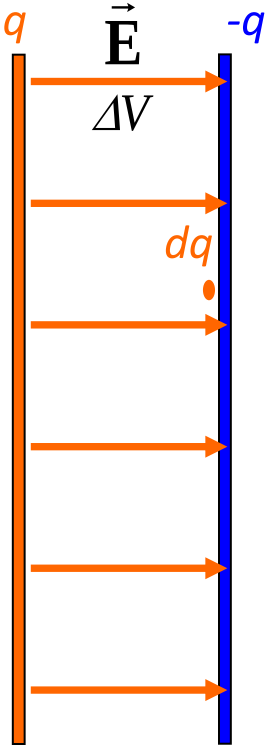
26.4 Energy Stored in a Charged Capacitor

To study this problem, recall that the work of a field force equals the electric potential energy loss (work done by the system)

$$W = -\Delta U = -Q\Delta V$$

and the work the battery does equals to the buildup of the electric potential energy (work done on the system)

$$W = \Delta U$$



The work necessary to transfer an increment of charge dq from the plate carrying charge -q to the plate carrying charge q (which is at the higher electric potential) is

$$dW = \Delta V dq = \frac{q}{C} dq$$

The total work required to charge the capacitor from q=0 to some final charge q=Q is

$$W = \int_0^Q dW = \int_0^Q \frac{q}{C} dq = \frac{Q^2}{2C}$$

26.4 Energy Stored in a Charged Capacitor

- The work done in charging the capacitor appears as electric potential energy U stored in the capacitor.

$$U = \frac{Q^2}{2C} \qquad \text{, using} \quad Q = C\Delta V$$

$$U = \frac{1}{2}Q\Delta V = \frac{1}{2}C(\Delta V)^2$$

- This applies to a capacitor of any geometry.
- The energy stored increases as the charge increases and as the potential difference increases.
- In practice, there is a maximum voltage before discharge occurs between the plates.

26.4 Energy Stored in a Charged Capacitor

- We can consider the energy stored in a capacitor as being stored in the electric field created between the plates as the capacitor is charged.
- For a parallel-plate capacitor ($\Delta V = Ed$ and $C = \varepsilon_0 A/d$), the energy can be expressed in terms of the field as

$$U = \tfrac{1}{2}\,(\varepsilon_0 A d)E^2$$

- It can also be expressed in terms of the energy density (energy per unit volume Ad)

$$\begin{aligned} u_E &= \text{U/volume} \\ &= \text{U/Ad} \\ &= \tfrac{1}{2}\,\varepsilon_0 E^2 \end{aligned}$$

The energy density in any electric field is proportional to the square of the magnitude of the electric field at a given point.

26.5 Capacitors with Dielectrics

- ✓ A dielectric is a nonconducting material that, when placed between the plates of a capacitor, increases the capacitance.
- ✓ Dielectrics include rubber, glass, and waxed paper.
- ✓ With a dielectric, the capacitance becomes

$$C = \kappa C_o$$

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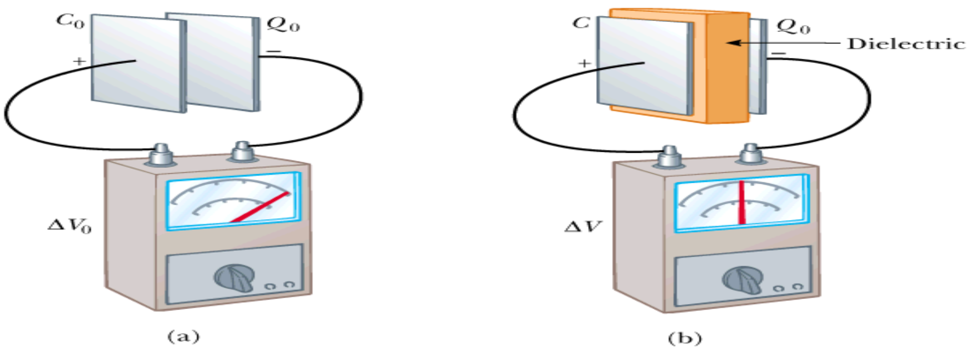
- ✓ κ is the dielectric constant of the material.
- ✓ The capacitance increases by the factor κ when the dielectric completely fills the region between the plates.



