Secant Method

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Outline

- Problems with Newton’s Method (NM).
- The secant method (SM).
- Comparison between NM and SM.
- Final Remarks

Rationale for the Secant Method

Problems with Newton's Method

- Newton's method is an extremely powerful technique, but it has a major weakness: the need to know the value of the derivative of f at each approximation.
- Frequently, $f'(x)$ is far more difficult and needs more arithmetic operations to calculate than $f(x)$.

Derivation of the Secant Method

$$f'(p_{n-1}) = \lim_{x \to p_{n-1}} \frac{f(x) - f(p_{n-1})}{x - p_{n-1}}.$$

Circumvent the Derivative Evaluation

If p_{n-2} is close to p_{n-1}, then

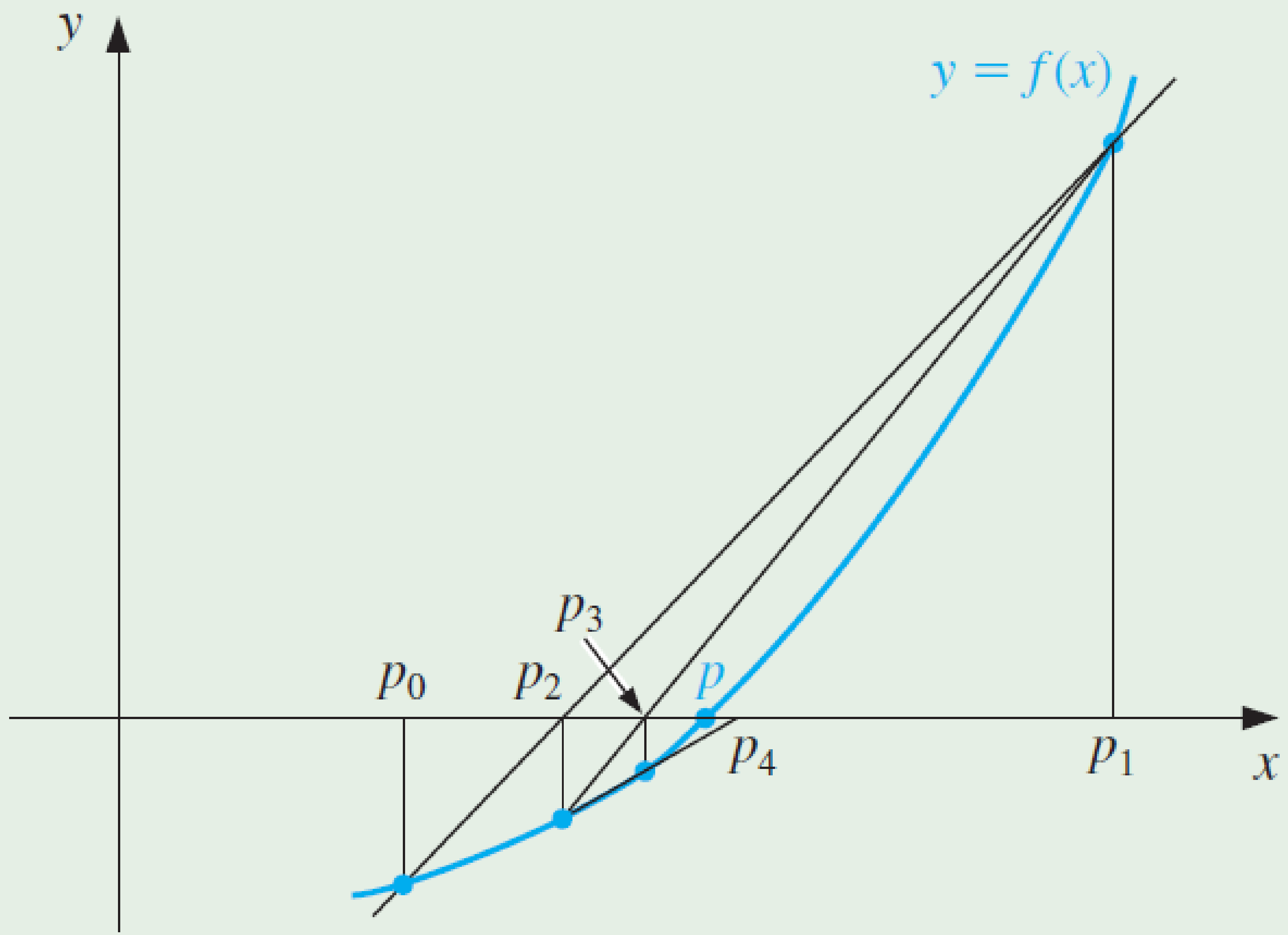
$$f'(p_{n-1}) \approx \frac{f(p_{n-2}) - f(p_{n-1})}{p_{n-2} - p_{n-1}} = \frac{f(p_{n-1}) - f(p_{n-2})}{p_{n-1} - p_{n-2}}.$$

