# *Chapter 2*

1) Suppose a population has N=4 elements.
   a) List all possible samples of size 2 if sampling with replacement.
   b) List all possible samples of size 3 if sampling without replacement. You only need to list the basic samples.

Repeat A) and B) if N=6.

2) For each of the following values of N and n, give the number of possible samples if sampling is done

   i) with replacement and ii) without replacement

a) N=6, n=2 b) N=5, n=3 c) N=10, n=3 d) N=8, n=2 e) N=100, n=10 f) N=100, n=50

3) Suppose in a population of 4 brothers and sisters, we determine the number of children that each one has obtaining

$X_1=5,\ X_2=3,\ X_3=6,\ X_4=1$

   a) Find the population mean and variance for the variable
   b) Find all possible with replacement samples of size 2. For each sample, find the sample mean
   c) Find the mean and variance of the distribution of the sample mean
   d) Verify the values in c) by appropriate formulas

4) Suppose we have the height (in cm) for a population of 5 plants of a certain type

$X_1=30,\ X_2=27,\ X_3=31,\ X_4=33,\ X_5=29$

   a) Find the population mean and variance for the variable
   b) Find the without replacement ( n=3 ) distribution of the sample mean
   c) Find the mean, variance and standard deviation of the sampling distribution of the sample mean
   d) Verify the formulas relating the variable's population mean and variance of the distribution of the sample mean

5) Suppose we have recorded whether or not an animal has a certain disease for a population of 6 animals:

$X_1$= yes, $X_2$= yes, $X_3$= no, X4= yes, $X_5$= no, $X_6$= yes.

   a) Find $\pi$, the population proportion with the disease.
   b) Find the without replacement ( n=4 ) distribution of p, the sample proportion with the disease.
   c) Find the with replacement ( n=4 ) distribution of p.
   d) For each b) and c), find the mean and variance of the sample distribution and verify the formulas relating these values to the population proportion.

## استخدام الآلة الحاسبة

| x | Freq |
|---|---|
| a | d |
| b | e |
| c | f |

لدينا جدول تكراري ونريد حساب المتوسط والانحراف المعياري والتباين ....

① تنظيف الشاشة والذاكرة :

Shift + CLR (9) ⟼ يظهر لنا Clear? 3:All نضغط على 3 ⟼ ثم نضغط على = مرتان

② إدراج عمود التكرار :

Shift + Mood ⟼ يظهر لنا 4: Stat نضغط على 4 ⟼ يظهر لنا Frequency? 1:ON نضغط على 1

③ حساب المتوسط والانحراف المعياري والتباين :

ⓘ Mood ⟼ يظهر لنا 3: Stat نضغط على 3 ⟼ يظهر لنا 1: 1-VaR نضغط على 1 ⟼ يظهر لنا

| X | FREQ |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| ⋮ | |

a=b=c ① ⟼ نضغط على AC

نحرك المؤشر الى اول قيمة ادخلناها وهي a ثم نحرك المؤشر الى خانة Freq ثم ندخل d=e=f

ⓒ Shift + stat (1) ⟼ يظهر لنا 5: Var نضغط على 5 ⟼ يظهر لنا

1:n 2:$\bar{x}$

3:$x\sigma n$ 4:$x\sigma n-1$

نضغط على 2 ثم مساواة تكون حسبنا متوسط

نضغط على 3 ثم مساواة تكون حسبنا الانحراف المعياري

ولحصول على التباين نضغط على $x^2$ ثم مساواة .

# Chapter 2

3) 5, 3, 6, 1 , $N=4$ , population is not normal

a) $\mu = 3.75$, $\sigma = 1.42024$, $\sigma^2 = 3.6875$ ( من الآلة الحاسبة )

b) with rep. $n=2 \Rightarrow N^n = 4^2 = 16$

| Samples | $\bar{X}$ |
|---|---|
| (5,5) | 10/2 = 5 |
| (5,3) | 8/2 = 4 |
| (5,6) | 11/2 |
| (5,1) | 6/2 = 3 |
| (3,3) | 9/2 |
| (3,5) | 8/2 = 4 |
| (3,6) | 9/2 |
| (3,1) | 4/2 = 2 |
| (6,6) | 12/2 = 6 |
| (6,5) | 11/2 |
| (6,3) | 9/2 |
| (6,1) | 7/2 |
| (1,1) | 2/2 = 1 |
| (1,5) | 6/2 = 3 |
| (1,3) | 4/2 = 2 |
| (1,6) | 7/2 |

c)

| $\bar{X}$ (كم مرة ليس عينة) | Freq. |
|---|---|
| 12/2 = 6 | 1 |
| 11/2 | 2 |
| 10/2 = 5 | 1 |
| 9/2 | 2 |
| 8/2 = 4 | 2 |
| 7/2 | 2 |
| 6/2 = 3 | 3 |
| 4/2 = 2 | 2 |
| 2/2 = 1 | 1 |
| المجموع | 16 |

