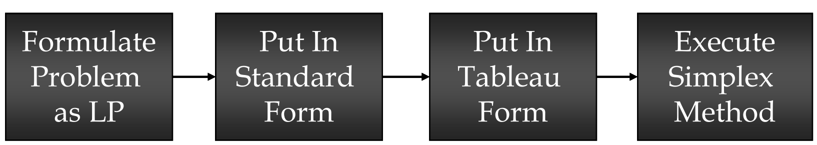
Linear Programming: The Simplex Method

- An Overview of the Simplex Method
- Standard Form
- Tableau Form
- Setting Up the Initial Simplex Tableau
- Improving the Solution
- Calculating the Next Tableau
- Solving a Minimization Problem
- Special Cases

Overview of the Simplex Method

- Steps Leading to the Simplex Method



Example: Initial Formulation

- A Minimization Problem

$$
\begin{aligned}
\text{MIN} \quad & 2x_1 - 3x_2 - 4x_3 \\
\text{s. t.} \quad & x_1 + x_2 + x_3 \leq 30 \\
& 2x_1 + x_2 + 3x_3 \geq 60 \\
& x_1 - x_2 + 2x_3 = 20 \\
& x_1, x_2, x_3 \geq 0
\end{aligned}
$$

Standard Form

- An LP is in standard form when:
  - All variables are non-negative
  - All constraints are equalities
- Putting an LP formulation into standard form involves:
  - Adding slack variables to "$\leq$" constraints
  - Subtracting surplus variables from "$\geq$" constraints.

Example: Standard Form

- Problem in Standard Form

$$
\begin{array}{llr}
\text{MIN} & 2x_1 - 3x_2 - 4x_3 & \\
\text{s. t.} & x_1 + x_2 + x_3 + s_1 & = 30 \\
 & 2x_1 + x_2 + 3x_3 - s_2 & = 60 \\
 & x_1 - x_2 + 2x_3 & = 20 \\
 & x_1, x_2, x_3, s_1, s_2 \geq 0 &
\end{array}
$$

Tableau Form

- A set of equations is in tableau form if for each equation:
  - its right hand side (RHS) is non-negative, and
  - there is a basic variable. (A basic variable for an equation is a variable whose coefficient in the equation is +1 and whose coefficient in all other equations of the problem is 0.)
- To generate an initial tableau form:
  - An artificial variable must be added to each constraint that does not have a basic variable.

Example: Tableau Form

- Problem in Tableau Form

$$
\begin{array}{llr}
\text{MIN} & 2x_1 - 3x_2 - 4x_3 + 0s_1 - 0s_2 + Ma_2 + Ma_3 & \\
\text{s. t.} & x_1 + x_2 + x_3 + s_1 & = 30 \\
 & 2x_1 + x_2 + 3x_3 - s_2 + a_2 & = 60 \\
 & x_1 - x_2 + 2x_3 + a_3 & = 20 \\
 & x_1, x_2, x_3, s_1, s_2, a_2, a_3 \geq 0 &
\end{array}
$$

Simplex Tableau

- The simplex tableau is a convenient means for performing the calculations required by the simplex method.

Setting Up Initial Simplex Tableau

- Step 1: If the problem is a minimization problem, multiply the objective function by -1.
- Step 2: If the problem formulation contains any constraints with negative right-hand sides, multiply each constraint by -1.
- Step 3: Add a slack variable to each $\leq$ constraint.
- Step 4: Subtract a surplus variable and add an artificial variable to each $\geq$ constraint.

Setting Up Initial Simplex Tableau

- Step 5: Add an artificial variable to each = constraint.
- Step 6: Set each slack and surplus variable's coefficient in the objective function equal to zero.
- Step 7: Set each artificial variable's coefficient in the objective function equal to -*M*, where *M* is a very large number.
- Step 8: Each slack and artificial variable becomes one of the basic variables in the initial basic feasible solution.

