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" chapter # 4 "
" part 2 "

$0 \le p(X=x_i) \le 1$

discrete random variable

as a Counter

Probability distribution which is $\sum_i p(X=x_i)=1$

$p(X=x_i) = p(X \le x_i) - p(X \le x_{i-1})$

i	x_i	$P(X=x_i)$	$P(X \le x_i)$	$x_i\,P(X=x_i)$
1	x_1	$P(X=x_1)$	$P(X \le x_1)$	$x_1\,P(X=x_1)$
2	x_2	$P(X=x_2)$	$P(X \le x_2)$	$x_2\,P(X=x_2)$
3	x_3	$P(X=x_3)$	$P(X \le x_3)$	$x_3\,P(X=x_3)$
⋮	⋮	⋮	⋮	⋮
		$\sum_i P(X=x_i)=1$		$\mu = \sum_i x_i P(X=x_i)$

① the value of c.d.f. for the last value of a random variable X always equals one

which is expected value of the random variable X

the cumulative distribution probability (c.d.f.)

$p(X \le x_i) = \sum_{j=1}^{i} p(X=x_j)$

and

$p(x_k \le X \le x_\ell) = p(X \le x_\ell) - p(X \le x_k)$

and

$p(X \le x_1) = p(X=x_1)$ (this is only true when $X=x_1$)

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Notes :

1) sometime we need for equality in marks ($<$, $>$) or may not need, so the following rulings illustate :

i) $p(a < X < b) = p(a+1 \leq X \leq b-1)$

ii) $p(a \leq X \leq b) = p(a-1 < X < b+1)$

2) cases use the complementary and c.d.f. :

i) $p(a < X) = 1 - p(X \leq a)$

ii) $p(a \leq X) = 1 - p(X < a) = 1 - p(X \leq a-1)$

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$X : x_1=0,\ x_2=1,\ x_3=2,\ x_4=3,\ x_5=4,\ x_6=5$

i	x_i	$P(X=x_i)$	$P(X\le x_i)$
1	$x_1=0$	$P(X=x_1)=.05$	$P(X\le x_1)=P(X=x_1)=.05$
2	$x_2=1$	$P(X=x_2)=.15$	$P(X\le x_2)=P(X=x_2)+P(X=x_1)=.15+.05=.2$
3	$x_3=2$	$P(X=x_3)=.15$	$P(X\le x_3)=P(X=x_3)+P(X=x_2)+P(X=x_1)=.15+.15+.05=.35$
4	$x_4=3$	$P(X=x_4)=.25$	$P(X\le x_4)=\sum_{j=1}^{4}P(X=x_j)=.25+.15+.15+.05=.6$
5	$x_5=4$	$P(X=x_5)=.3$	$P(X\le x_5)=\sum_{j=1}^{5}P(X=x_j)=.3+.25+.15+.15+.05=.9$
6	$x_6=5$	$P(X=x_6)=.1$	$P(X\le x_6)=\sum_{j=1}^{6}P(X=x_j)=1$ (always)
		$\sum_{i=1}^{6}P(X=x_i)=1$	

i	$x_i\,P(X=x_i)$
1	$(0)(.05)=0$
2	$(1)(.15)=.15$
3	$(2)(.15)=.3$
4	$(3)(.25)=.75$
5	$(4)(.3)=1.2$
6	$(5)(.1)=.5$
	$\sum_{i=1}^{6}x_i\,P(X=x_i)=2.9=\mu$

$$P(X\le x_i)=\begin{cases} 0, & x_i<0 \\ .05, & 0\le x_i<1 \\ .2, & 1\le x_i<2 \\ .35, & 2\le x_i<3 \\ .6, & 3\le x_i<4 \\ .9, & 4\le x_i<5 \\ 1, & 5\le x_i \end{cases}$$

1) $P(X=4)=.3$ where 4 has the highest probability which equals $.3$

$\therefore$ d

2) to find $P(X<3)$ we have two ways:

i) using probability distribution:

$P(X<3)=P(X=2)+P(X=1)+P(X=0)=.15+.15+.05=.35$

ii) using the c.d.f:

$P(X<3)=P(X\le 3-1)=P(X\le 2)=.35$

$\therefore$ b

3) to find $P(1\le X<4)$ we have two ways:

i) using probability distribution:

