

Ch 4: Potential Difference and Potential Energy – Part 2

Ch 25 on Book

<http://fac.ksu.edu.sa/fayalalhusan/course/91769>

Electric potential and potential energy due to point charges:

The electric potential between two arbitrary points A and B:

$$V_B - V_A = - \int_A^B \mathbf{E} \cdot d\mathbf{s}$$

field due to a point charge is given by: $\mathbf{E} = k_e q \hat{\mathbf{r}} / r^2$

$\hat{\mathbf{r}}$ is a unit vector directed from the charge to the point of concern.

Then the quantity $\mathbf{E} \cdot d\mathbf{s}$ can be expressed as:

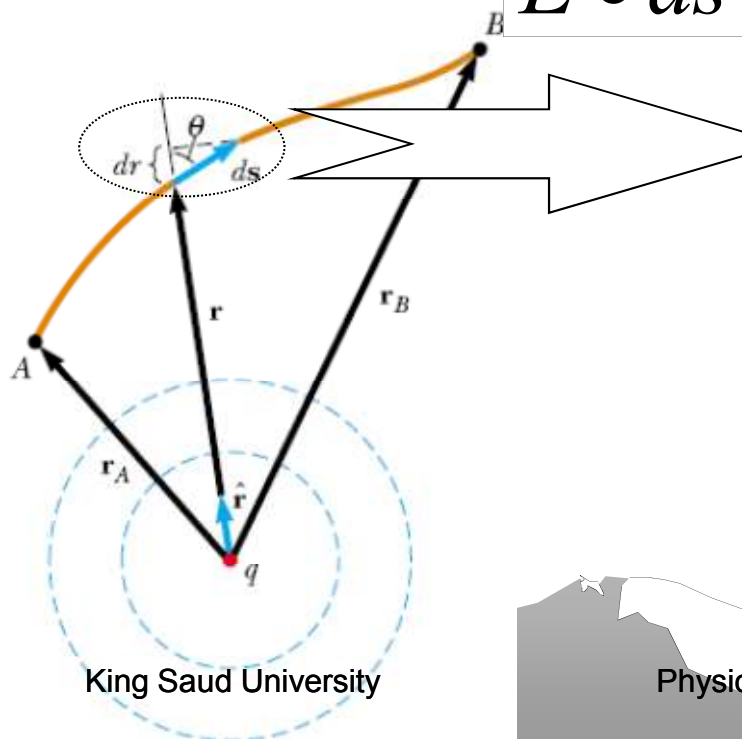
$$\mathbf{E} \cdot d\mathbf{s} = k_e \frac{q}{r^2} \hat{\mathbf{r}} \cdot d\mathbf{s}$$

$$\hat{\mathbf{r}} \cdot d\mathbf{s} = ds \cos \theta = dr$$

$$\mathbf{E} \cdot d\mathbf{s} = (k_e q / r^2) dr$$

$$V_B - V_A = -k_e q \int_{r_A}^{r_B} \frac{dr}{r^2} = \frac{k_e q}{r} \Big|_{r_A}^{r_B}$$

$$V_B - V_A = k_e q \left[\frac{1}{r_B} - \frac{1}{r_A} \right]$$



Cont.

That means that $E \cdot ds$ is independent of the path.

It is customary to choose the reference point of electric field to be zero at infinity [i.e. $V = 0$ when $r \rightarrow \infty$

the electric potential created by point charge at any distance r from the charge is

$$V = k_e \frac{q}{r}$$

If there is more than one charge

$$V = k_e \sum_i \frac{q_i}{r_i}$$

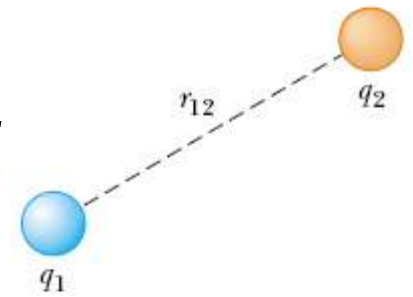
Potential energy of system of particles

We now consider the potential energy of a system of two charged particles.

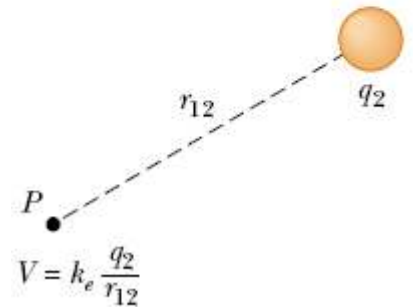
If V_2 is the electric potential at a point P due to charge q_2 , then the work an external agent must do to bring a second charge q_1 from infinity to P without acceleration is $q_1 V_2$.

