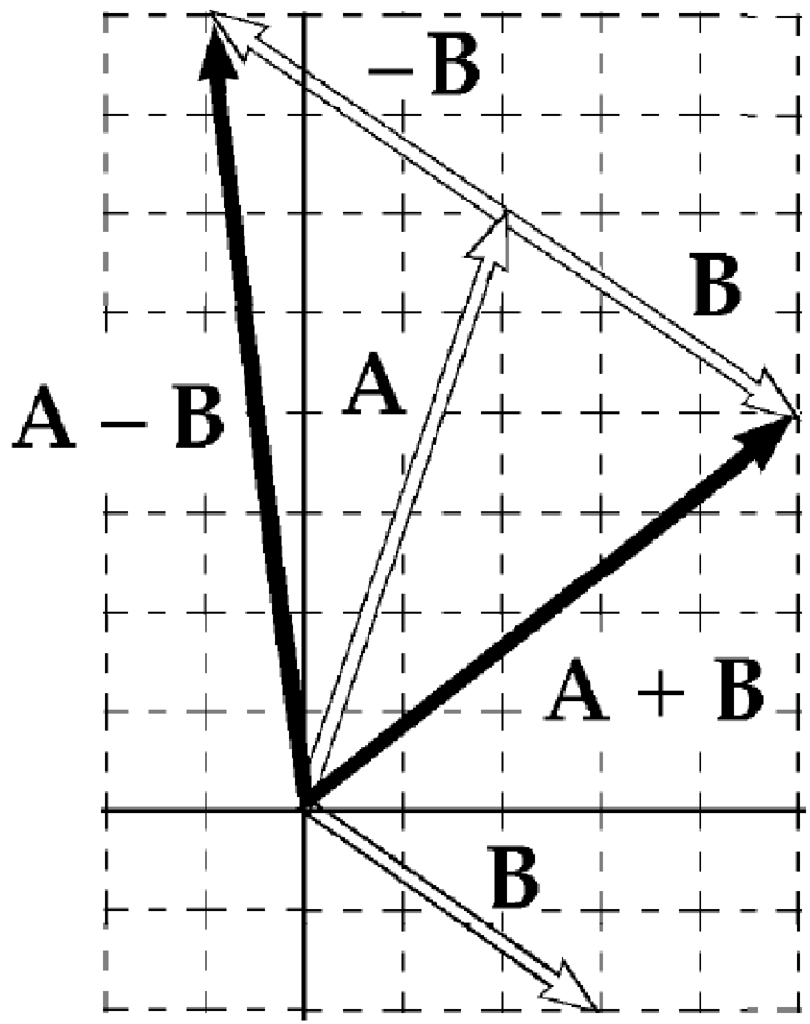
Chapter 1: Vectors

Tutorial



Coordinate systems

The polar coordinates of a point are $r = 5.5$ *m and* $\theta = 240^{o}$*. What are the Cartesian coordinates* of this point?

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$$x = r\cos\theta$$

$$= (5.50\text{ m})\cos 240°$$

$$(5.50\text{ m})(-0.5) = \boxed{-2.75\text{ m}}$$

$$y = r\sin\theta$$

$$= (5.50\text{ m})\sin 240°$$

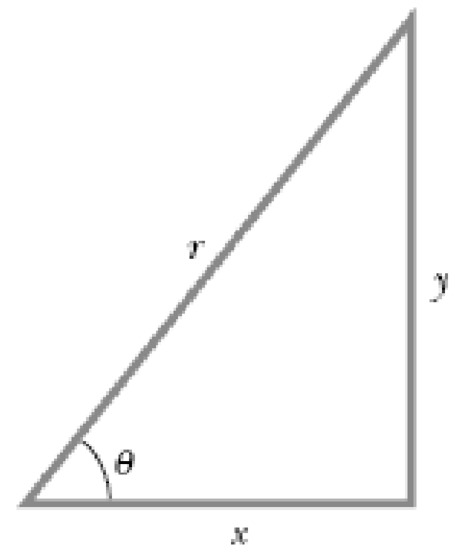
$$= (5.50\text{ m})(-0.866) = \boxed{-4.76\text{ m}}$$

$$r = \sqrt{x^2 + y^2}$$

$$\sin\theta = \frac{y}{r}$$

$$\cos\theta = \frac{x}{r}$$

$$\tan\theta = \frac{y}{x}$$



Coordinate systems

Convert the polar coordinates of the following points to *Cartesian coordinates:*