$P(1\le X<4)=P(X=1)+P(X=2)+P(X=3)=.15+.15+.25=.55$

ii) using the c.d.f:

$P(1\le X<4)=P(1-1<X\le 4-1)=P(0<X\le 3)=P(X\le 3)-P(X\le 0)$

$=.6-.05=.55$

$\therefore$ d

3 4) $\mu=2.9$ $\therefore$ b

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« Q # 29 »

$X: x_1=0,\ x_2=1,\ x_3=2,\ x_4=3,\ x_5=5$

i	x_i	$P(X=x_i)$	$P(X\le x_i)$
1	$0=x_1$	$P(X=x_1)=P(X\le x_1)=.12$	$.12$
2	$1=x_2$	$P(X=x_2)=P(X\le x_2)-P(X\le x_1)=.36-.12=.24$	$.36$
3	$2=x_3$	$P(X=x_3)=P(X\le x_3)-P(X\le x_2)=.72-.36=.36$	$.72$
4	$3=x_4$	$P(X=x_4)=P(X\le x_4)-P(X\le x_3)=.95-.72=.23$	$.95$
5	$5=x_5$	$P(X=x_5)=P(X\le x_5)-P(X\le x_4)=1-.95=.05$	1
		$\sum_{i=1}^{5} P(X=x_i)=1$	

$\mapsto$

i	$x_i\,P(X=x_i)$
1	$(0)(.12)=0$
2	$(1)(.24)=.24$
3	$(2)(.36)=.72$
4	$(3)(.23)=.69$
5	$(5)(.05)=.25$
	$\sum_{i=1}^{5} x_i\,P(X=x_i)=1.9=\mu$

$$,\ P(X\le x_i)=\begin{cases} 0, & x_i<0 \\ .12, & 0\le x_i<1 \\ .36, & 1\le x_i<2 \\ .72, & 2\le x_i<3 \\ .95, & 3\le x_i<5 \\ 1, & 5\le x_i \end{cases}$$

1) $P(X=2)=P(X=x_3)=.36$ $\therefore b$

2) $P(X=3)=.23$ $\therefore a$

3) to find $P(1<X\le 4)$ we have two ways:

i) using probability distribution:

$P(1<X\le 4)=P(X=2)+P(X=3)=.36+.23=.59$

ii) using c.d.f.:

$P(1<X\le 4)=P(X\le 4)-P(X\le 1)=.95-.36=.59$

$\therefore d$

4) $\mu=1.9$ $\therefore c$

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"Q#30"

$X: x_1=0, x_2=1, x_3=2, x_4=3, x_5=4$

i	x_i	$P(X=x_i)$	$P(X\le x_i)$	$x_i\,P(X=x_i)$
1	0	*.08	.08 ①	0
2	1	.19 ⑤	.27 ④	.19
3	2	*.31	.58 ②	.62
4	3	.32 ⑦	.9 ⑥	.96
5	4	.1 ⑧	1 ③	.4
		Sum = 1		Sum = 2.17

the given that we have : $P(X=0)=.08$, $P(X=2)=.31$, $P(X>2)=.42$, $P(1\le X\le 3)=.63$

* : is the given

①, ②, ③, ④, ⑤, ⑥, ⑦ and ⑧ : steps to fill in the table

for ① : we know that $P(X=x_1)=P(X\le x_1)$, so , $P(X\le x_1)=.08$

for ② : we have from the given $P(X>2)=.42$

$\Rightarrow 1-P(X\le 2)=.42$

$\Rightarrow P(X\le 2)=1-.42=.58$

for ③ : we know that the value of cdf for the last value of random variable X always equals one

for ④ : we have from the given $P(X=2)=.31$

$\Rightarrow P(X\le 2)-P(X\le 1)=.31$

$\Rightarrow .58-P(X\le 1)=.31$

$\Rightarrow P(X\le 1)=.27$

for ⑤ : we know that $P(X=1)=P(X\le 1)-P(X\le 0)$

$\Rightarrow P(X=1)=.27-.08=.31=.19$

for ⑥ : we have from the given $P(1\le X\le 3)=.63$

$\Rightarrow P(X\le 3)-P(X\le 1)=.63$

$\Rightarrow P(X\le 3)-.27=.63$

$\Rightarrow P(X\le 3)=.9$

for ⑦ : we know that $P(X=3)=P(X\le 3)-P(X\le 2)$

$\Rightarrow P(X=3)=.9-.58$

$\Rightarrow P(X=3)=.32$

for ⑧ : we have two ways to find $P(X=4)$ as following :

