GENERAL Introduction

Chapter 1 Introduction

- Body of Knowledge
- Problem Solving and Decision Making
- Quantitative Analysis and Decision Making
- Quantitative Analysis
- Models of Cost, Revenue, and Profit
- Management Science Techniques

Body of Knowledge

- Management science
  - Is an approach to decision making based on the scientific method
  - Makes extensive use of quantitative analysis
- The body of knowledge involving quantitative approaches to decision making is also referred to as
  - Operations research
  - Decision science
- It had its early roots in World War II and is flourishing in business and industry with the aid of computers

Problem Solving and Decision Making

- 7 Steps of Problem Solving

(First 5 steps are the process of decision making)
  - Define the problem.
  - Identify the set of alternative solutions.
  - Determine the criteria for evaluating alternatives.
  - Evaluate the alternatives.
  - Choose an alternative (make a decision).

  - Implement the chosen alternative.
  - Evaluate the results.

Quantitative Analysis and Decision Making

- Potential Reasons for a Quantitative Analysis Approach to Decision Making
  - The problem is complex.
  - The problem is very important.
  - The problem is new.
  - The problem is repetitive.

Quantitative Analysis

- Quantitative Analysis Process
  - Model Development
  - Data Preparation
  - Model Solution
  - Report Generation

Model Development

- Models are representations of real objects or situations
- Three forms of models are:
  - Iconic models - physical replicas (scalar representations) of real objects
  - Analog models - physical in form, but do not physically resemble the object being modeled
  - Mathematical models - represent real world problems through a system of mathematical formulas and expressions based on key assumptions, estimates, or statistical analyses

Advantages of Models

- Generally, experimenting with models (compared to experimenting with the real situation):
  - requires less time
  - is less expensive
  - involves less risk

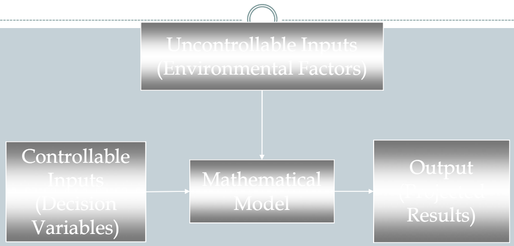
Mathematical Models

- Cost/benefit considerations must be made in selecting an appropriate mathematical model.
- Frequently a less complicated (and perhaps less precise) model is more appropriate than a more complex and accurate one due to cost and ease of solution considerations.

Mathematical Models

- Relate decision variables (controllable inputs) with fixed or variable parameters (uncontrollable inputs).
- Frequently seek to maximize or minimize some objective function subject to constraints.
- Are said to be stochastic if any of the uncontrollable inputs is subject to variation, otherwise are said to be deterministic.
- Generally, stochastic models are more difficult to analyze.
- The values of the decision variables that provide the mathematically-best output are referred to as the optimal solution for the model.

Transforming Model Inputs into Output



Example: Project Scheduling

Consider a construction company building a 250-unit apartment complex. The project consists of hundreds of activities involving excavating, framing, wiring, plastering, painting, landscaping, and more. Some of the activities must be done sequentially and others can be done simultaneously. Also, some of the activities can be completed faster than normal by purchasing additional resources (workers, equipment, etc.).

What is the best schedule for the activities and for which activities should additional resources be purchased?

Example: Project Scheduling

- Question:

  How could management science be used to solve this problem?
- Answer:

  Management science can provide a structured, quantitative approach for determining the minimum project completion time based on the activities' normal times and then based on the activities' expedited (reduced) times.

Example: Project Scheduling

- Question:

  What would be the uncontrollable inputs?
- Answer:
  - Normal and expedited activity completion times
  - Activity expediting costs
  - Funds available for expediting
  - Precedence relationships of the activities

Example: Project Scheduling

- Question:

  What would be the decision variables of the mathematical model? The objective function? The constraints?
- Answer:
  - Decision variables: which activities to expedite and by how much, and when to start each activity
  - Objective function: minimize project completion time
  - Constraints: do not violate any activity precedence relationships and do not expedite in excess of the funds available.

Example: Project Scheduling

- Question:

  Is the model deterministic or stochastic?
- Answer:

  Stochastic. Activity completion times, both normal and expedited, are uncertain and subject to variation. Activity expediting costs are uncertain. The number of activities and their precedence relationships might change before the project is completed due to a project design change.