Simplex Method

- Step 1: Determine Entering Variable
  - Identify the variable with the most positive value in the c_j - z_j row. (The entering column is called the pivot column.)
- Step 2: Determine Leaving Variable
  - For each positive number in the entering column, compute the ratio of the right-hand side values divided by these entering column values.
  - If there are no positive values in the entering column, STOP; the problem is unbounded.
  - Otherwise, select the variable with the minimal ratio. (The leaving row is called the pivot row.)

Simplex Method

- Step 3: Generate Next Tableau
  - Divide the pivot row by the pivot element (the entry at the intersection of the pivot row and pivot column) to get a new row. We denote this new row as (row *).
  - Replace each non-pivot row i with:

    [new row i] = [current row i] - [(a_{ij}) x (row *)],

    where a_{ij} is the value in entering column j of row i

Simplex Method

- Step 4: Calculate z_j Row for New Tableau
  - For each column j, multiply the objective function coefficients of the basic variables by the corresponding numbers in column j and sum them.

Simplex Method

- Step 5: Calculate c_j - z_j Row for New Tableau
  - For each column j, subtract the z_j row from the c_j row.
  - If none of the values in the c_j - z_j row are positive, GO TO STEP 1.
  - If there is an artificial variable in the basis with a positive value, the problem is infeasible. STOP.
  - Otherwise, an optimal solution has been found. The current values of the basic variables are optimal. The optimal values of the non-basic variables are all zero.
  - If any non-basic variable's c_j - z_j value is 0, alternate optimal solutions might exist. STOP.

Example: Simplex Method

- Solve the following problem by the simplex method:

$$\begin{array}{lrl}
\text{Max} & 12x_1 + 18x_2 + 10x_3 & \\
\text{s.t.} & 2x_1 + 3x_2 + 4x_3 & \leq 50 \\
 & x_1 - x_2 - x_3 & \geq 0 \\
 & x_2 - 1.5x_3 & \geq 0 \\
 & x_1, x_2, x_3 & \geq 0
\end{array}$$

Example: Simplex Method

- Writing the Problem in Tableau Form

  We can avoid introducing artificial variables to the second and third constraints by multiplying each by -1 (making them $\leq$ constraints). Thus, slack variables s_1, s_2, and s_3 are added to the three constraints.

$$\begin{array}{ll}
\text{Max} & 12x_1 + 18x_2 + 10x_3 + 0s_1 + 0s_2 + 0s_3 \\
\text{s.t.} & 2x_1 + 3x_2 + 4x_3 + s_1 = 50 \\
 & -x_1 + x_2 + x_3 + s_2 = 0 \\
 & -x_2 + 1.5x_3 + s_3 = 0 \\
 & x_1, x_2, x_3, s_1, s_2, s_3 \geq 0
\end{array}$$

Example: Simplex Method

■ Initial Simplex Tableau

Basis	c_B	x_1	x_2	x_3	s_1	s_2	s_3		
		12	18	10	0	0	0		
s_1	0	2	3	4	1	0	0	50	
s_2	0	-1	1	1	0	1	0	0	(* row)
s_3	0	0	-1	1.5	0	0	1	0	
z_j		0	0	0	0	0	0	0	
$c_j - z_j$		12	18	10	0	0	0		

Example: Simplex Method

■ Iteration 1

- Step 1: Determine the Entering Variable

  The most positive $c_j - z_j = 18$. Thus x_2 is the entering variable.

- Step 2: Determine the Leaving Variable

  Take the ratio between the right hand side and positive numbers in the x_2 column:

  $$50/3 = 16\ 2/3$$

  $$0/1 = 0 \longleftarrow \text{minimum}$$

  s_2 is the leaving variable and the 1 is the pivot element.

Example: Simplex Method

■ Iteration 1 (continued)

- Step 3: Generate New Tableau

  Divide the second row by 1, the pivot element. Call the "new" (in this case, unchanged) row the "* row".

  Subtract 3 x (* row) from row 1.

  Subtract -1 x (* row) from row 3.