1. *(2.5, 30^o)*
2. *(3.8, 120^o)*

$$x = r\cos\theta \qquad y = r\sin\theta$$

$$x_1 = (2.50\ \text{m})\cos 30.0°,\ y_1 = (2.50\ \text{m})\sin 30.0°$$

$$(x_1, y_1) = \boxed{(2.17,\ 1.25)\ \text{m}}$$

$$x_2 = (3.80\ \text{m})\cos 120°,\ y_2 = (3.80\ \text{m})\sin 120°$$

$$(x_2, y_2) = \boxed{(-1.90,\ 3.29)\ \text{m}}.$$

Can you calculate the distance between the above two points?

Coordinate systems

Convert the *Cartesian coordinates* of the following points to *polar coordinates:*

1. *(2, - 4)*
2. *(- 3, 3)*

$$r = \sqrt{x^2 + y^2}$$

$$r_1 = \sqrt{(2.00)^2 + (-4.00)^2} = \sqrt{20.0} = \boxed{4.47\ \text{m}}$$

$$\theta_1 = \tan^{-1}\left(-\frac{4.00}{2.00}\right) = \boxed{-63.4^\circ}$$

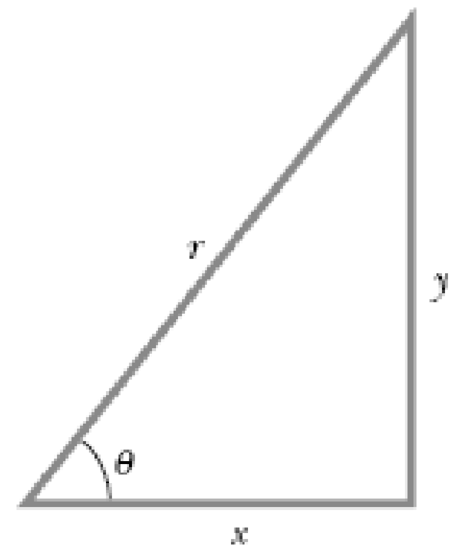
$$r_2 = \sqrt{(-3.00)^2 + (3.00)^2} = \sqrt{18.0} = \boxed{4.24\ \text{m}}$$

$$\theta_2 = \boxed{135^\circ} \text{ measured from the } +x \text{ axis.}$$

$$\sin\theta = \frac{y}{r}$$

$$\cos\theta = \frac{x}{r}$$

$$\tan\theta = \frac{y}{x}$$



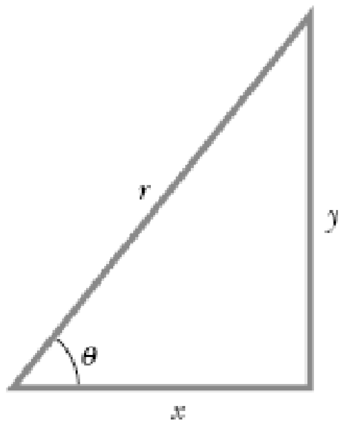
A pedestrian moves 6.00 km east and then 13.0 km north. Find the magnitude and direction of the resultant displacement vector using the graphical method.

