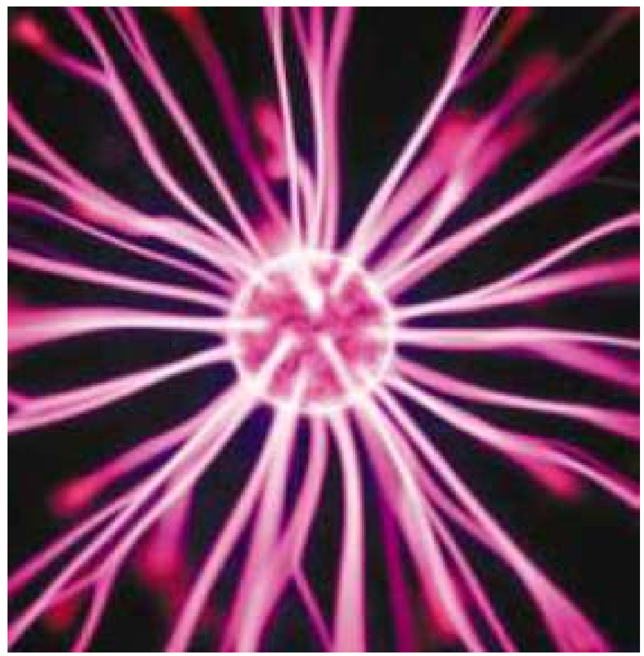
Ch 3: Gauss's Law (Electric Flux)

Ch 24 on book



Lecture Contents

•Electric Flux.

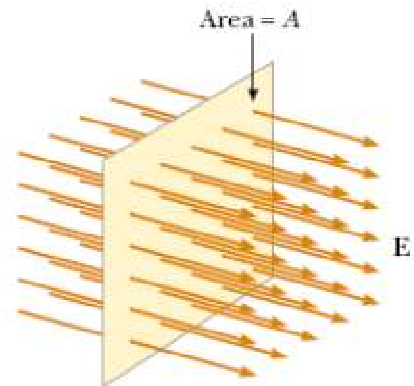
•Gauss' Law.

Revision and what's next?

- Electric force between charges.
- Electric field resulting from point charge and cont charge distribution.
- Here different ways for calc the electric field.
- Highly symmetric charge distribution, qualitative reasoning when dealing with complicated systems.

Electric Flux

- Electric field is proportional to the number of electric field lines.
- Electric field lines penetrating a **<u>perpendicular surface of area</u>** *A* .
- One can conclude that the total number of lines penetrating the surface is proportional to the product of *EA*.
- This product of *EA* is called ***<u>electric flux</u>*** **(Φ_E)**. The units of Φ_E is (N.m^2/C).



Electric Flux: Example

What is the electric flux through a sphere that has a radius of 1.00 m and carries a charge of +1.00 μC at its centre?

Solution:

The electric flux is required (Φ)?

$\Phi = EA$

$E = 8.99 \times 10^9 \times 1 \times 10^{-6} / 12$

$E = 8.99 \times 10^3$ N/C.

The area that the electric field lines penetrate is the surface area of the sphere of radius 1.00 m.

$A = 4 \pi r2 \rightarrow \mathrm{A} = 4 \pi (1)^2 = 12.57\ \mathrm{m}^2$.

Then $\Phi = 8.99 \times 10^3 \times 12.57 \rightarrow \Phi = 1.13 \times 10^5\ \mathrm{N.m^2/C}$.

Electric Flux

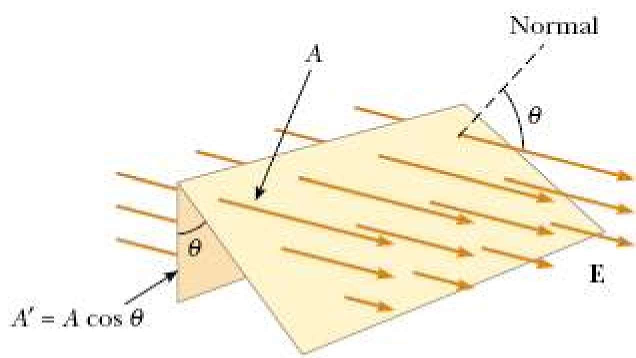
If the surface under consideration for calculating the electric flux is not perpendicular, i.e. has an angle θ with the norm (see Fig. 2), then the electric flux is less and can be evaluated as