  New rows 1, 2, and 3 are shown in the upcoming tableau.

Example: Simplex Method

■ Iteration 1 (continued)

- Step 4: Calculate z_j Row for New Tableau

  The new z_j row values are obtained by multiplying the c_B column by each column, element by element and summing.

  For example, $z_1 = 5(0) + -1(18) + -1(0) = -18$.

Example: Simplex Method

- Iteration 1 (continued)
  - Step 5: Calculate c_j - z_j Row for New Tableau

    The new c_j-z_j row values are obtained by subtracting z_j value in a column from the c_j value in the same column.

    For example, c_1-z_1 = 12 - (-18) = 30.

Example: Simplex Method

- Iteration 1 (continued) - New Tableau

		x_1	x_2	x_3	s_1	s_2	s_3		
Basis	c_B	12	18	10	0	0	0		
s_1	0	5	0	1	1	-3	0	50	(* row)
x_2	18	-1	1	1	0	1	0	0	
s_3	0	-1	0	2.5	0	1	1	0	
z_j		-18	18	18	0	18	0	0	
c_j - z_j		30	0	-8	0	-18	0		

Example: Simplex Method

- Iteration 2
  - Step 1: Determine the Entering Variable

    The most positive $c_j - z_j = 30$. x_1 is the entering variable.
  - Step 2: Determine the Leaving Variable

    Take the ratio between the right hand side and positive numbers in the x_1 column:

    $$10/5 = 2 \longleftarrow \text{minimum}$$

    There are no ratios for the second and third rows because their column elements (-1) are negative.

    Thus, s_1 (corresponding to row 1) is the leaving variable and 5 is the pivot element.

Example: Simplex Method

- Iteration 2 (continued)
  - Step 3: Generate New Tableau

    Divide row 1 by 5, the pivot element. (Call this new row 1 the "* row").

    Subtract (-1) x (* row) from the second row.

    Subtract (-1) x (* row) from the third row.
  - Step 4: Calculate z_j Row for New Tableau

    The new z_j row values are obtained by multiplying the c_B column by each column, element by element and summing.

    For example, $z_3 = .2(12) + 1.2(18) + .2(0) = 24$.

Example: Simplex Method

- Iteration 2 (continued)
  - Step 5: Calculate c_j - z_j Row for New Tableau

    The new c_j-z_j row values are obtained by subtracting z_j value in a column from the c_j value in the same column.

    For example, c_3-z_3 = 10 - (24) = -14.

    Since there are no positive numbers in the c_j - z_j row, this tableau is optimal. The optimal solution is: x_1 = 10; x_2 = 10; x_3 = 0; s_1 = 0; s_2 = 0 s_3 = 10, and the optimal value of the objective function is 300.

Example: Simplex Method

- Iteration 2 (continued) – Final Tableau

		x_1	x_2	x_3	s_1	s_2	s_3		
Basis	c_B	12	18	10	0	0	0		
x_1	12	1	0	.2	.2	-.6	0	10	(* row)
x_2	18	0	1	1.2	.2	.4	0	10	
s_3	0	0	0	2.7	.2	.4	1	10	
z_j		12	18	24	6	0	0	300	
c_j - z_j		0	0	-14	-6	0	0		

Special Cases

- Infeasibility
- Unboundedness
- Alternative Optimal Solution
- Degeneracy

Infeasibility

- <u>Infeasibility</u> is detected in the simplex method when an artificial variable remains positive in the final tableau.