$$r = \sqrt{x^2 + y^2}$$

$$\sin\theta = \frac{y}{r}$$

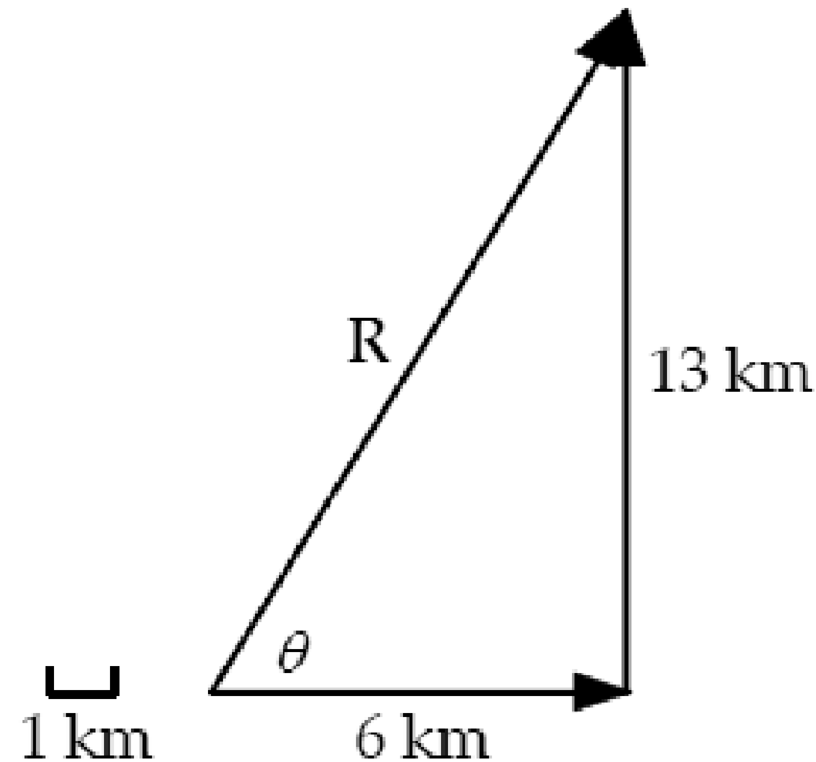
$$\cos\theta = \frac{x}{r}$$

$$\tan\theta = \frac{y}{x}$$



$$R = \boxed{14 \text{ km}}$$

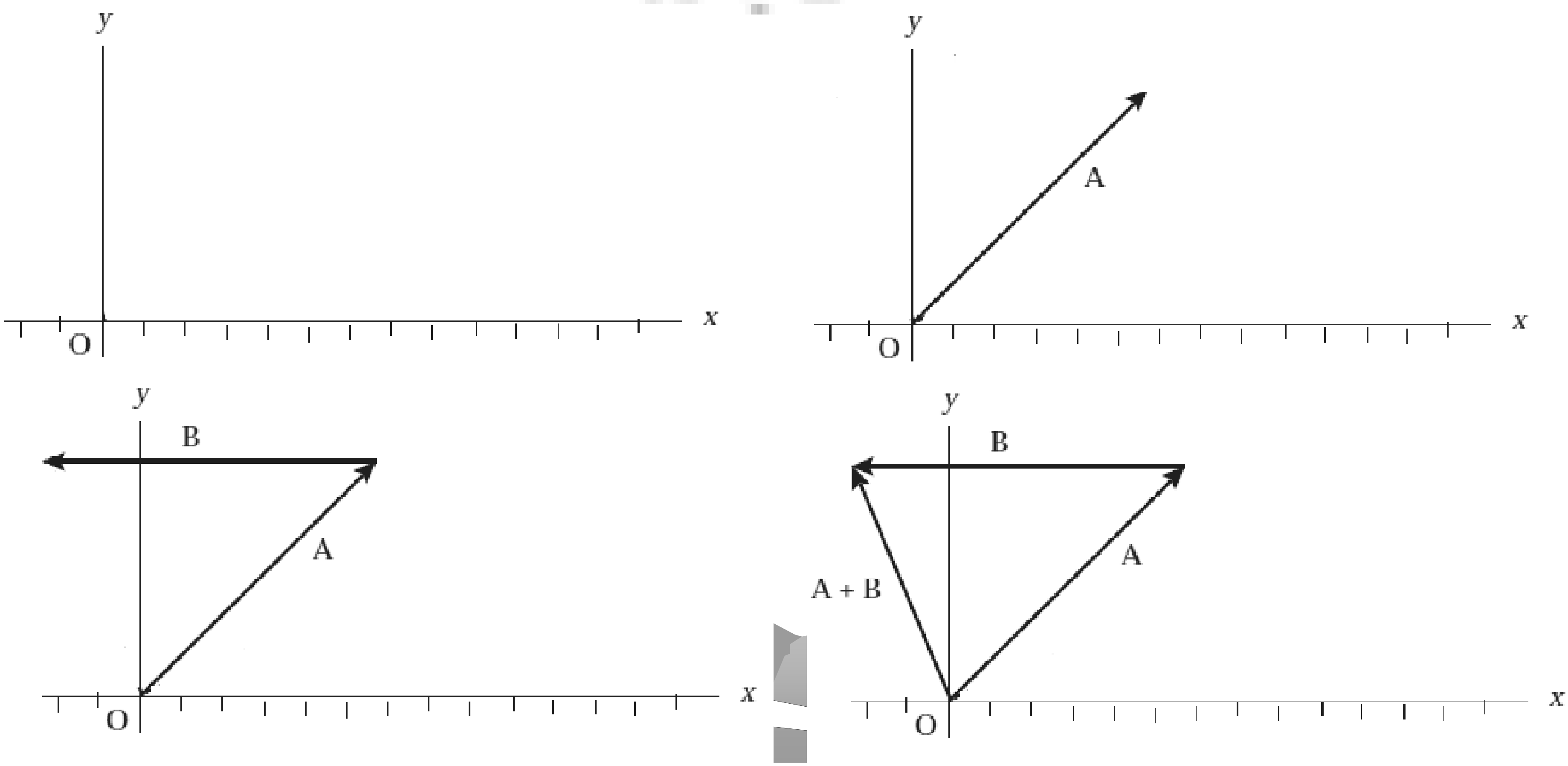
$$\theta = \boxed{65^\circ \text{ N of E}}$$