$\Phi = EA \cos \theta$

Max when $\theta = 0^o$

And zero when

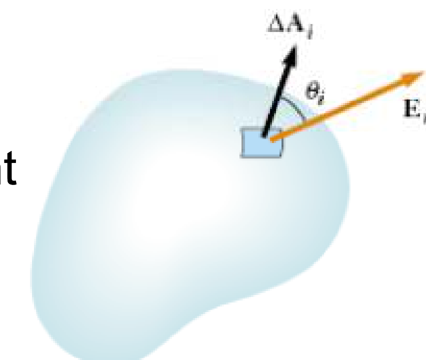
$\theta = 90^o$



Electric Flux: General View

•Surface divided up into a large number of small elements, each of area ΔAi. (variation in E neglected)

• vector ΔAi magnitude represents the area of the ith element of the surface and whose direction is defined to be *perpendicular* to the surface element. The electric field Ei at the location of this element makes an angle θi with the vector ΔAi. The electric flux $\Delta\Phi_E$ through this element is:



$$\Delta\Phi_E = E_i\,\Delta A_i \cos\theta_i = \mathbf{E}_i \cdot \Delta\mathbf{A}_i$$

Electric Flux: General View

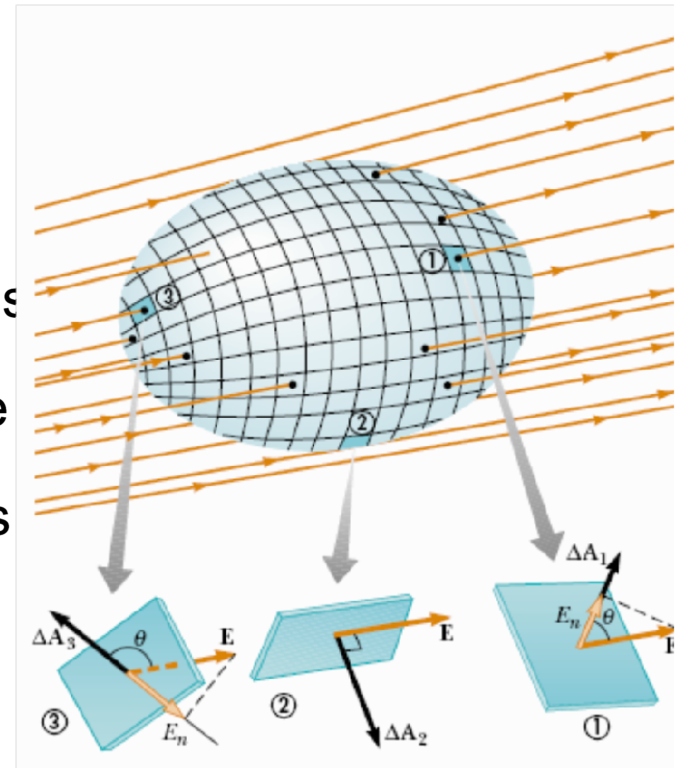
•The total flux is then

$$\Phi_E = \lim_{\Delta A_i \to 0} \sum \mathbf{E}_i \cdot \Delta \mathbf{A}_i = \int_{\text{surface}} \mathbf{E} \cdot d\mathbf{A}$$

•Φ_E depends on field pattern and the surface.

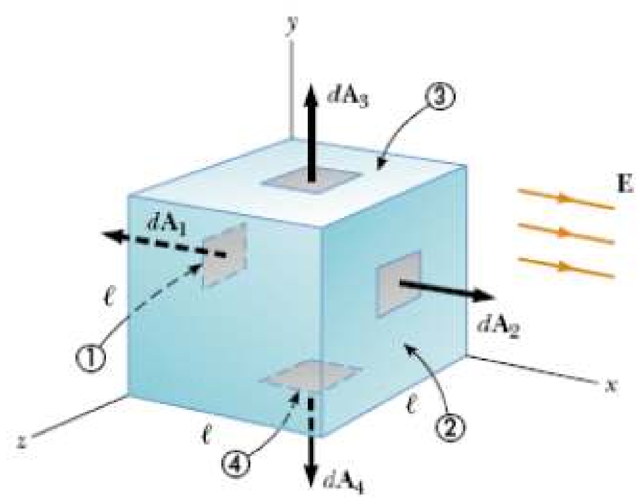
The net flux proportional net number of electric lines leaving a surface. (N lines leaving – N entering the surface). N leaving the flux is positive, N entering is negative.