1, $P(X=4)=1-(.32+.31+.19+.8)=1-.9=.1$

2, $P(X=4)=P(X\le 4)-P(X\le 3)=1-.9=.1$

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1) $p(X=1) = .19 \quad \therefore d$

2) $p(X=3) = .32 \quad \therefore c$

3) $p(X \le 2) = .58 \quad \therefore b$

4) $\mu = \sum_{i=1}^{5} x_i \, p(X = x_i) = 2.17 \quad \therefore e$

" Part 2 "

types of distributions

discrete distribution

Binomial distribution

(i) we have n trials which they are independent
(ii) each single trial has only two things :
  * success and its probability π
  ** failure and its probability $1-\pi$
(iii) the discrete random variable
X = # of successes in n trials

$\therefore$ from (i), (ii) and (iii) we have

$X \sim Bin(n, \pi)$

where the probability distribution function is

$$f_X(x) = P(X=x) = \begin{cases} \binom{n}{x} \pi^x (1-\pi)^{n-x} & , x = 0,1,\ldots,n \\ 0 & , \text{o.w.} \end{cases}$$

and the mean is $\mu = n\pi$
and the variance is $\sigma^2 = n\pi(1-\pi)$

Poisson distribution

(i) X = # of occurrences in time or space
(ii) λ : is the average of occurrences of X in time or space

$\therefore$ from (i) and (ii) we have
$X \sim pois(\lambda)$
where the probability distribution function is

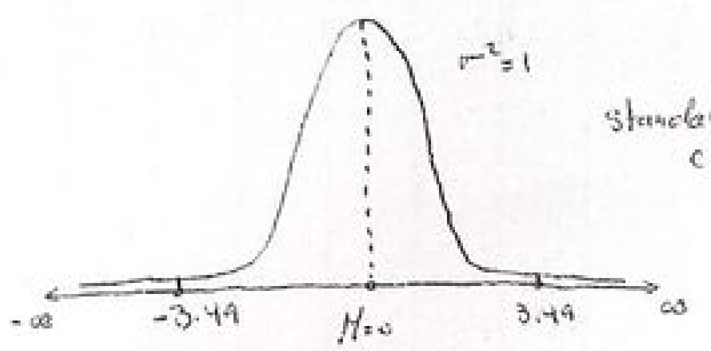
$$f_X(x) = P(X=x) = \begin{cases} \dfrac{e^{-\lambda}\lambda^x}{x!} & , x = 0,1,2,\ldots \\ 0 & , \text{o.w.} \end{cases}$$

and the mean is $\mu = \lambda$
and the variance is $\sigma^2 = \lambda$

continuous distribution

Standard normal distribution

$Z \sim N(0,1)$ — the mean μ ; the variance σ^2



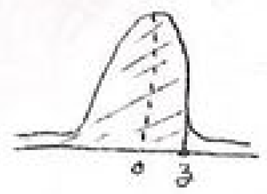
standard normal curve

which the Z has a table.

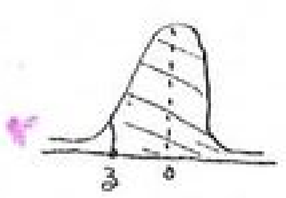
some rules for using a table of Z :

① $P(Z = z) = 0$, $\forall z$

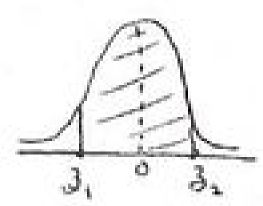
② $P(Z \le z) = P(Z < z)$
is the area to the left of z under Z



③ $P(Z \ge z) = 1 - P(Z \le z)$
is the area to the right of z under Z



④ $P(z_1 \le Z \le z_2) = P(Z \le z_2) - P(Z \le z_1)$
is the area between z_1 and z_2 under Z



⑤ $P(Z \ge 0) = P(Z \le 0) = .5$

⑥ * $P(Z \le z) = 0 \quad \forall z \le -3.49$
** $P(Z \le z) = 1 \quad \forall z \ge 3.49$

⑦ if X has a normal distribution $X \sim N(\mu, \sigma^2)$ — the mean ; the variance

to convert X from normal to standard normal Z, we use $Z = \dfrac{X-\mu}{\sigma} \sim N(0,1)$

and so we can use the table of Z

Binomial distribution :

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"Q # 31"

X = the number of people who use dental floss regularly of these 15 people.

