Motion in 2-d solutions

Projectile Motion:

P4.10

$$x = v_{xi}t = v_i \cos\theta_i t$$
$$x = (300\ \text{m/s})(\cos 55.0°)(42.0\ \text{s})$$
$$x = \boxed{7.23 \times 10^3\ \text{m}}$$
$$y = v_{yi}t - \frac{1}{2}gt^2 = v_i \sin\theta_i t - \frac{1}{2}gt^2$$
$$y = (300\ \text{m/s})(\sin 55.0°)(42.0\ \text{s}) - \frac{1}{2}(9.80\ \text{m/s}^2)(42.0\ \text{s})^2 = \boxed{1.68 \times 10^3\ \text{m}}$$

P4.11 (a) The mug leaves the counter horizontally with a velocity v_{xi} (say). If time t elapses before it hits the ground, then since there is no horizontal acceleration, $x_f = v_{xi}t$, i.e.,

$$t = \frac{x_f}{v_{xi}} = \frac{(1.40\ \text{m})}{v_{xi}}$$

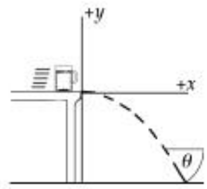


FIG. P4.11

In the same time it falls a distance of 0.860 m with acceleration downward of $9.80\ \text{m/s}^2$. Then

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2:\ 0 = 0.860\ \text{m} + \frac{1}{2}(-9.80\ \text{m/s}^2)\left(\frac{1.40\ \text{m}}{v_{xi}}\right)^2.$$

Thus,

$$v_{xi} = \sqrt{\frac{(4.90\ \text{m/s}^2)(1.96\ \text{m}^2)}{0.860\ \text{m}}} = \boxed{3.34\ \text{m/s}}.$$

(b) The vertical velocity component with which it hits the floor is

$$v_{yf} = v_{yi} + a_y t = 0 + (-9.80\ \text{m/s}^2)\left(\frac{1.40\ \text{m}}{3.34\ \text{m/s}}\right) = -4.11\ \text{m/s}.$$

Hence, the angle θ at which the mug strikes the floor is given by

$$\theta = \tan^{-1}\left(\frac{v_{yf}}{v_{xf}}\right) = \tan^{-1}\left(\frac{-4.11}{3.34}\right) = \boxed{-50.9°}.$$

P4.17 (a) $x_f = v_{xi}t = 8.00\cos 20.0°(3.00) = \boxed{22.6\text{ m}}$

(b) Taking y positive downwards,

$$y_f = v_{yi}t + \frac{1}{2}gt^2$$
$$y_f = 8.00\sin 20.0°(3.00) + \frac{1}{2}(9.80)(3.00)^2 = \boxed{52.3\text{ m}}.$$

(c) $10.0 = 8.00(\sin 20.0°)t + \frac{1}{2}(9.80)t^2$

$$4.90t^2 + 2.74t - 10.0 = 0$$
$$t = \frac{-2.74 \pm \sqrt{(2.74)^2 + 196}}{9.80} = \boxed{1.18\text{ s}}$$

Tangential and Radial Acceleration

P4.33 We assume the train is still slowing down at the instant in question.

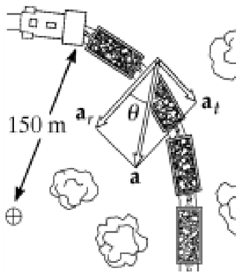


FIG. P4.33

$$a_c = \frac{v^2}{r} = 1.29\text{ m/s}^2$$
$$a_t = \frac{\Delta v}{\Delta t} = \frac{(-40.0\text{ km/h})(10^3\text{ m/km})\left(\frac{1\text{ h}}{3\,600\text{ s}}\right)}{15.0\text{ s}} = -0.741\text{ m/s}^2$$
$$a = \sqrt{a_c^2 + a_t^2} = \sqrt{(1.29\text{ m/s}^2)^2 + (-0.741\text{ m/s}^2)^2}$$

at an angle of $\tan^{-1}\left(\frac{|a_t|}{a_c}\right) = \tan^{-1}\left(\frac{0.741}{1.29}\right)$

$$\mathbf{a} = \boxed{1.48\text{ m/s}^2\text{ inward and }29.9°\text{ backward}}$$

P4.34 (a) $a_t = \boxed{0.600\text{ m/s}^2}$

(b) $a_r = \frac{v^2}{r} = \frac{(4.00\text{ m/s})^2}{20.0\text{ m}} = \boxed{0.800\text{ m/s}^2}$