باستخدام الآلة الحاسبة

$\mu_{\bar{X}} = 3.75$

$\sigma_{\bar{X}} = 1.35785 \Rightarrow \sigma_{\bar{X}}^2 = 1.84375$

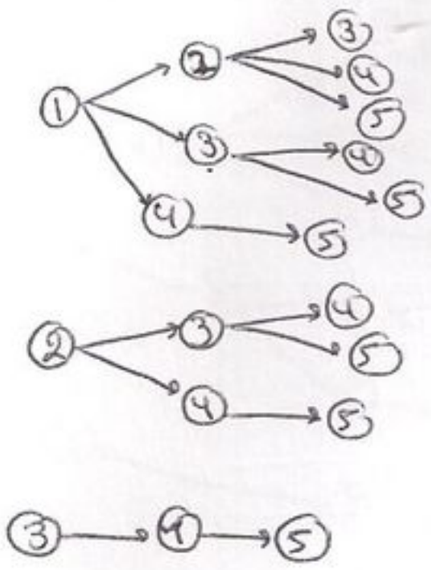
d) $\mu_{\bar{X}} = \mu = 3.75$

$$\sigma_{\bar{X}}^2 = \frac{\sigma^2}{n} = \frac{3.6875}{2} = 1.84375$$

4) ① ② ③ ④ ⑤
30, 27, 31, 33, 24 , $N=5$ , the Population is not normal

a) $\mu = 30$ , $\sigma = 2 \Rightarrow \sigma^2 = 4$ ( مقدار الحل )

b) without rep. $n=3$ $\binom{N}{n} = \binom{5}{3} = 10$



⇒

| samples | $\bar{X}$ |
|---|---|
| (30, 27, 31) | 88/3 |
| (30, 27, 33) | 90/3 = 30 |
| (30, 27, 29) | 86/3 |
| (30, 31, 33) | 94/3 |
| (30, 31, 29) | 90/3 = 30 |
| (30, 33, 29) | 92/3 |
| (27, 31, 33) | 91/3 |
| (27, 31, 29) | 87/3 = 29 |
| (27, 33, 29) | 89/3 |
| (31, 33, 29) | 93/3 = 31 |

c)

| $\bar{X}$ | Freq. |
|---|---|
| 94/3 | 1 |
| 93/3 = 31 | 1 |
| 92/3 | 1 |
| 91/3 | 1 |
| 90/3 = 30 | 2 |
| 89/3 | 1 |
| 88/3 | 1 |
| 87/3 = 29 | 1 |
| 86/3 | 1 |
| المجموع | 10 |