So, the success is the # of people who use dental floss regularly

$\Rightarrow$ probability of success $= \pi = \frac{25}{100} = .25$

and $n = 15$ $\therefore X \sim Bin(n = 15, \pi = .25)$

$$\therefore f_X(x) = P(X = x) = \begin{cases} \binom{n}{x} \pi^x (1-\pi)^{n-x} & , x = 0, 1, 2, \ldots, n \\ 0 & , o.w \end{cases}$$

$$= \begin{cases} \binom{15}{x} (.25)^x (1 - .25)^{15-x} & , x = 0, 1, 2, 3, \ldots, 15 \\ 0 & , o.w \end{cases}$$

$$= \begin{cases} \binom{15}{x} (.25)^x (.75)^{15-x} & , x = 0, 1, 2, \ldots, 15 \\ 0 & , o.w \end{cases}$$

Note: $\binom{n}{x} = C^n_x$

1) b

2) c

3) $P(X = 4) = \binom{15}{4} (.25)^4 (.75)^{15-4} = .2252$ $\therefore b$

4) $P(X \leq 1) = P(X = 1) + P(X = 0) = \binom{15}{1} (.25)^1 (.75)^{15-1} + \binom{15}{0} (.25)^0 (.75)^{15-0} = .0802$
$\therefore d$

5) $\mu = n\pi = (15)(.25) = 3.75$ $\therefore a$

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"Q # 32"

X = the number of children who have trouble saying some letters of these 8 children

So, the success is the # of children who have trouble saying some letters

$\Rightarrow$ the probability of success $= \pi = \frac{11}{100} = .11$

and $n = 8$ $\therefore X \sim Bin(n=8, \pi=.11)$

$$\therefore f_X(x) = P(X=x) = \begin{cases} \binom{n}{x} \pi^x (1-\pi)^{n-x}, & x = 0,1,\dots,n \\ 0, & o.w. \end{cases}$$

$$= \begin{cases} \binom{8}{x} .11^x (1-.11)^{8-x}, & x=0,1,2,\dots,8 \\ 0, & o.w. \end{cases} = \begin{cases} \binom{8}{x} (.11)^x (.89)^{8-x}, & x=0,1,\dots,8 \\ 0, & o.w. \end{cases}$$

1) a

2) d

3) $P(X=4) = \binom{8}{4} (.11)^4 (.89)^{8-4} = .0064$ $\therefore$ b

4) $P(X \leq 1) = P(X=1) + P(X=0) = \binom{8}{1} (.11)^1 (.89)^{8-1} + \binom{8}{0} (.11)^0 (.89)^{8-0} = .7829$

$\therefore$ a

5) $\mu = n\pi = (8)(.11) = .88$ $\therefore$ c

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"Q # 33"

X = the number of kidney transplants in year

the average number of kidney transplant in year $= \lambda = 5.4$

$$\therefore X \sim pois(\lambda = 5.4)$$

$$\therefore f_X(x) = P(X=x) = \begin{cases} \frac{e^{-\lambda}\lambda^x}{x!} &, x = 0,1,2,\dots \\ 0 &, o.w \end{cases}$$

$$= \begin{cases} \frac{e^{-5.4}(5.4)^x}{x!} &, x = 0,1,2,\dots \\ 0 &, o.w \end{cases}$$

1) $P(X=4) = \frac{e^{-5.4}(5.4)^4}{4!}$ $\therefore e$

2) $P(X \geq 2) = P(X=2) + P(X=3) + P(X=4) + P(X=5) + \dots$

or

$= 1 - P(X<2)$

$= 1 - P(X \leq 1)$

$= 1 - [P(X=1) + P(X=0)] = 1 - \left[\frac{e^{-5.4}(5.4)^1}{1!} + \frac{e^{-5.4}(5.4)^0}{0!}\right] = .9711$