Example: Infeasibility

- LP Formulation

$$\begin{aligned} \text{MAX} \quad & 2x_1 + 6x_2 \\ \text{s. t.} \quad & 4x_1 + 3x_2 \leq 12 \\ & 2x_1 + x_2 \geq 8 \\ & x_1, x_2 \geq 0 \end{aligned}$$

Example: Infeasibility

- Final Tableau

Basis	C_B	x_1	x_2	s_1	s_2	a_2	
		2	6	0	0	$-M$	
x_1	2	1	3/4	1/4	0	0	3
a_2	$-M$	0	-1/2	-1/2	-1	1	2
	z_j	2	$(1/2)M$ +3/2	$(1/2)M$ +1/2	M	$-M$	$-2M$ +6
	$c_j - z_j$	0	$-(1/2)M$ +9/2	$-(1/2)M$ -1/2	$-M$	0	

Example: Infeasibility

In the previous slide we see that the tableau is the final tableau because all c_j - z_j $\leq$ 0. However, an artificial
variable is still positive, so the problem is infeasible.

Unboundedness

- A linear program has an unbounded solution if all entries in an entering column are non-positive.

Example: Unboundedness

- LP Formulation

$$
\begin{array}{lrcl}
\text{MAX} & 2x_1 + 6x_2 & & \\
\text{s. t.} & 4x_1 + 3x_2 & \geq & 12 \\
 & 2x_1 + \quad x_2 & \geq & 8 \\
 & x_1, x_2 & \geq & 0
\end{array}
$$

Example: Unboundedness

- Final Tableau

Basis	c_B	x_1	x_2	s_1	s_2	
		3	4	0	0	
x_2	4	3	1	0	-1	8
s_1	0	2	0	1	-1	3
	z_j	12	4	0	-4	32
	$c_j - z_j$	-9	0	0	4	

Example: Unboundedness

In the previous slide we see that c_4 - z_4 = 4 (is positive), but its column is all non-positive. This indicates that the problem is unbounded.

Alternative Optimal Solution

- A linear program has alternate optimal solutions if the final tableau has a c_j - z_j value equal to 0 for a non-basic variable.

Example: Alternative Optimal Solution

- Final Tableau

Basis	c_B	x_1	x_2	x_3	s_1	s_2	s_3	s_4	
		2	4	6	0	0	0	0	
s_3	0	0	0	2	4	-2	1	0	8
x_2	4	0	1	2	2	-1	0	0	6
x_1	2	1	0	-1	1	2	0	0	4
s_4	0	0	0	1	3	2	0	1	12
	z_j	2	4	6	10	0	0	0	32
	$c_j - z_j$	0	0	0	-10	0	0	0	

Example: Alternative Optimal Solution

In the previous slide we see that the optimal solution is:

$x_1 = 4$, $x_2 = 6$, $x_3 = 0$, and $z = 32$

Note that x_3 is non-basic and its c_3 - z_3 = 0. This 0 indicates that if x_3 were increased, the value of the objective function would not change.

Another optimal solution can be found by choosing x_3 as the entering variable and performing one iteration of the simplex method. The new tableau on the next slide shows an alternative optimal solution is:

$x_1 = 7$, $x_2 = 0$, $x_3 = 3$, and $z = 32$

Example: Alternative Optimal Solution

- New Tableau

Basis	c_B	x_1	x_2	x_3	s_1	s_2	s_3	s_4	
		2	4	6	0	0	0	0	
s_3	0	0	-1	0	2	-1	1	0	2
x_3	6	0	.5	1	1	- .5	0	0	3
x_1	2	1	.5	0	2	1.5	0	0	7
s_4	0	0	- .5	0	2	2.5	0	1	9
	z_j	2	4	6	10	0	0	0	32
	c_j - z_j	0	0	0	-10	0	0	0	

Degeneracy

- A degenerate solution to a linear program is one in which at least one of the basic variables equals 0.
- This can occur at formulation or if there is a tie for the minimizing value in the ratio test to determine the leaving variable.
- When degeneracy occurs, an optimal solution may have been attained even though some $c_j – z_j > 0$.
- Thus, the condition that $c_j – z_j \leq 0$ is sufficient for optimality, but not necessary.

Simplex-Based Sensitivity Analysis and Duality

- Sensitivity Analysis with the Simplex Tableau
- Duality

Objective Function Coefficients and Range of Optimality

- The range of optimality for an objective function coefficient is the range of that coefficient for which the current optimal solution will remain optimal (keeping all other coefficients constant).
- The objective function value might change is this range.