باستخدام الآلة الحاسبة

$\mu_{\bar{X}} = 30$

$\sigma_{\bar{X}} = .816497$

$\Rightarrow \sigma_{\bar{X}}^2 = .66667$

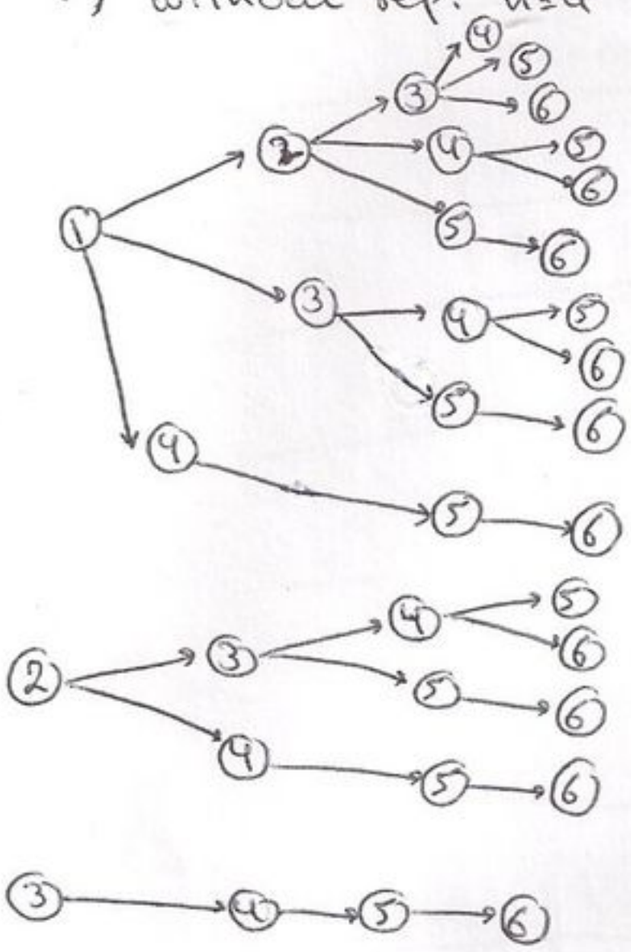
d) $\mu_{\bar{X}} = \mu = 30$

$$\sigma_{\bar{X}}^2 = \frac{\sigma^2}{n}\left(\frac{N-n}{N-1}\right) = \frac{4}{3}\left(\frac{5-3}{5-1}\right) = .66667$$

✓ 5) ① ② ③ ④ ⑤ ⑥
yes, yes, No, yes, No, yes , N=6 , the population is not normal

a) A = # of animal has disease in Population = 4 $\Rightarrow P = \frac{A}{N} = \frac{4}{6} \simeq .66667$

b) without rep. $n=4$ , $\binom{N}{n} = \binom{6}{4} = 15$



| Samples | $r = \frac{a}{n} = \frac{a}{4}$ |
|---|---|
| yyny | 3/4 |
| yynn | 2/4 |
| yyny | 3/4 |
| yyyn | 3/4 |
| yyyy | 4/4=1 |
| yyny | 3/4 |
| ynyn | 2/4 |
| ynyy | 3/4 |
| ynny | 2/4 |
| yyny | 3/4 |
| ynyn | 2/4 |
| ynyy | 3/4 |
| ynny | 2/4 |
| yyny | 3/4 |
| nyny | 2/4 |

a = # of animal has disease in sample

d)

| r | Freq. |
|---|---|
| 3/4 | 8 |
| 2/4 | 6 |
| 1 | 1 |
| المجموع | 15 |

باستخدام الآلة الحاسبة $\rightarrow$ $\mu_r = .66667$

$\sigma_r = .14907 \Rightarrow \sigma_r^2 = .02222$

وايضاً من الاجابة

$\mu_r = P$

$\sigma_r^2 = \frac{P(1-P)}{n}\left(\frac{N-n}{N-1}\right) = \frac{\left(\frac{4}{6}\right)\left(\frac{2}{6}\right)}{4}\left(\frac{6-4}{6-1}\right) = .02222$

c) with rep. $n=4 \Rightarrow N^n = 6^4 = 1296$

| a | $r = \frac{a}{n} = \frac{a}{4}$ | $F = C_a^n (A)^a (N-A)^{n-a} = C_a^4 (4)^a (2)^{4-a}$ |
|---|---|---|
| 0 | 0/4 = 0 | $C_0^4 (4)^0 (2)^4 = 16$ |
| 1 | 1/4 | $C_1^4 (4)^1 (2)^3 = 128$ |
| 2 | 2/4 | $C_2^4 (4)^2 (2)^2 = 384$ |
| 3 | 3/4 | $C_3^4 (4)^3 (2)^1 = 512$ |
| 4 | 4/4 = 1 | $C_4^4 (4)^4 (2)^0 = 256$ |

d)

| r | Freq. |
|---|---|
| 0 | 16 |
| 1/4 | 128 |
| 2/4 | 384 |
| 3/4 | 512 |
| 1 | 256 |
| المجموع | 1296 |

بالاستخدام الآلة الحاسبة

$M_r = .66667$

$\sigma_r = .23570$

$\Rightarrow \sigma_r^2 = .05556$

والتأكد من الإجابة

$M_r = P = .66667$

$\sigma_r^2 = \frac{P(1-P)}{n} = \frac{(\frac{4}{6})(\frac{2}{6})}{4} = .05556$