$\therefore b$

3)

let Y = the number of kidney transplants in 3 year

and let λ' is the average number of kidney transplant in 3 year

for X : 1 year $\longrightarrow \lambda = 5.4$

for Y : 3 year $\longrightarrow \lambda'$

$\Rightarrow (\lambda')(1\text{ year}) = (5.4)(3\text{ year}) \Rightarrow \lambda' = \frac{(3)(5.4)}{1} = 16.2$

$\therefore Y \sim pois(\lambda' = 16.2)$ $\therefore C$

and $f_Y(y) = \begin{cases} \frac{e^{-16.2}(16.2)^y}{y!} &, y = 0,1,2,\dots \\ 0 &, o.w \end{cases}$

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"Q # 34"

the average number of deaths per year from cancer $= \lambda = 3$

$\Rightarrow X =$ the number of deaths per year form cancer

$\therefore X \sim pois(\lambda = 3)$

$$\Rightarrow f_X(x) = p(X = x) = \begin{cases} \dfrac{e^{-3}(3)^x}{x!}, & x = 0, 1, 2, \dots \\ 0, & o.w \end{cases}$$

not more than 2 $\equiv$ at most

a) $p(X \le 2) = p(X=2) + p(X=1) + p(X=0)$

$$= \frac{e^{-3}(3)^2}{2!} + \frac{e^{-3}(3)^1}{1!} + \frac{e^{-3}(3)^0}{0!} = .423$$

at least 2

b) $p(X \ge 2) = p(X=2) + p(X=3) + p(X=4) + \dots$

or

$= 1 - p(X < 2)$

$= 1 - p(X \le 1) = 1 - [p(X=1) + p(X=0)]$

$$= 1 - \left[\frac{e^{-3}(3)^1}{1!} + \frac{e^{-3}(3)^0}{0!}\right] = .8$$

exactly

c) $p(X = 3) = \dfrac{e^{-3}(3)^3}{3!} = .224$

d) let $Y =$ the number of deaths for 4 months

and let $\lambda' =$ the average number of deaths for 4 months

for X: 1 year $= 12 \times 1$ months $\longrightarrow \lambda = 3$

for Y: 4 months $\longrightarrow \lambda'$

$\Rightarrow (\lambda')(12\text{ months}) = (3)(4\text{ months}) \Rightarrow \lambda' = \dfrac{(3)(4)}{12} = 1$

$\therefore Y \sim pois(\lambda' = 1)$

$$\Rightarrow f_Y(y) = p(Y = y) = \begin{cases} \dfrac{e^{-1}(1)^y}{y!}, & y = 0, 1, 2, \dots \\ 0, & o.w \end{cases}$$

exactly

$\therefore p(Y = 4) = \dfrac{e^{-1}(1)^4}{4!} = .01$

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"Q # 35"

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X = the number of food poisoning cases in a month

the average number of food poisoning cases in a month = $\lambda = 5.6$

$\therefore X \sim pois(\lambda = 5.6)$

$\therefore f_X(x) = p(X = x) = \begin{cases} \dfrac{e^{-5.6}(5.6)^x}{x!} & , x = 0,1,2,\dots \\ 0 & , o.w \end{cases}$

1) d

2) $p(X=7) = \dfrac{e^{-5.6}(5.6)^7}{7!} = .1267 \quad \therefore C$

3)

let Y = the number of food poisoning cases in 3 months

and let λ' = the average number of food poisoning cases in 3 months

for X : 1 month $\longrightarrow \lambda = 5.6$

for Y : 3 month $\longrightarrow \lambda'$

$\Rightarrow (\lambda')(1\text{ month}) = (5.6)(3\text{ months}) \Rightarrow \lambda' = \dfrac{(5.6)(3)}{1} = 16.8$

$\therefore Y \sim pois(\lambda' = 16.8) \Rightarrow f_Y(y) = \begin{cases} \dfrac{e^{-16.8}(16.8)^y}{y!} & , y = 0,1,2,\dots \\ 0 & , o.w \end{cases}$

and the expected $= \mu = \lambda' = 16.8 \quad \therefore c$

4) let H = the number of food poisoning cases in $\frac{1}{2}$ month

and let λ'' = the average number of food poisoning cases in $\frac{1}{2}$ month

for X : 1 month $\longrightarrow \lambda = 5.6$

for H : $\frac{1}{2}$ month $\longrightarrow \lambda''$

$\Rightarrow \lambda'' = (5.6)(\frac{1}{2}) = 2.8 \Rightarrow H \sim pois(\lambda'' = 2.8)$

