Chapter 27 Current and resistance

27.1 Electric Current

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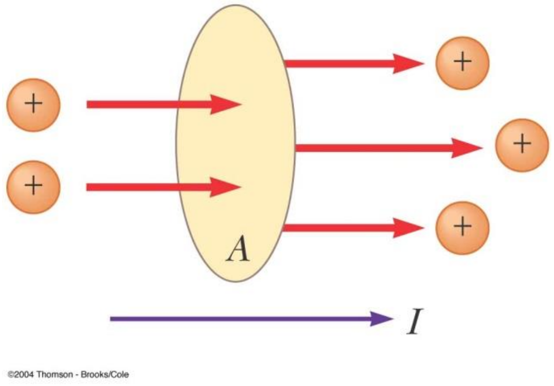
27.1 Electric Current

Consider a system of electric charges in motion. Whenever there is a net flow of charge through some region, a current, *I*, is said to exist.
Assume charges are moving perpendicular to a surface of area A. If ΔQ is the amount of charge that passes through A in time Δ t, then the average current is

$$I_{av} = \frac{\Delta Q}{\Delta t}$$

If the rate at which the charge flows varies with time, the instantaneous current, I, can be found

$$I \equiv \frac{dQ}{dt}$$



The SI unit of current is the ampere (A)

1 A = 1 C / s

27.1 Electric Current

It is conventional to assign to the current the same direction as the flow of positive charge.

The direction of the current is opposite the direction of flow of electrons.

It is common to refer to a moving charge (positive or negative) as a mobile charge carrier. For example, the mobile charge carriers in a metal are electrons.

27.1 Electric Current

If:

- There are charged particles move through a conductor of cross-sectional area A.
- n is the number of charge carriers per unit volume.
- $nA\Delta x$ is the total number of charge carriers.

Then, the total charge is the number of carriers times the charge per carrier, q

$$\Delta Q = (nA\Delta x)q$$

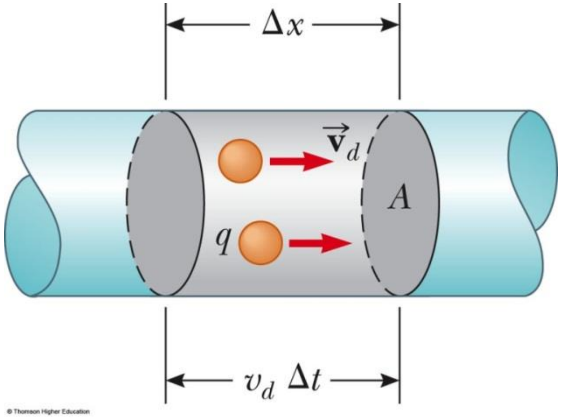
The drift speed, v_d, is the speed at which the carriers move

$$v_d = \Delta x / \Delta t \text{ and } \Delta x = v_d \Delta t$$

$$\rightarrow \Delta Q = (nAv_d\Delta t)q$$

Finally, current,

$$I_{\text{ave}} = \Delta Q/\Delta t = nqv_dA$$



27.1 Electric Current

Example 27.1 Drift Speed in a Copper Wire

The 12-gauge copper wire in a typical residential building has a cross-sectional area of $3.31 \times 10^{-6}\ \text{m}^2$. If it carries a current of 10.0 A, what is the drift speed of the electrons? Assume that each copper atom contributes one free electron to the current. The density of copper is $8.95\ \text{g/cm}^3$.

Solution From the periodic table of the elements in Appendix C, we find that the molar mass of copper is 63.5 g/mol. Recall that 1 mol of any substance contains Avogadro's number of atoms (6.02×10^{23}). Knowing the density of copper, we can calculate the volume occupied by 63.5 g (= 1 mol) of copper:

$$V = \frac{m}{\rho} = \frac{63.5\ \text{g}}{8.95\ \text{g/cm}^3} = 7.09\ \text{cm}^3$$

Because each copper atom contributes one free electron to the current, we have

$$n = \frac{6.02 \times 10^{23}\ \text{electrons}}{7.09\ \text{cm}^3}\left(\frac{1.00 \times 10^6\ \text{cm}^3}{1\ \text{m}^3}\right)$$
$$= 8.49 \times 10^{28}\ \text{electrons/m}^3$$