Vectors

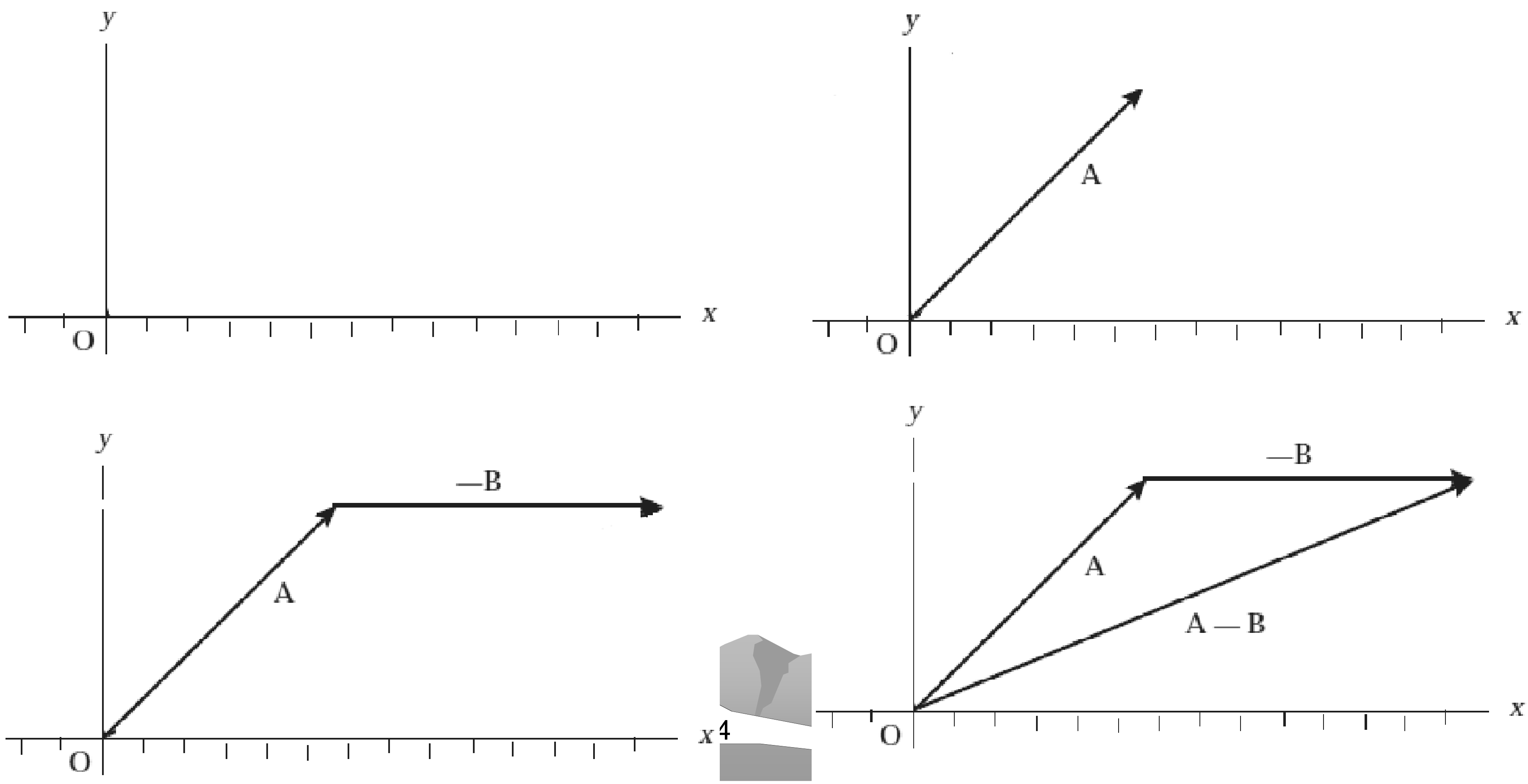
Vector **A** has a magnitude of 8.00 units and makes an angle of 45.0° with the positive x axis. Vector **B** also has a magnitude of 8.00 units and is directed along the negative x axis. Using graphical methods, find (a) the vector sum **A** + **B** and (b) the vector difference **A** − **B**.