$$\Phi_E = \oint \mathbf{E} \cdot d\mathbf{A} = \oint E_n \, dA$$



Example

•Consider a uniform electric field *E* oriented in the *x* direction. Find the net electric flux through the surface of a cube of edge length *l*, oriented as shown



Example

•Consider a uniform electric field E oriented in the x direction. Find the net electric flux through the surface of a cube of edge length l, oriented as shown

The net flux is the sum of all the fluxes through all the faces of the cube.

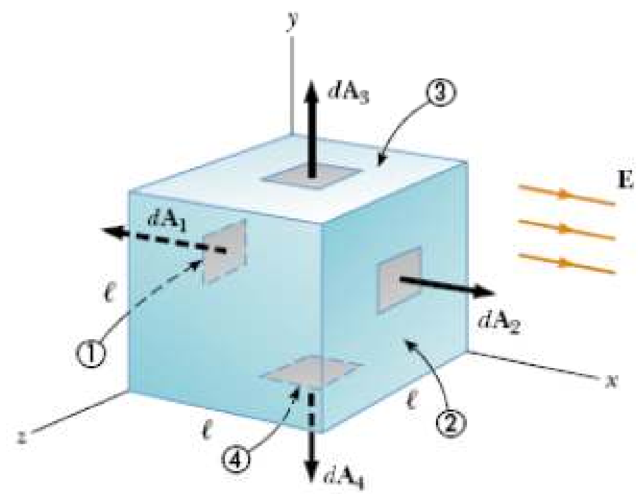
The flux through the surfaces 3, 4, and the two unnumbered faces is zero because E is perpendicular to dA.

For face 1, E is constant and the flux

For face 2, E is constant and the flux

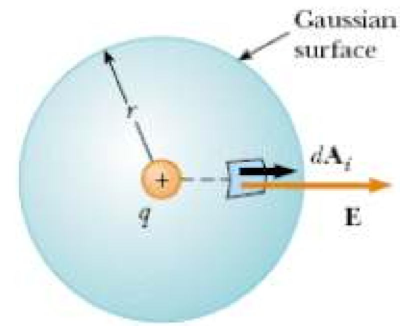
The net flux is $\Phi = -El^2 + El^2 + 0+0+0+0 \rightarrow$

$\Phi = 0$



Gauss' Law

- It is the relationship between the net flux through a closed surface (often called Gaussian surface) and the charge enclosed by the surface.

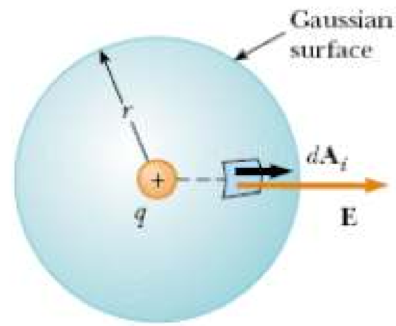


- Consider a point charge on the centre of a sphere as shown, *E* is parallel to dA_i. $\mathbf{E}\cdot\Delta\mathbf{A}_i = E\,\Delta A_i$
- And electric flux is (*E* is const)