26.5 Capacitors with Dielectrics

- For a parallel-plate capacitor,

$$C_o = \varepsilon_o(A/d); \quad C=\kappa C_o \quad \rightarrow \quad C = \kappa\varepsilon_o(A/d)$$

- In theory, d could be made very small to create a very large capacitance but in practice, there is a limit to d. d is limited by the electric discharge that could occur though the dielectric medium separating the plates.

- For a given d, the maximum voltage that can be applied to a capacitor without causing a discharge depends on the dielectric strength of the material.

- If magnitude of the electric field in the dielectric exceeds the dielectric strength, then the insulating properties break down and the dielectric begins to conduct.

26.5 Capacitors with Dielectrics

Dielectrics provide the following advantages:

- ✓ Increase in capacitance.
- ✓ Increase the maximum operating voltage.
- ✓ Possible mechanical support between the plates, which allows the plates to be close together without touching, thereby decreases d and increases C

26.5 Capacitors with Dielectrics

Table 26.1

Approximate Dielectric Constants and Dielectric Strengths of Various Materials at Room Temperature

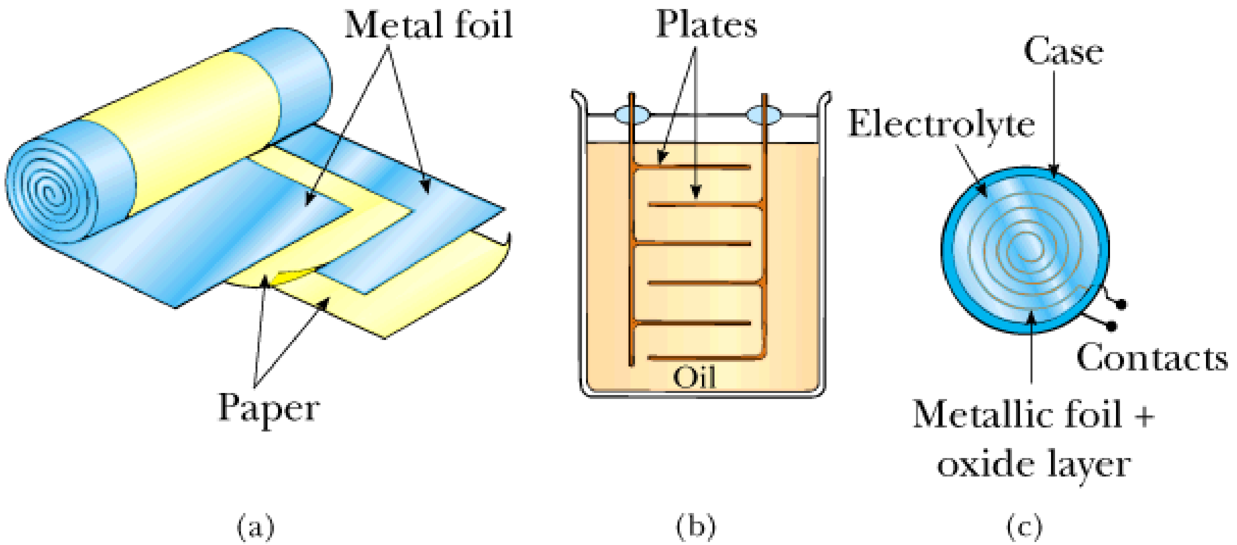
Material	Dielectric Constant κ	Dielectric Strength[a] (10^6 V/m)
Air (dry)	1.000 59	3
Bakelite	4.9	24
Fused quartz	3.78	8
Mylar	3.2	7
Neoprene rubber	6.7	12
Nylon	3.4	14
Paper	3.7	16
Paraffin-impregnated paper	3.5	11
Polystyrene	2.56	24
Polyvinyl chloride	3.4	40
Porcelain	6	12
Pyrex glass	5.6	14
Silicone oil	2.5	15
Strontium titanate	233	8
Teflon	2.1	60
Vacuum	1.000 00	—
Water	80	—

"Dielectric strength" is the maximum field in the dielectric before breakdown.
(a spark or flow of charge)

$$E_{max} = V_{max} / d$$

26.5 Capacitors with Dielectrics

Types of Capacitors



(a) A tubular capacitor, whose plates are separated by paper and then rolled into a cylinder. (b) A high-voltage capacitor consisting of many parallel plates separated by insulating oil. (c) An electrolytic capacitor.