From Equation 27.4, we find that the drift speed is

$$v_d = \frac{I}{nqA}$$

where q is the absolute value of the charge on each electron. Thus,

$$v_d = \frac{I}{nqA}$$
$$= \frac{10.0\ \text{C/s}}{(8.49 \times 10^{28}\ \text{m}^{-3})(1.60 \times 10^{-19}\ \text{C})(3.31 \times 10^{-6}\ \text{m}^2)}$$
$$= \boxed{2.22 \times 10^{-4}\ \text{m/s}}$$

27.2 Resistance

We found that the electric field inside a conductor is zero. However, this statement is true only if the conductor is in static equilibrium. In this section, we will see what happens when the charges in the conductor are not in equilibrium, in which case there is an electric field in the conductor.

Consider a conductor of cross-sectional area A carrying a current I.
The current density J in the conductor is defined as the current per unit area. Because the current $I = nAqv_dA$, the current density is

$$J = \frac{I}{A} = nqv_d$$

J has SI units of A/m^2.

This expression is valid only if the current density is uniform and only if the surface of cross-sectional area A is perpendicular to the direction of the current. current density is a vector quantity:

$$\vec{J} = qn\overrightarrow{v_d}$$

27.2 Resistance

$$\vec{J} = qn\overrightarrow{v_d}$$

we see that current density is in the direction of charge motion for positive charge carriers and opposite the direction of motion for negative charge carriers.

A current density J and an electric field E are established in a conductor whenever a potential difference is maintained across the conductor.
If the potential difference is constant, then the current also is constant. In some materials, the current density is proportional to the electric field:

$$J = \sigma E \quad (*)$$

where the constant of proportionality σ is called the conductivity of the conductor. Materials that obey Equation * are said to follow Ohm's law. More specifically, Ohm's law states that ;
for many materials (including most metals), the ratio of the current density to the electric field is a constant σ that is independent of the electric field producing the current.

27.2 Resistance

Materials that obey Ohm's law and hence demonstrate this simple relationship between E and J are said to be ohmic,

Materials that do not obey Ohm's law are said to be non-ohmic

27.2 Resistance

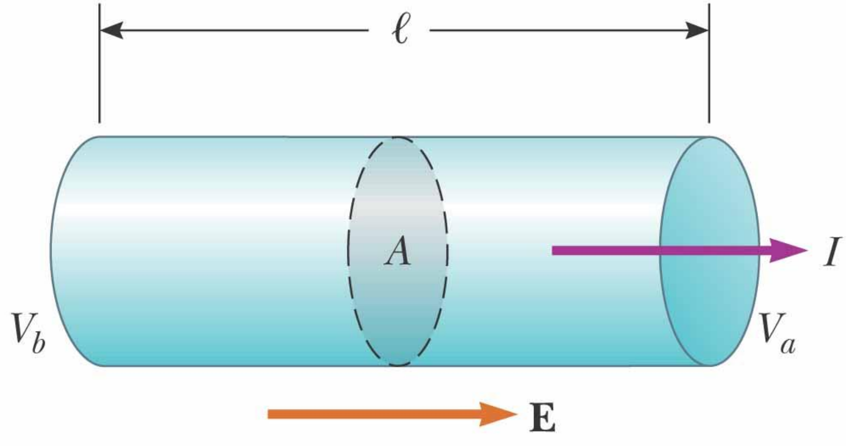
We can obtain a form of Ohm's law useful in practical applications by considering a segment of straight wire of uniform cross-sectional area A and length ℓ. If the field is assumed to be uniform, the potential difference is related to the field through the relationship

$$\Delta V = E\ell$$

$$J = \sigma E = \sigma \frac{\Delta V}{\ell}$$

$$\Delta V = \frac{\ell}{\sigma} J = \left(\frac{\ell}{\sigma A}\right) I$$

$$R \equiv \frac{\ell}{\sigma A} \equiv \frac{\Delta V}{I}$$



27.2 Resistance

$$\rho\,(\text{resistivity}) = \frac{1}{\sigma}$$

$$R \equiv \frac{\ell}{\sigma A} \equiv \rho\frac{\ell}{A} \equiv \frac{\Delta V}{I}$$