Objective Function Coefficients and Range of Optimality

- Given an optimal tableau, the range of optimality for c_k can be calculated as follows:
  - Change the objective function coefficient to c_k in the c_j row.
  - If x_k is basic, then also change the objective function coefficient to c_k in the c_B column and recalculate the z_j row in terms of c_k.
  - Recalculate the c_j - z_j row in terms of c_k. Determine the range of values for c_k that keep all entries in the c_j - z_j row less than or equal to 0.

Objective Function Coefficients and Range of Optimality

- If c_k changes to values outside the range of optimality, a new c_j - z_j row may be generated. The simplex method may then be continued to determine a new optimal solution.

Shadow Price

- A shadow price for a constraint is the increase in the objective function value resulting from a one unit increase in its right-hand side value.
- Shadow prices and dual prices on *The Management Scientist* output are the same thing for maximization problems and negative of each other for minimization problems.

Shadow Price

- Shadow prices are found in the optimal tableau as follows:
  - "less than or equal to" constraint -- z_j value of the corresponding slack variable for the constraint
  - "greater than or equal to" constraint -- negative of the z_j value of the corresponding surplus variable for the constraint
  - "equal to" constraint -- z_j value of the corresponding artificial variable for the constraint.

Right-Hand Side Values and Range of Feasibility

- The <u>range of feasibility</u> for a right hand side coefficient is the range of that coefficient for which the shadow price remains unchanged.
- The <u>range of feasibility</u> is also the range for which the current set of basic variables remains the optimal set of basic variables (although their values change.)

Right-Hand Side Values and Range of Feasibility

- The range of feasibility for a right-hand side coefficient of a "less than or equal to" constraint, b_k, is calculated as follows:
  - Express the right-hand side in terms of Δb_k by adding Δb_k times the column of the k-th slack variable to the current optimal right hand side.
  - Determine the range of Δb_k that keeps the right-hand side greater than or equal to 0.
  - Add the original right-hand side value b_k (from the original tableau) to these limits for Δb_k to determine the range of feasibility for b_k.

Right-Hand Side Values and Range of Feasibility

- The range of feasibility for "greater than or equal to" constraints is similarly found except one subtracts Δb_k times the current column of the k-th surplus variable from the current right hand side.
- For equality constraints this range is similarly found by adding Δb_k times the current column of the k-th artificial variable to the current right hand side. Otherwise the procedure is the same.

Simultaneous Changes

- For simultaneous changes of two or more objective function coefficients the 100% rule provides a guide to whether the optimal solution changes.
- It states that as long as the sum of the percent changes in the coefficients from their current value to their maximum allowable increase or decrease does not exceed 100%, the solution will not change.
- Similarly, for shadow prices, the 100% rule can be applied to changes in the the right hand side coefficients.

Canonical Form

- A maximization linear program is said to be in canonical form if all constraints are "less than or equal to" constraints and the variables are non-negative.
- A minimization linear program is said to be in canonical form if all constraints are "greater than or equal to" constraints and the variables are non-negative.

Canonical Form

- Convert any linear program to a maximization problem in canonical form as follows:
  - minimization objective function:
    multiply it by -1
  - "less than or equal to" constraint:
    leave it alone
  - "greater than or equal to" constraint:
    multiply it by -1
  - "equal to" constraint:
    form two constraints, one "less than or equal to", the other "greater or equal to"; then multiply this "greater than or equal to" constraint by -1.

Primal and Dual Problems

- Every linear program (called the <u>primal</u>) has associated with it another linear program called the <u>dual</u>.
- The dual of a maximization problem in canonical form is a minimization problem in canonical form.
- The rows and columns of the two programs are interchanged and hence the objective function coefficients of one are the right hand side values of the other and vice versa.

Primal and Dual Problems

- The optimal value of the objective function of the primal problem equals the optimal value of the objective function of the dual problem.
- Solving the dual might be computationally more efficient when the primal has numerous constraints and few variables.