A + B



Vectors

$\mathbf{A} - \mathbf{B}$



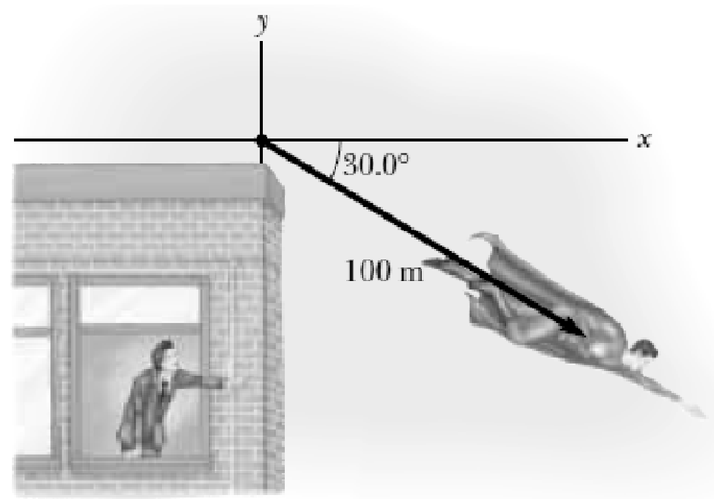
Find the horizontal and vertical components of the 100-m displacement of a superhero who flies from the top of a tall building following the path shown in Fig. P3.18.

- This is like Convert from polar coordinate to Cartesian (retangular) coordinate.

$x = r \cos \theta$

$y = r \sin \theta$

But what is θ ?



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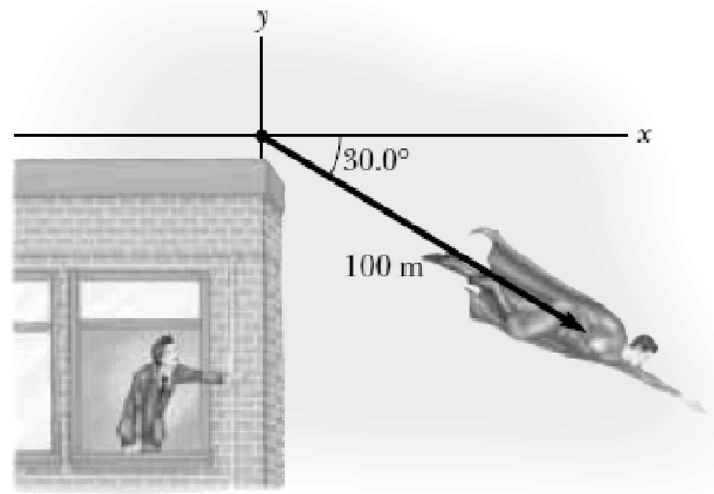