$$\Phi_E = \oint \mathbf{E}\cdot d\mathbf{A} = \oint E\,dA = E\oint dA$$

Gauss' Law

•The surface integral of the sphere is



$$\oint dA = A = 4\pi r^2$$

•The net electric flux is

$$\Phi_E = \frac{k_e q}{r^2}(4\pi r^2) = 4\pi k_e q$$

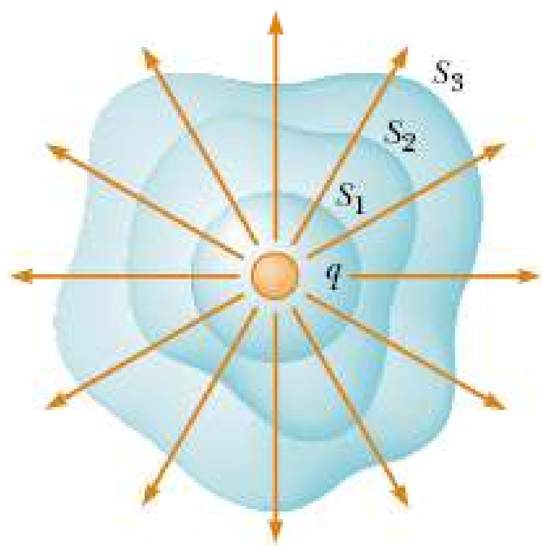
•Recalling that Coulomb const k_e is $k_e = 1/4\pi\epsilon_0$,
Then the electric flux can be calculated as

$$\Phi_E = \frac{q}{\epsilon_0}$$

What's that mean?

- The electric flux is proportional to the charge (q) inside the sphere and is not a function of r.
- A_{sphere} is proportional to r^2 and E is proportional to $1/r^2$. Dependency on r cancels.
- Consider several surfaces surrounding a point charge.

$$\Phi_E = \oint \mathbf{E} \cdot d\mathbf{A} = \oint E_n \, dA$$



- Lines passing through the sphere is the same as the lines passing through the nonspherical shapes.

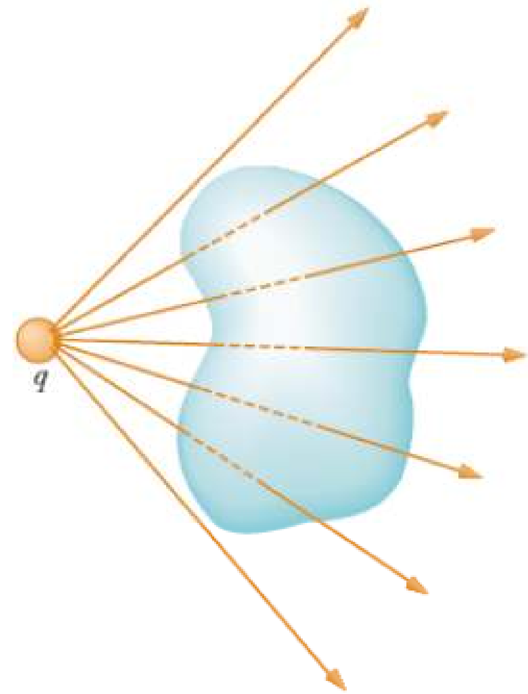
Conclusion

•The net flux through any closed surface surrounding a point charge *q* is given by

•q / ε_o and it is independent of the shape of that surface.

What if the charge outside the surface?

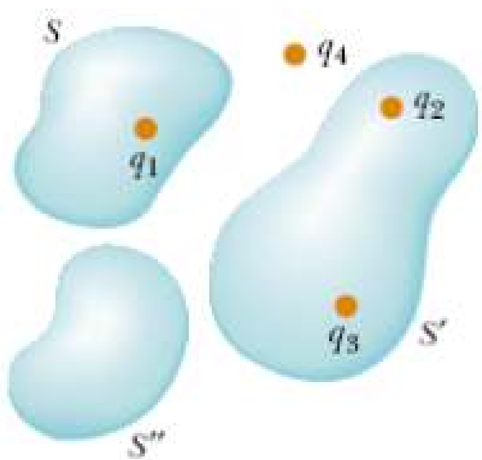
- $N_{\text{enter}} = N_{\text{exit}}$
- Conclude: the net electric flux through a closed surface that surrounds NO CHARGE is ***<u>zero</u>***.
- How easily the cube question if we know this fact before, is it????



Extend our argument

- Many point charges.
- Continuous distribution of charges.
- Make use of the Superposition principle

$$\oint \mathbf{E} \cdot d\mathbf{A} = \oint (\mathbf{E}_1 + \mathbf{E}_2 + \cdots) \cdot d\mathbf{A}$$



Gauss' Law

- States that the net electric flux through a closed surface can be evaluated as:

$$\Phi_E = \oint E \bullet dA = \frac{q_{in}}{\varepsilon_0}$$

- It can be used to evaluate *E*. Its applicability is (limited) when highly symmetric situation is present.
- Choose the Gaussian surface carefully to simplify the above equation.
- Zero Electric flux does not means zero Electric field.

