Interpolation & Polynomial Approximation
Divided Differences

Dr. Feras Fraige

Predict the value of f(1.1) using Newton forward divided difference based on data points given.

x	$f(x)$
1.0	0.7651977
1.3	0.6200860
1.6	0.4554022
1.9	0.2818186
2.2	0.1103623

		First divided differences	Second divided differences	Third divided differences	Fourth divided differences
1.0	0.7651977				
		−0.4837057			
1.3	0.6200860		−0.1087339		
		−0.5489460		0.0658784	
1.6	0.4554022		−0.0494433		0.0018251
		−0.5786120		0.0680685	
1.9	0.2818186		0.0118183		
		−0.5715210			
2.2	0.1103623				

$h = x_1 - x_0 = 1.3 - 1 = 0.3$

$S = (x - x_0)/h = (1.1 - 1)/0.3$

$S = 1/3$

- If an approximation to *f (1.1) is required, the reasonable choice for the nodes would* be $x0 = 1.0$, $x1 = 1.3$, $x2 = 1.6$, $x3 = 1.9$, *and* $x4 = 2.2$ *since this choice makes the* earliest possible use of the data points closest to $x = 1.1$, *and also makes use of the fourth* divided difference. This implies that $h = 0.3$ *and* $s = 1/3$, *so the Newton forward divided difference* formula is used with the divided differences that have a *solid underline (___) in* Table

		First divided differences	Second divided differences	Third divided differences	Fourth divided differences
1.0	0.7651977				
		−0.4837057			
1.3	0.6200860		−0.1087339		
		−0.5489460		0.0658784	
1.6	0.4554022		−0.0494433		0.0018251
		−0.5786120		0.0680685	
1.9	0.2818186		0.0118183		
		−0.5715210			
2.2	0.1103623				

$$
\begin{aligned}
P_n(x) = P_n(x_0 + sh) &= f[x_0] + sh\,f[x_0, x_1] + s(s-1)h^2 f[x_0, x_1, x_2] \\
&\quad + \cdots + s(s-1)\cdots(s-n+1)h^n f[x_0, x_1, \ldots, x_n] \\
&= f[x_0] + \sum_{k=1}^{n} s(s-1)\cdots(s-k+1)h^k f[x_0, x_1, \ldots, x_k].
\end{aligned}
$$

$$
\begin{aligned}
P_4(1.1) &= P_4\left(1.0 + \frac{1}{3}(0.3)\right) \\
&= 0.7651977 + \frac{1}{3}(0.3)(-0.4837057) + \frac{1}{3}\left(-\frac{2}{3}\right)(0.3)^2(-0.1087339) \\
&\quad + \frac{1}{3}\left(-\frac{2}{3}\right)\left(-\frac{5}{3}\right)(0.3)^3(0.0658784) \\
&\quad + \frac{1}{3}\left(-\frac{2}{3}\right)\left(-\frac{5}{3}\right)\left(-\frac{8}{3}\right)(0.3)^4(0.0018251) \\
&= 0.7196460.
\end{aligned}
$$

Predict the value of f(2) using Newton backward divided difference based on data points given.

- To approximate a value when *x is close to the end of the tabulated values, say,* $x = 2.0$*, we* would again like to make the earliest use of the data points closest to *x. This requires using* the Newton backward divided-difference formula with $s = -2/3$ and the divided differences in Table that have a *wavy underline ( ). Notice that the fourth divided difference* is used in both formulas.

$h = x_1 - x_0 = 1.3 - 1 = 0.3$

$S = (x - x_n)/h = (2 - 2.2)/0.3$

$S = -2/3$

$$
\begin{aligned}
P_4(2.0) &= P_4\left(2.2 - \frac{2}{3}(0.3)\right) \\
&= 0.1103623 - \frac{2}{3}(0.3)(-0.5715210) - \frac{2}{3}\left(\frac{1}{3}\right)(0.3)^2(0.0118183) \\
&\quad - \frac{2}{3}\left(\frac{1}{3}\right)\left(\frac{4}{3}\right)(0.3)^3(0.0680685) - \frac{2}{3}\left(\frac{1}{3}\right)\left(\frac{4}{3}\right)\left(\frac{7}{3}\right)(0.3)^4(0.0018251) \\
&= 0.2238754. \qquad \square
\end{aligned}
$$