Primal and Dual Variables

- The dual variables are the "value per unit" of the corresponding primal resource, i.e. the shadow prices. Thus, they are found in the z_j row of the optimal simplex tableau.
- If the dual is solved, the optimal primal solution is found in z_j row of the corresponding surplus variable in the optimal dual tableau.
- The optimal value of the primal's slack variables are the negative of the c_j - z_j entries in the optimal dual tableau for the dual variables.

Example: Jonni’s Toy Co.

Jonni's Toy Co. produces stuffed toy animals and is gearing up for the Christmas rush by hiring temporary workers giving it a total production crew of 30 workers. Jonni's makes two sizes of stuffed animals. The profit, the production time and the material used per toy animal is summarized on the next slide. Workers work 8 hours per day and there are up to 2000 pounds of material available daily.

What is the optimal daily production mix?

Example: Jonni's Toy Co.

Toy Size	Unit Profit	Production Time (hrs.)	Material Used Per Unit (lbs.)
Small	$3	.10	1
Large	$8	.30	2

Example: Jonni's Toy Co.

- LP Formulation

  x_1 = number of small stuffed animals produced daily

  x_2 = number of large stuffed animals produced daily

$$
\begin{array}{llrl}
\text{Max} & 3x_1 + 8x_2 & & \\
\text{s.t.} & .1x_1 + .3x_2 & \le & 240 \\
 & x_1 + 2x_2 & \le & 2000 \\
 & x_1, x_2 & \ge & 0
\end{array}
$$

Example: Jonni's Toy Co.

- Simplex Method: First Tableau

		x_1	x_2	s_1	s_2	
Basis	c_B	3	8	0	0	
s_1	0	.1	.3	1	0	240
s_2	0	1	2	0	1	2000
z_j		0	0	0	0	0
$c_j - z_j$		3	8	0	0	

Example: Jonni's Toy Co.

- Simplex Method: Second Tableau

		x_1	x_2	s_1	s_2	
Basis	c_B	3	8	0	0	
x_2	8	1/3	1	10/3	0	800
s_2	0	1/3	0	-20/3	1	400
z_j		8/3	8	80/3	0	6400
$c_j - z_j$		1/3	0	-80/3	0	

Example: Jonni's Toy Co.

- Simplex Method: Third Tableau

		x_1	x_2	s_1	s_2	
Basis	c_B	3	8	0	0	
x_2	8	0	1	10	-1	400
x_1	3	1	0	-20	3	1200
	z_j	3	8	20	1	6800
	$c_j - z_j$	0	0	-20	-1	

Example: Jonni's Toy Co.

- Optimal Solution
  - Question:

    How many animals of each size should be produced daily and what is the resulting daily profit?
  - Answer:

    Produce 1200 small animals and 400 large animals daily for a total profit of $6,800.

Example: Jonni's Toy Co.

- Range of Optimality for c_1 (small animals)
  Replace 3 by c_1 in the objective function row and c_B column. Then recalculate z_j and c_j - z_j rows.

z_j	c_1	8	80 -20c_1		-8 +3c_1		3200 + 1200c_1
c_j - z_j	0	0	-80 +20c_1		8 -3c_1		

For the c_j - z_j row to remain non-positive, $8/3 \leq c_1 \leq 4$

Example: Jonni's Toy Co.

- Range of Optimality for c_2 (large animals)
  Replace 8 by c_2 in the objective function row and c_B column. Then recalculate z_j and c_j - z_j rows.

z_j	3	c_2	-60 +10c_2	9 -c_2	3600 + 400c_2
c_j - z_j	0	0	60 -10c_2	-9 +c_2	

For the c_j - z_j row to remain non-positive, $6 \leq c_2 \leq 9$

Example: Jonni's Toy Co.