Example: Project Scheduling

- Question:

  Suggest assumptions that could be made to simplify the model.
- Answer:

  Make the model deterministic by assuming normal and expedited activity times are known with certainty and are constant. The same assumption might be made about the other stochastic, uncontrollable inputs.

Data Preparation

- Data preparation is not a trivial step, due to the time required and the possibility of data collection errors.
- A model with 50 decision variables and 25 constraints could have over 1300 data elements!
- Often, a fairly large data base is needed.
- Information systems specialists might be needed.

Model Solution

- The analyst attempts to identify the alternative (the set of decision variable values) that provides the "best" output for the model.
- The "best" output is the optimal solution.
- If the alternative does not satisfy all of the model constraints, it is rejected as being infeasible, regardless of the objective function value.
- If the alternative satisfies all of the model constraints, it is feasible and a candidate for the "best" solution.

Model Solution

- One solution approach is trial-and-error.
  - Might not provide the best solution
  - Inefficient (numerous calculations required)
- Special solution procedures have been developed for specific mathematical models.
  - Some small models/problems can be solved by hand calculations
  - Most practical applications require using a computer

Computer Software

- A variety of software packages are available for solving mathematical models.
  - Spreadsheet packages such as *Microsoft Excel*
  - *The formal languages (LINGO, AMPL, GAMS, ..)*

Model Testing and Validation

- Often, goodness/accuracy of a model cannot be assessed until solutions are generated.
- Small test problems having known, or at least expected, solutions can be used for model testing and validation.
- If the model generates expected solutions, use the model on the full-scale problem.
- If inaccuracies or potential shortcomings inherent in the model are identified, take corrective action such as:
  - Collection of more-accurate input data
  - Modification of the model

Report Generation

- A managerial report, based on the results of the model, should be prepared.
- The report should be easily understood by the decision maker.
- The report should include:
  - the recommended decision
  - other pertinent information about the results (for example, how sensitive the model solution is to the assumptions and data used in the model)

Implementation and Follow-Up

- Successful implementation of model results is of critical importance.
- Secure as much user involvement as possible throughout the modeling process.
- Continue to monitor the contribution of the model.
- It might be necessary to refine or expand the model.

Example: Austin Auto Auction

An auctioneer has developed a simple mathematical model for deciding the starting bid he will require when auctioning a used automobile.

Essentially, he sets the starting bid at seventy percent of what he predicts the final winning bid will (or should) be. He predicts the winning bid by starting with the car's original selling price and making two deductions, one based on the car's age and the other based on the car's mileage.

The age deduction is \$800 per year and the mileage deduction is \$.025 per mile.

Example: Austin Auto Auction

- Question:

Develop the mathematical model that will give the starting bid (B) for a car in terms of the car's original price (P), current age (A) and mileage (M).

Example: Austin Auto Auction

- Answer:

The expected winning bid can be expressed as:

$$P - 800(A) - .025(M)$$

The entire model is:

$$B = .7(\text{expected winning bid})$$
$$B = .7(P - 800(A) - .025(M))$$

$$B = .7(P) - 560(A) - .0175(M)$$

Example: Austin Auto Auction

- Question:

Suppose a four-year old car with 60,000 miles on the odometer is up for auction. If its original price was \$12,500, what starting bid should the auctioneer require?

- Answer:

$B = .7(12{,}500) - 560(4) - .0175(60{,}000) =$ \$5460

Example: Austin Auto Auction

- Question:

  The model is based on what assumptions?

- Answer:

  The model assumes that the only factors influencing the value of a used car are the original price, age, and mileage (not condition, rarity, or other factors).

  Also, it is assumed that age and mileage devalue a car in a linear manner and without limit. (Note, the starting bid for a very old car might be negative!)

Example: Iron Works, Inc.

Iron Works, Inc. (IWI) manufactures two products made from steel and just received this month's allocation of *b* pounds of steel. It takes a_1 pounds of steel to make a unit of product 1 and it takes a_2 pounds of steel to make a unit of product 2.

Let x_1 and x_2 denote this month's production level of product 1 and product 2, respectively. Denote by p_1 and p_2 the unit profits for products 1 and 2, respectively.

The manufacturer has a contract calling for at least *m* units of product 1 this month. The firm's facilities are such that at most *u* units of product 2 may be produced monthly.

Example: Iron Works, Inc.

- Mathematical Model
  - The total monthly profit =

    (profit per unit of product 1)
    x (monthly production of product 1)
    + (profit per unit of product 2)
    x (monthly production of product 2)
    $= p_1x_1 + p_2x_2$

    We want to maximize total monthly profit:

    Max $p_1x_1 + p_2x_2$

Example: Iron Works, Inc.