$$\sigma = \frac{1}{\rho} = \frac{\ell}{RA}$$

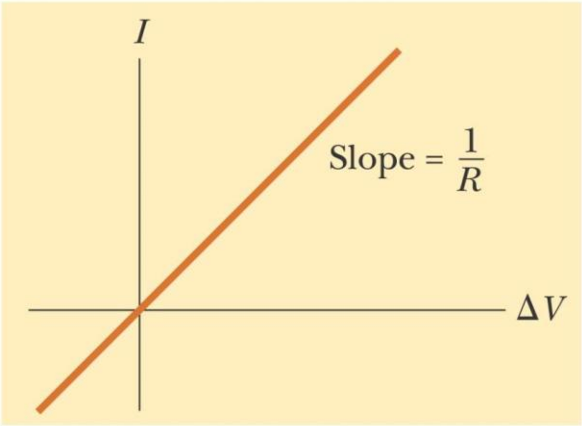
From this result we see that resistance has SI units of volts per ampere. One volt per ampere is defined to be 1 ohm (Ω):

$$1\ \Omega \equiv \frac{1\ \text{V}}{1\ \text{A}}$$

$$\boxed{I = \frac{V}{R}; \quad V = IR; \quad R = \frac{V}{I}}$$

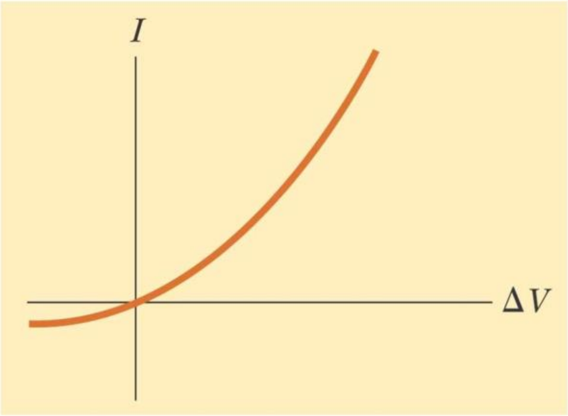
Increasing ΔV increases I
Increasing R decreases I

27.2 Resistance



(a)

(a) The current–potential difference curve for an ohmic material. The curve is linear, and the slope is equal to the inverse of the resistance of the conductor.



(b)

(b) A nonlinear current–potential difference curve for a semiconducting diode. This device does not obey Ohm's law.

EXAMPLE 27.2 The Resistance of a Conductor

Calculate the resistance of an aluminum cylinder that is 10.0 cm long and has a cross-sectional area of $2.0\text{x}\ 10^{-4}$ m^2. Repeat the calculation for a cylinder of the same dimensions and made of glass having a resistivity of $3\text{x}10^{10}\ \Omega$.m

$$R = \rho\frac{\ell}{A} = (2.82 \times 10^{-8}\ \Omega\cdot\text{m})\left(\frac{0.100\ \text{m}}{2.00 \times 10^{-4}\ \text{m}^2}\right)$$

$$= \boxed{1.41 \times 10^{-5}\ \Omega}$$

Similarly, for glass we find that

$$R = \rho\frac{\ell}{A} = (3.0 \times 10^{10}\ \Omega\cdot\text{m})\left(\frac{0.100\ \text{m}}{2.00 \times 10^{-4}\ \text{m}^2}\right)$$

$$= \boxed{1.5 \times 10^{13}\ \Omega}$$

EXAMPLE 27.3 The Resistance of Nichrome Wire

(a) Calculate the resistance per unit length of a 22-gauge Nichrome wire, which has a radius of 0.321 mm.

$$A = \pi r^2 = \pi(0.321 \times 10^{-3}\ \text{m})^2 = 3.24 \times 10^{-7}\ \text{m}^2$$

The resistivity of Nichrome is $1.5 \times 10^{-6}\ \Omega\cdot\text{m}$ (see Table 27.1). Thus, we can use Equation 27.11 to find the resistance per unit length:

$$\frac{R}{\ell} = \frac{\rho}{A} = \frac{1.5 \times 10^{-6}\ \Omega\cdot\text{m}}{3.24 \times 10^{-7}\ \text{m}^2} = \boxed{4.6\ \Omega/\text{m}}$$