- Range of Optimality
  - Question: Will the solution change if the profit on small animals is increased by \$.75? Will the objective function value change?
  - Answer: If the profit on small stuffed animals is changed to \$3.75, this is within the range of optimality and the optimal solution will not change. However, since x_1 is a basic variable at positive value, changing its objective function coefficient will change the value of the objective function to 3200 + 1200(3.75) = 7700.

Example: Jonni's Toy Co.

- Range of Optimality
  - Question: Will the solution change if the profit on large animals is increased by \$.75? Will the objective function value change?
  - Answer: If the profit on large stuffed animals is changed to \$8.75, this is within the range of optimality and the optimal solution will not change. However, since x_2 is a basic variable at positive value, changing its objective function coefficient will change the value of the objective function to 3600 + 400(8.75) = 7100.

Example: Jonni's Toy Co.

- Range of Optimality and 100% Rule
  - Question: Will the solution change if the profits on both large and small animals are increased by $.75? Will the value of the objective function change?
  - Answer: If both the profits change by $.75, since the maximum increase for c_1 is $1 (from $3 to $4) and the maximum increase in c_2 is $1 (from $8 to $9), the overall sum of the percent changes is (.75/1) + (.75/1) = 75% + 75% = 150%. This total is greater than 100%; both the optimal solution and the value of the objective function change.

Example: Jonni's Toy Co.

- Shadow Price
  - Question: The unit profits do not include a per unit labor cost. Given this, what is the maximum wage Jonni should pay for overtime?
  - Answer: Since the unit profits do not include a per unit labor cost, man-hours is a sunk cost. Thus the shadow price for man-hours gives the maximum worth of man-hours (overtime). This is found in the z_j row in the s_1 column (since s_1 is the slack for man-hours) and is $20.

Example: Prime the Cannons!

■ LP Formulation

$$
\begin{aligned}
\text{Max} \quad & 2x_1 + x_2 + 3x_3 \\
\text{s.t.} \quad & x_1 + 2x_2 + 3x_3 \le 15 \\
& 3x_1 + 4x_2 + 6x_3 \ge 24 \\
& x_1 + x_2 + x_3 = 10 \\
& x_1, x_2, x_3 \ge 0
\end{aligned}
$$

Example: Prime the Cannons!

■ Primal in Canonical Form

- Constraint (1) is a "$\le$" constraint. Leave it alone.
- Constraint (2) is a "$\ge$" constraint. Multiply it by -1.
- Constraint (3) is an "=" constraint. Rewrite this as two constraints, one a "$\le$", the other a "$\ge$" constraint. Then multiply the "$\ge$" constraint by -1.

(result on next slide)

Example: Prime the Cannons!

■ Primal in Canonical Form (continued)

$$
\begin{array}{llcl}
\text{Max} & 2x_1 + x_2 + 3x_3 & & \\
\text{s.t.} & x_1 + 2x_2 + 3x_3 & \leq & 15 \\
 & -3x_1 - 4x_2 - 6x_3 & \leq & -24 \\
 & x_1 + x_2 + x_3 & \leq & 10 \\
 & -x_1 - x_2 - x_3 & \leq & -10 \\
 & x_1, x_2, x_3 \geq 0 & &
\end{array}
$$

Example: Prime the Cannons!

■ Dual of the Canonical Primal

- There are four dual variables, U_1, U_2, U_3', U_3''.
- The objective function coefficients of the dual are the RHS of the primal.
- The RHS of the dual is the objective function coefficients of the primal.
- The rows of the dual are the columns of the primal.

(result on next slide)

Example: Prime the Cannons!

- Dual of the Canonical Primal (continued)

$$\text{Min} \quad 15U_1 - 24U_2 + 10U_3' - 10U_3''$$

$$\begin{aligned} \text{s.t.} \quad U_1 - 3U_2 + U_3' - U_3'' &\geq 2 \\ 2U_1 - 4U_2 + U_3' - U_3'' &\geq 1 \\ 3U_1 - 6U_2 + U_3' - U_3'' &\geq 3 \end{aligned}$$

$$U_1, U_2, U_3', U_3'' \geq 0$$