$\Rightarrow f_H(h) = p(H = h) = \begin{cases} \dfrac{e^{-2.8}(2.8)^h}{h!} & , h = 0,1,\dots \\ 0 & , o.w \end{cases}$

$\therefore b$

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"Q # 26"

the average number of breast cancer cases in a year = $\lambda = 6$

$\Rightarrow$ X = the # of breast cancer cases in a year

$\therefore X \sim pois(\lambda = 6)$

$$\Rightarrow f_X(x) = \begin{cases} \frac{e^{-6}(6)^x}{x!} & , x = 0, 1, 2, \ldots \\ 0 & , o.w \end{cases}$$

a) $P(X=0) = \frac{e^{-6}(6)^0}{0!} = e^{-6}$

b) let Y = the # of breast cancer cases in 6 months.

and let λ' = the average # of breast cancer cases in 6 months

for X : 1 year = 1×12 months $\longrightarrow \lambda = 6$

for Y : 6 months $\longrightarrow \lambda'$

$$\Rightarrow \lambda' = \frac{(6)(6)}{12} = 3$$

$$\therefore Y \sim pois(\lambda' = 3) \Rightarrow f_Y(y) = \begin{cases} \frac{e^{-3}(3)^y}{y!} & , y = 0, 1, 2, \ldots \\ 0 & , o.w \end{cases}$$

at most

$$P(Y \leq 2) = P(Y=2) + P(Y=1) + P(Y=0)$$
$$= \frac{e^{-3}(3)^2}{2!} + \frac{e^{-3}(3)^1}{1!} + \frac{e^{-3}(3)^0}{0!}$$

Standard normal distribution :

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"Q * 39"

we have here Z is a standard normal distribution i.e
$Z \sim N(0,1)$
so we can use the table of Z

1) $P(Z \geq 1.34) = 1 - P(Z \leq 1.34)$
$= 1 - .9099$
$= .0901$
$\therefore a$

	.04
1.3	.9099

2) $P(Z \leq -.78) = .2177$
$\therefore b$

	.08
-0.7	.2177

3) $P(-1.89 \leq Z \leq 2.56) = P(Z \leq 2.56) - P(Z \leq -1.89)$
$= .9948 - .0294$
$= .9654$
$\therefore C$

	.06
2.5	.9943

	.09
-1.8	.0294

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"Q # 40"

$Z \sim N(0,1)$

So, we can use the table of Z

a) $P(Z \le 1.36) = .9131$

.06
1.3 — .9131

b) $P(Z \ge 2.4) = 1 - P(Z \le 2.4)$
$= 1 - .9918$
$= .0082$

.00
2.4 — .9918

c) $P(Z \le 1.81) = .9649$

.01
1.8 — .9649

d) $P(Z \ge 2.7) = 1 - P(Z \le 2.7)$
$= 1 - .9965$
$= .0035$

.00
2.7 — .9965

e) $P(-1.2 < Z < 2.1) = P(Z < 2.1) - P(Z < -1.2)$
$= .9821 - .1151$
$= .8670$

.00
2.1 — .9821

.00
-1.2 — .1151

f) $P(Z = 1.4) = 0$ Because Z is a continuous distribution

g) $P(-2.36 \le Z \le 1.45) = P(Z \le 1.45) - P(Z \le -2.36)$
$= .9265 - .0091$
$= .9174$

.05
1.4 — .9265

.06
-2.3 — .0091

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"Q # 41"

we have Z is a standard normal distribution

i.e $Z \sim N(0,1)$

so we can use the table of Z.