(c) $a = \sqrt{a_t^2 + a_r^2} = \boxed{1.00\text{ m/s}^2}$

$\theta = \tan^{-1}\frac{a_r}{a_t} = \boxed{53.1°\text{ inward from path}}$

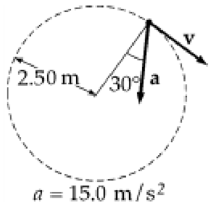
P4.35 $r = 2.50\text{ m}$, $a = 15.0\text{ m/s}^2$

(a) $a_c = a\cos 30.0° = (15.0\text{ m/s}^2)(\cos 30°) = \boxed{13.0\text{ m/s}^2}$

(b) $a_c = \dfrac{v^2}{r}$

so $v^2 = ra_c = 2.50\text{ m}(13.0\text{ m/s}^2) = 32.5\text{ m}^2/\text{s}^2$

$v = \sqrt{32.5}\text{ m/s} = \boxed{5.70\text{ m/s}}$



$a = 15.0\text{ m/s}^2$

FIG. P4.35

(c) $a^2 = a_t^2 + a_r^2$

so $a_t = \sqrt{a^2 - a_r^2} = \sqrt{(15.0\text{ m/s}^2)^2 - (13.0\text{ m/s}^2)} = \boxed{7.50\text{ m/s}^2}$

P4.36 (a) See figure to the right.

(b) The components of the 20.2 and the 22.5 m/s^2 along the rope together constitute the centripetal acceleration:

$$a_c = (22.5\text{ m/s}^2)\cos(90.0° - 36.9°) + (20.2\text{ m/s}^2)\cos 36.9° = \boxed{29.7\text{ m/s}^2}$$

(c) $a_c = \dfrac{v^2}{r}$ so $v = \sqrt{a_c r} = \sqrt{29.7\text{ m/s}^2\,(1.50\text{ m})} = 6.67\text{ m/s}$ tangent to circle

$\mathbf{v} = \boxed{6.67\text{ m/s at }36.9°\text{ above the horizontal}}$

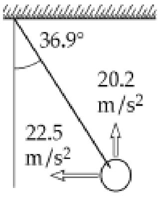


FIG. P4.36

Linear Momentum

P9.1 $m = 3.00\text{ kg}$, $\mathbf{v} = (3.00\hat{\mathbf{i}} - 4.00\hat{\mathbf{j}})\text{ m/s}$

(a) $\mathbf{p} = m\mathbf{v} = (9.00\hat{\mathbf{i}} - 12.0\hat{\mathbf{j}})\text{ kg}\cdot\text{m/s}$

Thus, $\boxed{p_x = 9.00\text{ kg}\cdot\text{m/s}}$

and $\boxed{p_y = -12.0\text{ kg}\cdot\text{m/s}}$

(b) $p = \sqrt{p_x^2 + p_y^2} = \sqrt{(9.00)^2 + (12.0)^2} = \boxed{15.0\text{ kg}\cdot\text{m/s}}$

$\theta = \tan^{-1}\left(\dfrac{p_y}{p_x}\right) = \tan^{-1}(-1.33) = \boxed{307°}$

P9.4 (a) For the system of two blocks $\Delta p = 0$,

or $\quad p_i = p_f$

Therefore, $\quad 0 = Mv_m + (3M)(2.00 \text{ m/s})$

Solving gives $\quad v_m = \boxed{-6.00 \text{ m/s}}$ (motion toward the left).

(b) $\quad \dfrac{1}{2}kx^2 = \dfrac{1}{2}Mv_M^2 + \dfrac{1}{2}(3M)v_{3M}^2 = \boxed{8.40 \text{ J}}$

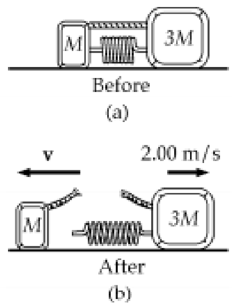


FIG. P9.4

P9.7 (a) $\quad I = \int F dt =$ area under curve

$$I = \frac{1}{2}\left(1.50 \times 10^{-3} \text{ s}\right)(18\,000 \text{ N}) = \boxed{13.5 \text{ N}\cdot\text{s}}$$

(b) $\quad F = \dfrac{13.5 \text{ N}\cdot\text{s}}{1.50 \times 10^{-3} \text{ s}} = \boxed{9.00 \text{ kN}}$