This work represents a transfer of energy into the system and the energy appears in the system as potential energy U when the particles are separated by a distance r_{12} . Therefore, we can express the potential energy of the system as:

$$U = k_e \frac{q_1 q_2}{r_{12}}$$



(a)



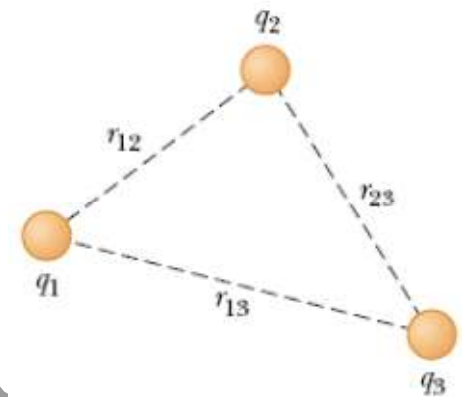
(b)

Potential energy of system of particles

If the system consists of more than two charged particles, we can obtain the total potential energy by calculating U for every pair of charges and summing the terms algebraically.

U for the three charges system can be expressed as:

$$U = k_e \left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}} \right)$$



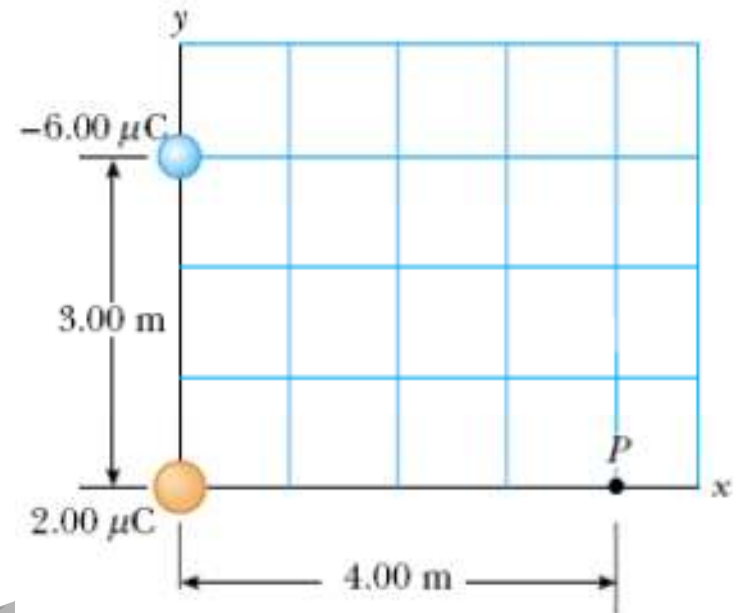
Example

A charge $q_1 = 2.00 \mu\text{C}$ is located at the origin, and a charge $q_2 = -6.00 \mu\text{C}$ is located at $(0, 3.00) \text{ m}$, as shown in the Figure below.

Find the total electric potential due to these charges at the point P , whose coordinates are $(4.00, 0) \text{ m}$.

Find the change in potential energy of the system of two charges plus a charge $q_3 = 3.00 \mu\text{C}$ as the latter charge moves from infinity to point P

$$\begin{aligned} V_P &= k_e \left(\frac{q_1}{r_1} + \frac{q_2}{r_2} \right) \\ V_P &= (8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \\ &\quad \times \left(\frac{2.00 \times 10^{-6} \text{ C}}{4.00 \text{ m}} - \frac{6.00 \times 10^{-6} \text{ C}}{5.00 \text{ m}} \right) \\ &= -6.29 \times 10^8 \text{ V} \end{aligned}$$



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$$\Delta U = q_3 V_P - 0 = (3.00 \times 10^{-6} \text{ C})(-6.29 \times 10^3 \text{ V})$$

$$= -1.89 \times 10^{-2} \text{ J}$$

had asked to find the change in potential energy when *all three* charges start out infinitely far apart and are then brought to the positions in Figure 25.12b, we would need to calculate the change as follows, using Equation 25.14:

$$\begin{aligned} U &= k_e \left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}} \right) \\ &= (8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2) \\ &\quad \times \left(\frac{(2.00 \times 10^{-6} \text{ C})(-6.00 \times 10^{-6} \text{ C})}{3.00 \text{ m}} \right. \\ &\quad + \frac{(2.00 \times 10^{-6} \text{ C})(3.00 \times 10^{-6} \text{ C})}{4.00 \text{ m}} \\ &\quad \left. + \frac{(3.00 \times 10^{-6} \text{ C})(-6.00 \times 10^{-6} \text{ C})}{5.00 \text{ m}} \right) \\ &= -5.48 \times 10^{-2} \text{ J} \end{aligned}$$