Example 3

A spherical gaussian surface surrounds a point charge q. Describe what happens to the total flux through the surface if

(A) the charge is tripled,

(B) the radius of the sphere is doubled,

(C) the surface is changed to a cube, and

(D) the charge is moved to another location inside the surface.

A. The flux will triple.
B. The flux remains the same as it is not a function of r.
C. The flux will stay the same as it is not a function of surface shape.
D. It remains the same. However, it will be difficult to evaluate the electric field as it will vary.

Application of Gauss' Law

- Used for highly symmetry situations to calculate *E.*
- Take advantage of the symmetry to assume *E* is constant.
- The Gaussian surface should satisfy one:

1. The value of the electric field can be argued by symmetry to be constant over the surface.
2. The dot product in Gauss' Law Equation can be expressed as a simple algebraic product *E dA* because *E* and *d*A are parallel.
3. The dot product in Gauss' Law Equation is zero because *E* and *d*A are perpendicular.
4. The field can be argued to be zero over the surface.

Example 4

Starting from Gauss' Law, calculate the electric field due to an isolated point charge (*q*).

1. Select a suitable Gaussian surface. (Sphere concentric with the charge).
2. *E* is constant at the surface area of the sphere.
3. Surface area of the sphere $4\pi r^2$.

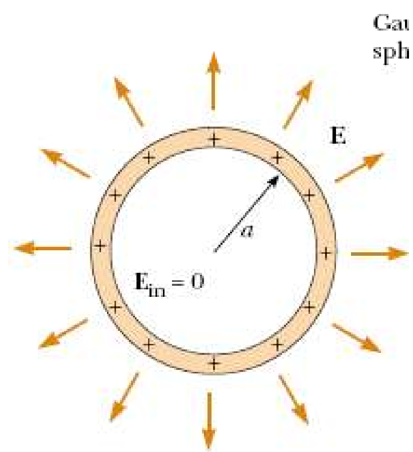
$$\oint E\,dA = E\oint dA = E(4\pi r^2) = \frac{q}{\epsilon_0}$$

$$E = \frac{q}{4\pi\epsilon_0 r^2} = \boxed{k_e\frac{q}{r^2}}$$

Example 5

A thin spherical shell of radius *a* has a total charge *Q* distributed uniformly over its surface (see below). Find the electric field at points:

(1) Outside and (2) Inside the shell.

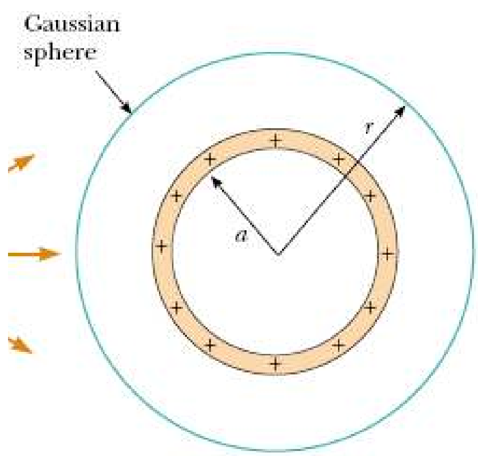


(a)

Example 5-1

A thin spherical shell of radius *a* has a total charge *Q* distributed uniformly over its surface (see below). Find the electric field at points:

(1) Outside the shell.



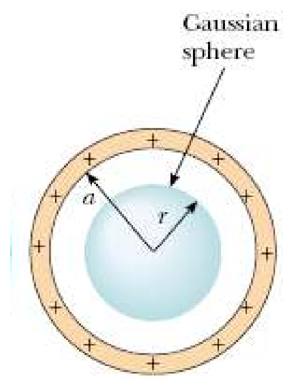
(b)

$$E = \boxed{k_e \frac{Q}{r^2}} \qquad (\text{for } r > a)$$

Example 5-2

A thin spherical shell of radius *a* has a total charge *Q* distributed uniformly over its surface (see below). Find the electric field at points:

(2) Inside the shell.



(c)