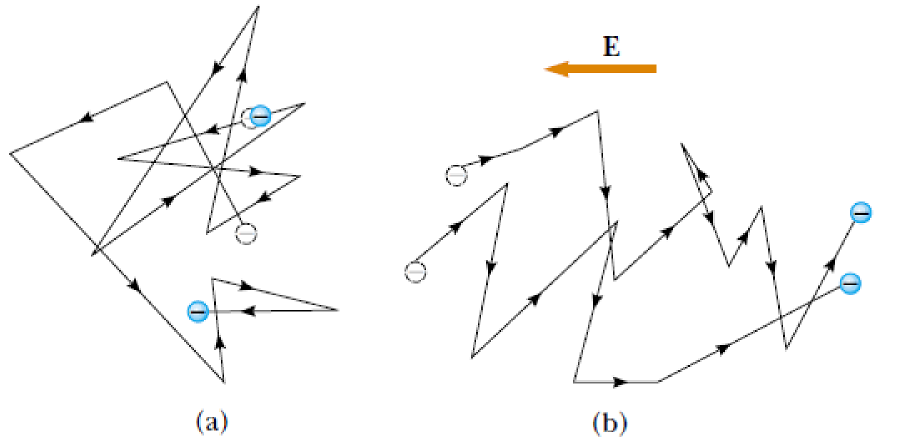
(b) If a potential difference of 10 V is maintained across a 1.0-m length of the Nichrome wire, what is the current in the wire?

$$I = \frac{\Delta V}{R} = \frac{10\ \text{V}}{4.6\ \Omega} = \boxed{2.2\ \text{A}}$$

27.3 A Model for Electrical Conduction

Conductor → atoms + free electrons (*conduction* electrons)
Conduction electrons bound to free atoms (not part of a solid)
Conduction electrons gain mobility when the free atoms condense into a solid.
No electric field→ conduction electrons move in random directions through the conductor with average speeds between collisions on the order of 10^6 m/s, no current (no net flow of charge) in the conductor because $v_d=0$

If electric field is applied, the free electrons drift slowly in a direction opposite that of the electric field, with an average $v_d=10^{-4}$ m/s.



27.3 A Model for Electrical Conduction

Assume that:

1- The motion of an electron after a collision is independent of its motion before the collision.

2- The excess energy acquired by the electrons in the electric field is lost to the atoms of the conductor when the electrons and atoms collide.

3- The energy given up to the atoms increases their vibrational energy, and this causes the temperature of the conductor to increase.

When a free electron of mass m_e and charge q =(-e) is subjected to an electric field E, it experiences a force F=qE. Because this force is related to the acceleration of the electron through Newton's second law, $F=m_e a$, we conclude that the acceleration of the electron is

$$\mathbf{a} = \frac{q\mathbf{E}}{m_e}$$

27.3 A Model for Electrical Conduction

The time interval between tow successive collisions starts at t=0 when the electron's initial velocity is v_i and ends at time t when electron's final velocity is v_f.

$$\mathbf{v}_f = \mathbf{v}_i + \mathbf{a}t = \mathbf{v}_i + \frac{q\mathbf{E}}{m_e}t$$

- We now take the average value of v_f over all possible collision times t and all possible values of v_i.
- The average value of v_i is zero (are randomly distributed over all possible values)
- The term (qE/me)t is the velocity change of the electron due to the electric field during one trip between atoms. Its average is (qE/me)τ, where τ is the *average time interval between successive collisions.*

Because the average value of v_f is equal to the drift velocity, we have

$$\overline{\mathbf{v}_f} = \mathbf{v}_d = \frac{q\mathbf{E}}{m_e}\tau$$

27.3 A Model for Electrical Conduction

We can relate this expression for drift velocity to the current in the conductor

$$J = nqv_d = \frac{nq^2E}{m_e}\tau$$

Comparing this expression with Ohm's law, J =σE, we obtain the following relationships for conductivity and resistivity of a conductor:

$$\sigma = \frac{nq^2\tau}{m_e} \qquad \rho = \frac{1}{\sigma} = \frac{m_e}{nq^2\tau}$$

conductivity and resistivity do not depend on the strength of the electric field. This feature is characteristic of a conductor obeying Ohm's law.