(c) From the graph, we see that $F_{\max} = \boxed{18.0 \text{ kN}}$

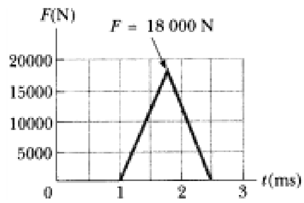


FIG. P9.7

P9.9

$$\Delta \mathbf{p} = \mathbf{F}\Delta t$$

$$\Delta p_y = m\left(v_{fy} - v_{iy}\right) = m(v\cos 60.0°) - mv\cos 60.0° = 0$$

$$\Delta p_x = m(-v\sin 60.0° - v\sin 60.0°) = -2mv\sin 60.0°$$

$$= -2(3.00 \text{ kg})(10.0 \text{ m/s})(0.866)$$

$$= -52.0 \text{ kg}\cdot\text{m/s}$$

$$F_{\text{ave}} = \frac{\Delta p_x}{\Delta t} = \frac{-52.0 \text{ kg}\cdot\text{m/s}}{0.200 \text{ s}} = \boxed{-260 \text{ N}}$$

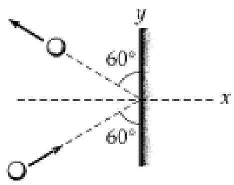


FIG. P9.9

P9.15

$$(200 \text{ g})(55.0 \text{ m/s}) = (46.0 \text{ g})v + (200 \text{ g})(40.0 \text{ m/s})$$

$$v = \boxed{65.2 \text{ m/s}}$$

P9.25 At impact, momentum of the clay-block system is conserved, so:

$$mv_1 = (m_1 + m_2)v_2$$

After impact, the change in kinetic energy of the clay-block-surface system is equal to the increase in internal energy:

$$\frac{1}{2}(m_1 + m_2)v_2^2 = f_f d = \mu(m_1 + m_2)gd$$

$$\frac{1}{2}(0.112 \text{ kg})v_2^2 = 0.650(0.112 \text{ kg})\left(9.80 \text{ m/s}^2\right)(7.50 \text{ m})$$

$v_2^2 = 95.6 \text{ m}^2/\text{s}^2 \qquad v_2 = 9.77 \text{ m/s}$

$\left(12.0 \times 10^{-3} \text{ kg}\right)v_1 = (0.112 \text{ kg})(9.77 \text{ m/s}) \qquad v_1 = \boxed{91.2 \text{ m/s}}$

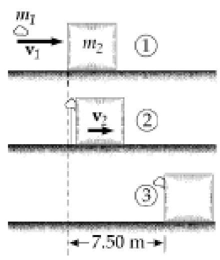


FIG. P9.25

P9.29 $p_{xf} = p_{xi}$

$mv_O \cos 37.0° + mv_Y \cos 53.0° = m(5.00 \text{ m/s})$

$0.799v_O + 0.602v_Y = 5.00 \text{ m/s}$ (1)

$p_{yf} = p_{yi}$

$mv_O \sin 37.0° - mv_Y \sin 53.0° = 0$

$0.602v_O = 0.799v_Y$ (2)

Solving (1) and (2) simultaneously,

$\boxed{v_O = 3.99 \text{ m/s}}$ and $\boxed{v_Y = 3.01 \text{ m/s}}$.

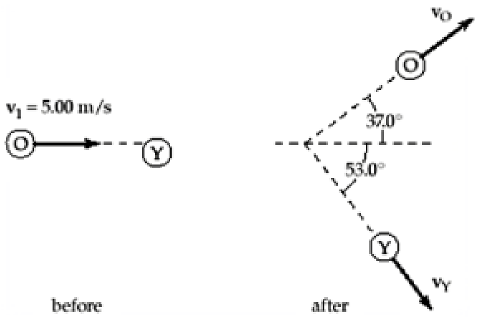


FIG. P9.29

P9.33 By conservation of momentum for the system of the two billiard balls (with all masses equal),

$$5.00 \text{ m/s} + 0 = (4.33 \text{ m/s})\cos 30.0° + v_{2fx}$$

$$v_{2fx} = 1.25 \text{ m/s}$$

$$0 = (4.33 \text{ m/s})\sin 30.0° + v_{2fy}$$

$$v_{2fy} = -2.16 \text{ m/s}$$

$$\mathbf{v}_{2f} = \boxed{2.50 \text{ m/s at } -60.0°}$$

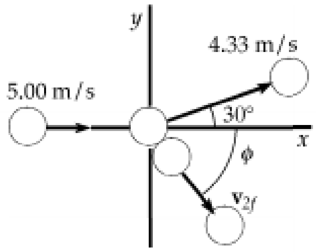


FIG. P9.33