- Mathematical Model (continued)
  - The total amount of steel used during monthly production equals:

    (steel required per unit of product 1)
    x (monthly production of product 1)
    + (steel required per unit of product 2)
    x (monthly production of product 2)
    $= a_1x_1 + a_2x_2$

  This quantity must be less than or equal to the allocated *b* pounds of steel:

  $$a_1x_1 + a_2x_2 \leq b$$

Example: Iron Works, Inc.

- Mathematical Model (continued)
  - The monthly production level of product 1 must be greater than or equal to m :

$$x_1 \geq m$$

  - The monthly production level of product 2 must be less than or equal to u :

$$x_2 \leq u$$

  - However, the production level for product 2 cannot be negative:

$$x_2 \geq 0$$

Example: Iron Works, Inc.

- Mathematical Model Summary

$$\begin{array}{lrcl} \text{Max} & p_1x_1 + p_2x_2 & & \\ \text{s.t.} & a_1x_1 + a_2x_2 & \leq & b \\ & x_1 & \geq & m \\ & x_2 & \leq & u \\ & x_2 & \geq & 0 \end{array}$$

Objective Function

Constraints

"Subject to"

Example: Iron Works, Inc.

- Question:

Suppose b = 2000, a_1 = 2, a_2 = 3, m = 60, u = 720, p_1 = 100, p_2 = 200. Rewrite the model with these specific values for the uncontrollable inputs.

Example: Iron Works, Inc.

- Answer:

Substituting, the model is:

$$\begin{array}{lrcl} \text{Max} & 100x_1 + 200x_2 & & \\ \text{s.t.} & 2x_1 + 3x_2 & \leq & 2000 \\ & x_1 & \geq & 60 \\ & x_2 & \leq & 720 \\ & x_2 & \geq & 0 \end{array}$$

Example: Iron Works, Inc.

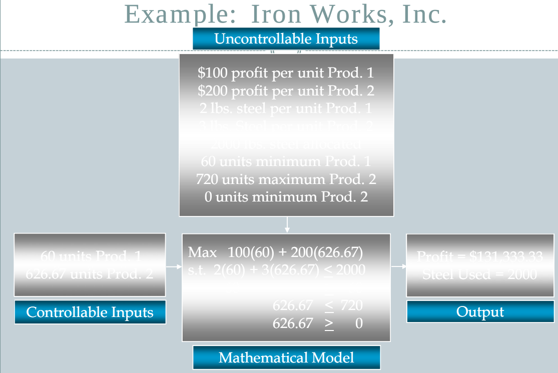
- Question:

The optimal solution to the current model is $x_1 = 60$ and $x_2 = 626\ 2/3$. If the product were engines, explain why this is not a true optimal solution for the "real-life" problem.

- Answer:

One cannot produce and sell 2/3 of an engine. Thus the problem is further restricted by the fact that both x_1 and x_2 must be integers. They could remain fractions if it is assumed these fractions are work in progress to be completed the next month.

Example: Iron Works, Inc.



Example: Ponderosa Development Corp.

Ponderosa Development Corporation (PDC) is a small real estate developer that builds only one style house. The selling price of the house is $115,000.

Land for each house costs $55,000 and lumber, supplies, and other materials run another $28,000 per house. Total labor costs are approximately $20,000 per house.

Example: Ponderosa Development Corp.

Ponderosa leases office space for $2,000 per month. The cost of supplies, utilities, and leased equipment runs another $3,000 per month.

The one salesperson of PDC is paid a commission of $2,000 on the sale of each house. PDC has seven permanent office employees whose monthly salaries are given on the next slide.

Example: Ponderosa Development Corp.

Employee	Monthly Salary
President	$10,000
VP, Development	6,000
VP, Marketing	4,500
Project Manager	5,500
Controller	4,000
Office Manager	3,000
Receptionist	2,000

Example: Ponderosa Development Corp.

- Question:

  Identify all costs and denote the marginal cost and marginal revenue for each house.
- Answer:

  The monthly salaries total $35,000 and monthly office lease and supply costs total another $5,000. This $40,000 is a monthly fixed cost.

  The total cost of land, material, labor, and sales commission per house, $105,000, is the marginal cost for a house.

  The selling price of $115,000 is the marginal revenue per house.

Example: Ponderosa Development Corp.