Note: $p(Z \le Z_\alpha) = \alpha$

a) $Z_{.5} \Rightarrow p(Z \le Z_{.5}) = .5$

$.00$

$\therefore \quad \pm 0.0 \leftarrow .5000 \qquad \therefore Z_{.5} = 0.00$

b) $Z_{.95} \Rightarrow p(Z \le Z_{.95}) = .95$

$.04 \qquad\qquad .05$

$1.6 \leftarrow .9495 \quad .9500 \quad .9505$

$.0005 \qquad .0005$

$Z_{.95} = \frac{1.64 + 1.65}{2} = 1.645$

or $Z_{.95} = -Z_{1-.95} = -Z_{.05}$, Now we want to find $Z_{.05} \Rightarrow p(Z \le Z_{.05}) = .05$

$.04 \qquad\qquad .05$

$-1.6 \leftarrow .0505 \quad .0500 \quad .0495$

$.0005 \qquad .0005$

$Z_{.05} = \frac{(-1.64)+(-1.65)}{2} = -1.645 \Rightarrow -Z_{.05} = -(-1.645) = 1.645 = Z_{.95}$

يكتب في
ذا الهامش

c) $Z_{.025} \Rightarrow P(Z \le Z_{.025}) = .025$

$.06$
$\uparrow$
$-1.9 \longleftarrow .0250$

$\therefore Z_{.025} = -1.96$

or

$Z_{.025} = -Z_{1-.025} = -Z_{.975}$, Now we want to find $Z_{.975} \Rightarrow P(Z \le Z_{.9}) = .975$

$.06$
$\uparrow$
$1.9 \longleftarrow .9750$

$\therefore Z_{.975} = 1.96 \Rightarrow -Z_{.975} = -1.96 = Z_{.025}$

d) $Z_{.975} = 1.96$ where we find it in part c

لا يكتب في
هذا الهامش

"Q# 42"

$X \sim N(\mu = 25, \sigma^2 = (2))$

1) $P(X < 21) = P\left(\underbrace{\frac{X-25}{2}}_{=Z} < \frac{21-25}{2}\right) = P(Z < -2.00) = .0228$

$.00$

$\therefore d$ $\qquad -2.0 \quad .0228$

2) $P(19 < X < 28) = P\left(\frac{19-25}{2} < \underbrace{\frac{X-25}{2}}_{=Z} < \frac{28-25}{2}\right)$

$= P(-3.00 < Z < 1.50) = P(Z < 1.50) - P(Z < -3.00)$

$= .9332 - .0013$ $\qquad .00$

$= .9319$ $\qquad 1.5 \quad .9332$

$\therefore b$ $\qquad .00$

$-3.0 \quad .0013$

3) $P(X > x) = .2578 \Rightarrow 1 - P(X \leq x) = .2578$

$\Rightarrow P(X \leq x) = .7422$

$\Rightarrow P\left(\frac{X-25}{2} \leq \frac{x-25}{2}\right) = .7422$

$\Rightarrow P\left(Z \leq \frac{x-25}{2}\right) = .7422$

$.05$

$0.6 \leftarrow .7422$

$\Rightarrow \frac{x-25}{2} = .65 \Rightarrow (2)(.65) = x - 25 \Rightarrow 1.3 = x - 25 \Rightarrow x = 1.3 + 25 = 26.3$

$\therefore c$

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يكتب في
الهامش

"Q # 43"

$X \sim N(\mu = 40, \sigma^2 = 100 = (10)^2)$

1) $P(X \le 25) = P(\overset{=Z}{\frac{X-40}{10}} \le \frac{25-40}{10}) = P(Z \le -1.50) = .0668$

.00
-1.5 .0668

∴ b

2) $P(20 \le X \le 70) = P(\frac{20-40}{10} \le \overset{=Z}{\frac{X-40}{10}} \le \frac{70-40}{10})$

$= P(-2.00 \le Z \le 3.00) = P(Z \le 3.00) - P(Z \le -2.00)$

$= .9987 - .0228$

$= .9759$

∴ d

.00
3.0 .9987

.00
-2.0 .0228

3) $P(X > x) = .791 \Rightarrow 1 - P(X \le x) = .791$

$\Rightarrow P(X \le x) = .2090$

$\Rightarrow P(\overset{=Z}{\frac{X-40}{10}} \le \frac{x-40}{10}) = .2090 \Rightarrow P(Z \le \frac{x-40}{10}) = .2090$

.01
-0.8 ← .2090

$\Rightarrow \frac{x-40}{10} = -.81 \Rightarrow x = (10)(-.81) + 40 = 31.9$

∴ c