The average time interval τ between collisions is related to the average distance between collisions ℓ (*the mean free path*) and the average speed through the expression

$$\tau = \frac{\ell}{\bar{v}}$$

Example 27.5 Electron Collisions in a Wire

(A) Using the data and results from Example 27.1 and the classical model of electron conduction, estimate the average time interval between collisions for electrons in household copper wiring.

Solution From Equation 27.17, we see that

$$\tau = \frac{m_e}{nq^2\rho}$$

where $\rho = 1.7 \times 10^{-8}\ \Omega \cdot \text{m}$ for copper and the carrier density is $n = 8.49 \times 10^{28}$ electrons/m^3 for the wire described in Example 27.1. Substitution of these values into the expression above gives

$$\tau = \frac{9.11 \times 10^{-31}\ \text{kg}}{(8.49 \times 10^{28}\ \text{m}^{-3})(1.6 \times 10^{-19}\ \text{C})^2\,(1.7 \times 10^{-8}\ \Omega \cdot \text{m})}$$

$$= \boxed{2.5 \times 10^{-14}\ \text{s}}$$

(B) Assuming that the average speed for free electrons in copper is 1.6×10^6 m/s and using the result from part (A), calculate the mean free path for electrons in copper.

Solution From Equation 27.18,

$$\ell = \bar{v}\tau = (1.6 \times 10^6\ \text{m/s})(2.5 \times 10^{-14}\ \text{s})$$

$$= \boxed{4.0 \times 10^{-8}\ \text{m}}$$

27.4 Resistance and Temperature

Over a limited temperature range, the resistivity of a metal varies approximately linearly with temperature according to the expression

$$\rho = \rho_0[1 + \alpha(T - T_0)]$$

where ρ is the resistivity at some temperature T (in degrees Celsius), ρ_0 is the resistivity at some reference temperature T_0 (usually taken to be 20°C), and α is the temperature coefficient of resistivity.

$$\alpha = \frac{1}{\rho_0}\frac{\Delta\rho}{\Delta T}$$

The unit of α is degrees Celsius^{-1} [$(^{o}C)^{-1}$]

where $\Delta\rho = \rho - \rho_0$ is the change in resistivity in the temperature interval $\Delta T = T - T_0$.

27.4 Resistance and Temperature

Because resistance is proportional to resistivity

$$R = \rho \frac{L}{A}$$

For most materials, the resistance R changes in proportion to the initial resistance R_0 and to the change in temperature **Δ**T.

$$R = R_o\left[1 + \alpha\left(T - T_0\right)\right]$$

Change in resistance:

$$\Delta R = \alpha R_0 \Delta T$$

The temperature coefficient of resistance α, a is the change in resistance per unit resistance per unit degree change of temperature

$$\alpha = \frac{\Delta R}{R_0 \Delta T} \quad (°\mathrm{C})$$

27.4 Resistance and Temperature

Factors Affecting Resistance

$$R = \rho \frac{L}{A}$$

1. The length L of the material. Longer materials have greater resistance.

2. The cross-sectional area A of the material. Larger areas offer less resistance.

3. The temperature T of the material. The higher temperatures usually result in higher resistances.

4. The kind of material.

27.4 Resistance and Temperature

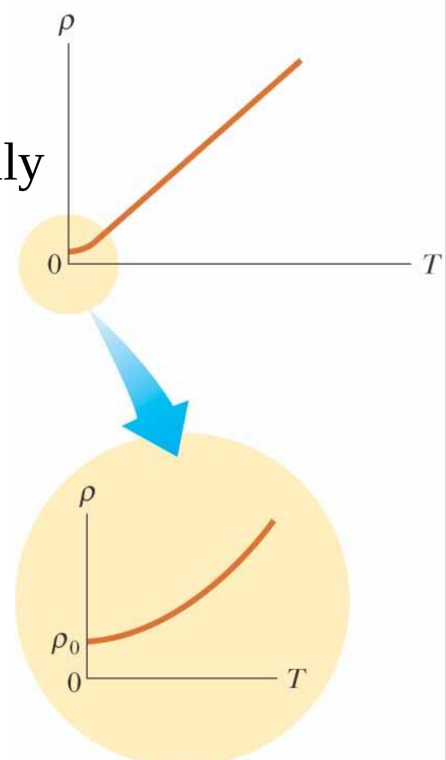
For some metals, the resistivity is nearly proportional to the temperature.