$x = r \cos \theta$

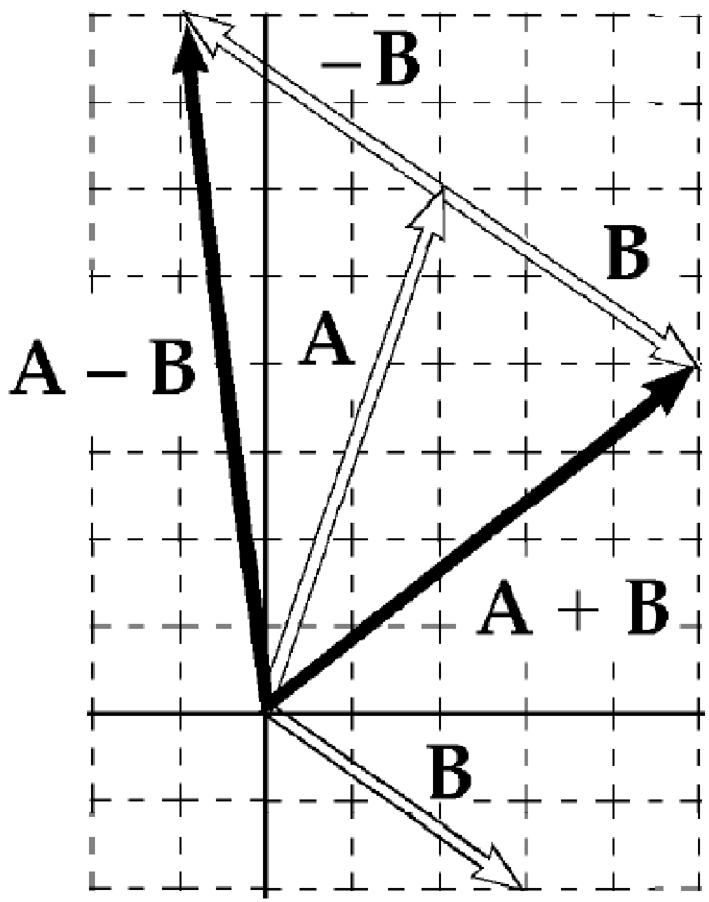
$y = r \sin \theta$

$\theta = 360 - 30 = 330$

$x = 100 \cos 330 = 86.6$ m

$y = 100 \sin 330 = -50$ m

Given the vectors $\mathbf{A} = 2.00\hat{\mathbf{i}} + 6.00\hat{\mathbf{j}}$ and $\mathbf{B} = 3.00\hat{\mathbf{i}} - 2.00\hat{\mathbf{j}}$, (a) draw the vector sum $\mathbf{C} = \mathbf{A} + \mathbf{B}$ and the vector difference $\mathbf{D} = \mathbf{A} - \mathbf{B}$. (b) Calculate $\mathbf{C}$ and $\mathbf{D}$, first in terms of unit vectors and then in terms of polar coordinates, with angles measured with respect to the $+x$ axis.

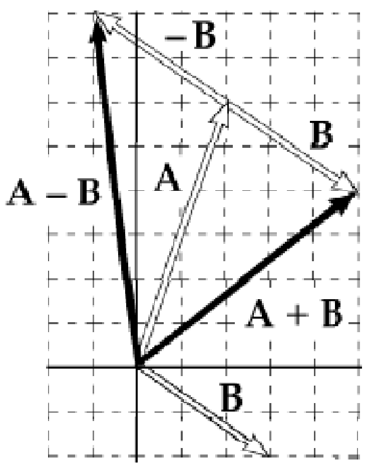


Given the vectors $\mathbf{A} = 2.00\hat{\mathbf{i}} + 6.00\hat{\mathbf{j}}$ and $\mathbf{B} = 3.00\hat{\mathbf{i}} - 2.00\hat{\mathbf{j}}$, (a) draw the vector sum $\mathbf{C} = \mathbf{A} + \mathbf{B}$ and the vector difference $\mathbf{D} = \mathbf{A} - \mathbf{B}$. (b) Calculate $\mathbf{C}$ and $\mathbf{D}$, first in terms of unit vectors and then in terms of polar coordinates, with angles measured with respect to the $+x$ axis.

$$\mathrm{C} = \mathbf{A} + \mathrm{B} = 2.00\hat{\mathbf{i}} + 6.00\hat{\mathbf{j}} + 3.00\hat{\mathbf{i}} - 2.00\hat{\mathbf{j}} = \boxed{5.00\hat{\mathbf{i}} + 4.00\hat{\mathbf{j}}}$$

$$\mathrm{C} = \sqrt{25.0 + 16.0} \text{ at } \tan^{-1}\left(\frac{4}{5}\right) = \boxed{6.40 \text{ at } 38.7°}$$

$$\mathrm{D} = \mathbf{A} - \mathrm{B} = 2.00\hat{\mathbf{i}} + 6.00\hat{\mathbf{j}} - 3.00\hat{\mathbf{i}} + 2.00\hat{\mathbf{j}} = \boxed{-1.00\hat{\mathbf{i}} + 8.00\hat{\mathbf{j}}}$$



$$\mathrm{D} = \sqrt{(-1.00)^2 + (8.00)^2} \text{ at } \tan^{-1}\left(\frac{8.00}{-1.00}\right)$$

$$\mathrm{D} = 8.06 \text{ at } (180° - 82.9°) = \boxed{8.06 \text{ at } 97.2°}$$