Using this approximation for $f'(p_{n-1})$ in Newton's formula gives

$$p_n = p_{n-1} - \frac{f(p_{n-1})(p_{n-1} - p_{n-2})}{f(p_{n-1}) - f(p_{n-2})}$$

This technique is called the Secant method

Secant Method: Using Successive Secants



The Secant Method

$$p_n = p_{n-1} - \frac{f(p_{n-1})(p_{n-1} - p_{n-2})}{f(p_{n-1}) - f(p_{n-2})}$$

Procedure

- Starting with the two initial approximations p_0 and p_1, the approximation p_2 is the x-intercept of the line joining $(p_0, f(p_0))$ and $(p_1, f(p_1))$.
- The approximation p_3 is the x-intercept of the line joining $(p_1, f(p_1))$ and $(p_2, f(p_2))$, and so on.
- Note that only one function evaluation is needed per step for the Secant method after p_2 has been determined.
- In contrast, each step of Newton's method requires an evaluation of both the function and its derivative.

Comparing the Secant & Newton's Methods

Example: $f(x) = \cos x - x$

Use the Secant method to find a solution to $x = \cos x$, and compare the approximations with those given by Newton's method with $p_0 = \pi/4$.

Formula for the Secant Method

We need two initial approximations. Suppose we use $p_0 = 0.5$ and $p_1 = \pi/4$. Succeeding approximations are generated by the formula

$$p_n = p_{n-1} - \frac{(p_{n-1} - p_{n-2})(\cos p_{n-1} - p_{n-1})}{(\cos p_{n-1} - p_{n-1}) - (\cos p_{n-2} - p_{n-2})}, \quad \text{for } n \geq 2.$$

Comparing the Secant & Newton's Methods

Newton's Method for $f(x) = \cos(x) - x$, $p_0 = \frac{\pi}{4}$

n	p_{n-1}	$f(p_{n-1})$	$f'(p_{n-1})$	p_n	$\lvert p_n - p_{n-1}\rvert$
1	0.78539816	-0.078291	-1.707107	0.73953613	0.04586203
2	0.73953613	-0.000755	-1.673945	0.73908518	0.00045096
3	0.73908518	-0.000000	-1.673612	0.73908513	0.00000004
4	0.73908513	-0.000000	-1.673612	0.73908513	0.00000000

- An excellent approximation is obtained with $n = 3$.
- Because of the agreement of p_3 and p_4 we could reasonably expect this result to be accurate to the places listed.

Comparing the Secant & Newton's Methods

Secant Method for $f(x) = \cos(x) - x$, $p_0 = 0.5$, $p_1 = \frac{\pi}{4}$

n	p_{n-2}	p_{n-1}	p_n	$\lvert p_n - p_{n-1}\rvert$
2	0.500000000	0.785398163	0.736384139	0.0490140246
3	0.785398163	0.736384139	0.739058139	0.0026740004
4	0.736384139	0.739058139	0.739085149	0.0000270101
5	0.739058139	0.739085149	0.739085133	0.0000000161

- Comparing results, we see that the Secant Method approximation p_5 is accurate to the tenth decimal place, whereas Newton's method obtained this accuracy by p_3.
- Here, the convergence of the Secant method is much faster than functional iteration but slightly slower than Newton's method.

The Secant Method

Final Remarks

- The Secant method and Newton's method are often used to refine an answer obtained by another technique (such as the Bisection Method).
- Both methods require good first approximations but generally give rapid acceleration.