A nonlinear region always exists at very low temperatures.

The resistivity usually reaches some finite value as the temperature approaches absolute zero.

The residual resistivity near absolute zero is caused primarily by the collisions of electrons with impurities and imperfections in the metal.

High temperature resistivity (the linear region) is predominantly characterized by collisions between the electrons and the metal atoms.



EXAMPLE 27.6 A Platinum Resistance Thermometer

A resistance thermometer, which measures temperature by measuring the change in resistance of a conductor, is made from platinum and has a resistance of 50.0 Ω at 20.0°C. When immersed in a vessel containing melting indium, its resistance increases to 76.8 . Calculate the melting point of the indium.

$$\Delta T = \frac{R - R_0}{\alpha R_0} = \frac{76.8\ \Omega - 50.0\ \Omega}{[3.92 \times 10^{-3}\ (°\mathrm{C})^{-1}](50.0\ \Omega)} = 137°\mathrm{C}$$

Because $T_0 = 20.0°\mathrm{C}$, we find that T, the temperature of the melting indium sample, is 157°C.

27.6 Electrical Power

Assume a circuit as shown

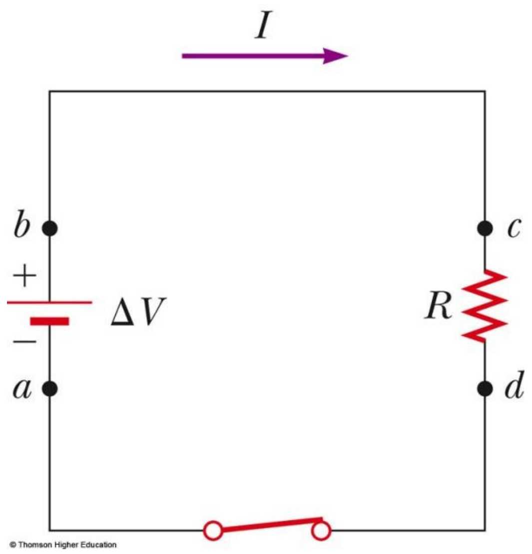
As a charge moves from a to b, the electric potential energy of the system increases by QΔV.

The chemical energy in the battery must decrease by this same amount (QΔV).

As the charge moves through the resistor (c to d), the system loses this electric potential energy during collisions of the electrons with the atoms of the resistor.

This energy is transformed into internal energy in the resistor.

Corresponds to increased vibrational motion of the atoms in the resistor.



27.6 Electrical Power

The resistor is normally in contact with the air, so its increased temperature will result in a transfer of energy by heat into the air.

The resistor also emits thermal radiation.

After some time interval, the resistor reaches a constant temperature.
The input of energy from the battery is balanced by the output of energy by heat and radiation.

The rate at which the system loses potential energy as the charge passes through the resistor is equal to the rate at which the system gains internal energy in the resistor.

The power is the rate at which the energy is delivered to the resistor

27.6 Electrical Power

The rate at which the charge Q loses potential energy in going through the resistor is

$$\frac{\Delta U}{\Delta t} = \frac{\Delta Q}{\Delta t}\Delta V = I\,\Delta V$$

The power is given by the equation:

$$\mathcal{P} = I\,\Delta V$$

Applying Ohm's Law, alternative expressions can be found:

$$\mathcal{P} = I^2 R = \frac{(\Delta V)^2}{R}$$

Units: I is in A, R is in Ω, V is in V, and $\mathcal{P}$ is in W

EXAMPLE 27.7 Power in an Electric Heater

An electric heater is constructed by applying a potential difference of 120 V to a Ni-chrome wire that has a total resistance of 8.0 Ω . Find the current carried by the wire and the power rating of the heater.

Solution Because $\Delta V = IR$, we have

$$I = \frac{\Delta V}{R} = \frac{120\ \text{V}}{8.00\ \Omega} = \boxed{15.0\ \text{A}}$$

We can find the power rating using the expression $\mathcal{P} = I^2R$:

$$\mathcal{P} = I^2R = (15.0\ \text{A})^2(8.00\ \Omega) = \boxed{1.80\ \text{kW}}$$

If we doubled the applied potential difference, the current would double but the power would be 4 times larger because

$$\mathcal{P} = (\Delta V)^2/R.$$