Example 26.6 A Paper-Filled Capacitor

A parallel-plate capacitor has plates of dimensions 2.0 cm by 3.0 cm separated by a 1.0-mm thickness of paper.

(A) Find its capacitance.

Solution Because $\kappa = 3.7$ for paper (see Table 26.1), we have

$$C = \kappa \frac{\epsilon_0 A}{d}$$

$$= 3.7\left(\frac{(8.85 \times 10^{-12}\, \mathrm{C^2/N \cdot m^2})(6.0 \times 10^{-4}\, \mathrm{m^2})}{1.0 \times 10^{-3}\, \mathrm{m}}\right)$$

$$= 20 \times 10^{-12}\, \mathrm{F} = \boxed{20\ \mathrm{pF}}$$

(B) What is the maximum charge that can be placed on the capacitor?

Solution From Table 26.1 we see that the dielectric strength of paper is 16×10^6 V/m. Because the thickness of the paper is 1.0 mm, the maximum voltage that can be applied before breakdown is

$$\Delta V_{\max} = E_{\max} d = (16 \times 10^6\ \mathrm{V/m})(1.0 \times 10^{-3}\ \mathrm{m})$$

$$= 16 \times 10^3\ \mathrm{V}$$

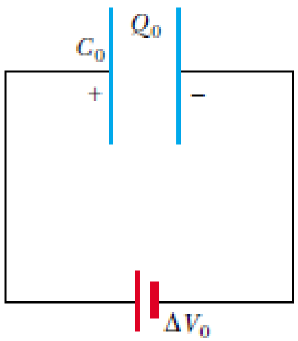
Hence, the maximum charge is

$$Q_{\max} = C\, \Delta V_{\max} = (20 \times 10^{-12}\, \mathrm{F})(16 \times 10^3\, \mathrm{V})$$

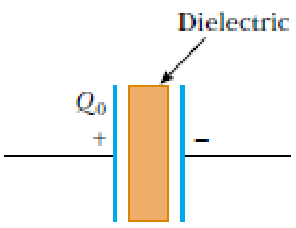
$$= \boxed{0.32\ \mu\mathrm{C}}$$

Example 26.7 Energy Stored Before and After

A parallel-plate capacitor is charged with a battery to a charge Q_0, as shown in Figure 26.20a. The battery is then removed, and a slab of material that has a dielectric constant κ is inserted between the plates, as shown in Figure 26.20b. Find the energy stored in the capacitor before and after the dielectric is inserted.



(a)



(b)

Figure 26.20 (Example 26.7) (a) A battery charges up a parallel-plate capacitor. (b) The battery is removed and a slab of dielectric material is inserted between the plates.

Solution From Equation 26.11, we see that the energy stored in the absence of the dielectric is

$$U_0 = \boxed{\frac{Q_0^2}{2C_0}}$$

After the battery is removed and the dielectric inserted, the *charge on the capacitor remains the same.* Hence, the energy stored in the presence of the dielectric is

$$U = \frac{Q_0^2}{2C}$$

But the capacitance in the presence of the dielectric is $C = \kappa C_0$, so U becomes

$$U = \frac{Q_0^2}{2\kappa C_0} = \boxed{\frac{U_0}{\kappa}}$$

Because $\kappa > 1$, the final energy is less than the initial energy. We can account for the "missing" energy by noting that the dielectric, when inserted, is pulled into the device (see Section 26.7). An external agent must do negative work to keep the dielectric from accelerating. This work is simply the difference $U - U_0$. (Alternatively, the positive work done by the system on the external agent is $U_0 - U$.)