- Question:

  Write the monthly cost function $c(x)$, revenue function $r(x)$, and profit function $p(x)$.
- Answer:

  $c(x)$ = variable cost + fixed cost = $105{,}000x + 40{,}000$

  $r(x) = 115{,}000x$

  $p(x) = r(x) - c(x) = 10{,}000x - 40{,}000$

Example: Ponderosa Development Corp.

- Question:

  What is the breakeven point for monthly sales of the houses?
- Answer:

  $r(x) = c(x)$

  $115{,}000x = 105{,}000x + 40{,}000$

  Solving, $x = 4$.

Example: Ponderosa Development Corp.

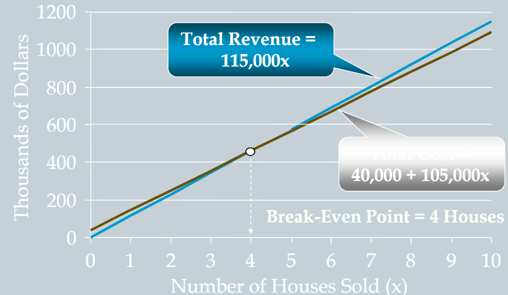
- Question:

  What is the monthly profit if 12 houses per month are built and sold?
- Answer:

  p (12) = 10,000(12) - 40,000 = $80,000 monthly profit

Example: Ponderosa Development Corp.

- Graph of Break-Even Analysis



Example: Ponderosa Development Corp.

- A <u>spreadsheet software package</u> such as <u>*Microsoft Excel*</u> can be used to perform a quantitative analysis of Ponderosa Development Corporation.
- We will enter the <u>problem data</u> in the top portion of the spreadsheet.
- The bottom of the spreadsheet will be used for <u>model development</u>.

Example: Ponderosa Development Corp.

- Formula

	A	B
1	PROBLEM DATA	
2	Fixed Cost	$40,000
3	Variable Cost Per Unit	$105,000
4	Selling Price Per Unit	$115,000
5	MODEL	
6	Sales Volume	
7	Total Revenue	=B4*B6
8	Total Cost	=B2+B3*B6
9	Total Profit (Loss)	=B7-B8

Example: Ponderosa Development Corp.

- Question

  What is the monthly profit if 12 houses per month are built and sold?

Example: Ponderosa Development Corp.

- Spreadsh

	A	B
1	PROBLEM DATA	
2	Fixed Cost	$40,000
3	Variable Cost Per Unit	$105,000
4	Selling Price Per Unit	$115,000
5	MODEL	
6	Sales Volume	12
7	Total Revenue	$1,380,000
8	Total Cost	$1,300,000
9	Total Profit (Loss)	$80,000

Example: Ponderosa Development Corp.

- Question:

  What is the breakeven point for monthly sales of the houses?

- Spreadsheet Solution:
  - One way to determine the break-even point using a spreadsheet is to use the Goal Seek tool.
  - *Microsoft Excel* 's Goal Seek tool allows the user to determine the value for an input cell that will cause the output cell to equal some specified value.
  - In our case, the goal is to set Total Profit to zero by seeking an appropriate value for Sales Volume.

Example: Ponderosa Development Corp.

- Spreadsheet Solution: Goal Seek Approach to Determining Break-Even Point

Using *Excel* 's **Goal Seek** Tool

Step 1: Select the **Tools** pull-down menu

Step 2: Choose the **Goal Seek** option

Step 3: When the **Goal Seek dialog box** appears:

Enter B9 in the **Set cell** box

Enter 0 in the **To value** box

Enter B6 in the **By changing cell** box

Select **OK**

Example: Ponderosa Development Corp.

- Spreadsheet Solution: Goal Seek Approach to Determining Break-Even Point

Example: Ponderosa Development Corp.

- Spreadsheet Solution: Goal Seek Approach to
  Even Point

	A	B
1	PROBLEM DATA	
2	Fixed Cost	$40,000
3	Variable Cost Per Unit	$105,000
4	Selling Price Per Unit	$115,000
5	MODEL	
6	Sales Volume	4
7	Total Revenue	$460,000
8	Total Cost	$460,000
9	Total Profit (Loss)	$0

Management Science Techniques

- Linear Programming
- Integer Linear Programming
- Network Models
- PERT/CPM
- Inventory models
- Queuing Models
- Simulation
- Decision Analysis
- Goal Programming
- Analytic Hierarchy Process
- Forecasting
- Markov-Process Models
- Dynamic Programming

End of Chapter 1